# Introduction to Electric Circuits

JAMES A. SVOBODA
RICHARD C. DORF

9th edition

9TH EDITION

# Introduction to Electric Circuits

James A. Svoboda
*Clarkson University*

Richard C. Dorf
*University of California*

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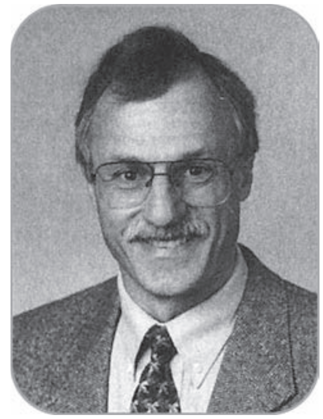
The scientific nature of the ordinary man
Is to go on out and do the best he can.
—John Prine

But, Captain, I cannot change the laws of physics.
—Lt. Cmdr. Montogomery Scott (Scotty), USS *Enterprise*

*Dedicated to our grandchildren:*

Ian Christopher Boilard, Kyle Everett Schafer, and Graham Henry Schafer
and
Heather Lynn Svoboda, James Hugh Svoboda, Jacob Arthur Leis,
Maxwell Andrew Leis, and Jack Mandlin Svoboda

# About the Authors



**James A. Svoboda** is an associate professor of electrical and computer engineering at Clarkson University, where he teaches courses on topics such as circuits, electronics, and computer programming. He earned a PhD in electrical engineering from the University of Wisconsin at Madison, an MS from the University of Colorado, and a BS from General Motors Institute.

Sophomore Circuits is one of Professor Svoboda's favorite courses. He has taught this course to 6,500 undergraduates at Clarkson University over the past 35 years. In 1986, he received Clarkson University's Distinguished Teaching Award.

Professor Svoboda has written several research papers describing the advantages of using nullors to model electric circuits for computer analysis. He is interested in the way technology affects engineering education and has developed several software packages for use in Sophomore Circuits.

**Richard C. Dorf**, professor of electrical and computer engineering at the University of California, Davis, teaches graduate and undergraduate courses in electrical engineering in the fields of circuits and control systems. He earned a PhD in electrical engineering from the U.S. Naval Postgraduate School, an MS from the University of Colorado, and a BS from Clarkson University. Highly concerned with the discipline of electrical engineering and its wide value to social and economic needs, he has written and lectured internationally on the contributions and advances in electrical engineering.

Professor Dorf has extensive experience with education and industry and is professionally active in the fields of robotics, automation, electric circuits, and communications. He has served as a visiting professor at the University of Edinburgh, Scotland, the Massachusetts Institute of Technology, Stanford University, and the University of California at Berkeley.

A Fellow of the Institute of Electrical and Electronic Engineers and the American Society for Engineering Education, Dr. Dorf is widely known to the profession for his *Modern Control Systems*, twelfth edition (Pearson, 2011) and *The International Encyclopedia of Robotics* (Wiley, 1988). Dr. Dorf is also the coauthor of *Circuits, Devices and Systems* (with Ralph Smith), fifth edition (Wiley, 1992). Dr. Dorf edited the widely used *Electrical Engineering Handbook*, third edition (CRC Press and IEEE press), published in 2011. His latest work is Technology Ventures, fourth edition (McGraw-Hill 2013).

# Preface

The central theme of *Introduction to Electric Circuits* is the concept that electric circuits are part of the basic fabric of modern technology. Given this theme, we endeavor to show how the analysis and design of electric circuits are inseparably intertwined with the ability of the engineer to design complex electronic, communication, computer, and control systems as well as consumer products.

## Approach and Organization

This book is designed for a one- to three-term course in electric circuits or linear circuit analysis and is structured for maximum *flexibility*. The flowchart in Figure 1 demonstrates alternative chapter organizations that can accommodate different course outlines without disrupting continuity.

The presentation is geared to readers who are being exposed to the basic concepts of electric circuits for the first time, and the scope of the work is broad. Students should come to the course with the basic knowledge of differential and integral calculus.

This book endeavors to prepare the reader to solve realistic problems involving electric circuits. Thus, circuits are shown to be the results of real inventions and the answers to real needs in industry, the office, and the home. Although the tools of electric circuit analysis may be partially abstract, electric circuits are the building blocks of modern society. The analysis and design of electric circuits are critical skills for all engineers.

## What's New in the 9th Edition

### Revisions to Improve Clarity

Chapter 10, covering AC circuits, has been largely rewritten to improve clarity of exposition. In addition, revisions have been made through the text to improve clarity. Sometimes these revisions are small, involving sentences or paragraphs. Other larger revisions involved pages or even entire sections. Often these revisions involve examples. Consequently, the 9th edition contains 36 new examples.

### More Problems

The 9th edition contains 180 new problems, bringing the total number of problems to more than 1,400. This edition uses a variety of problem types and they range in difficulty from simple to challenging, including:

- Straightforward analysis problems.
- Analysis of complicated circuits.
- Simple design problems. (For example, given a circuit and the specified response, determine the required *RLC* values.)
- Compare and contrast, multipart problems that draw attention to similarities or differences between two situations.
- MATLAB and PSpice problems.
- Design problems. (Given some specifications, devise a circuit that satisfies those specifications.)
- How Can We Check . . . ? (Verify that a solution is indeed correct.)

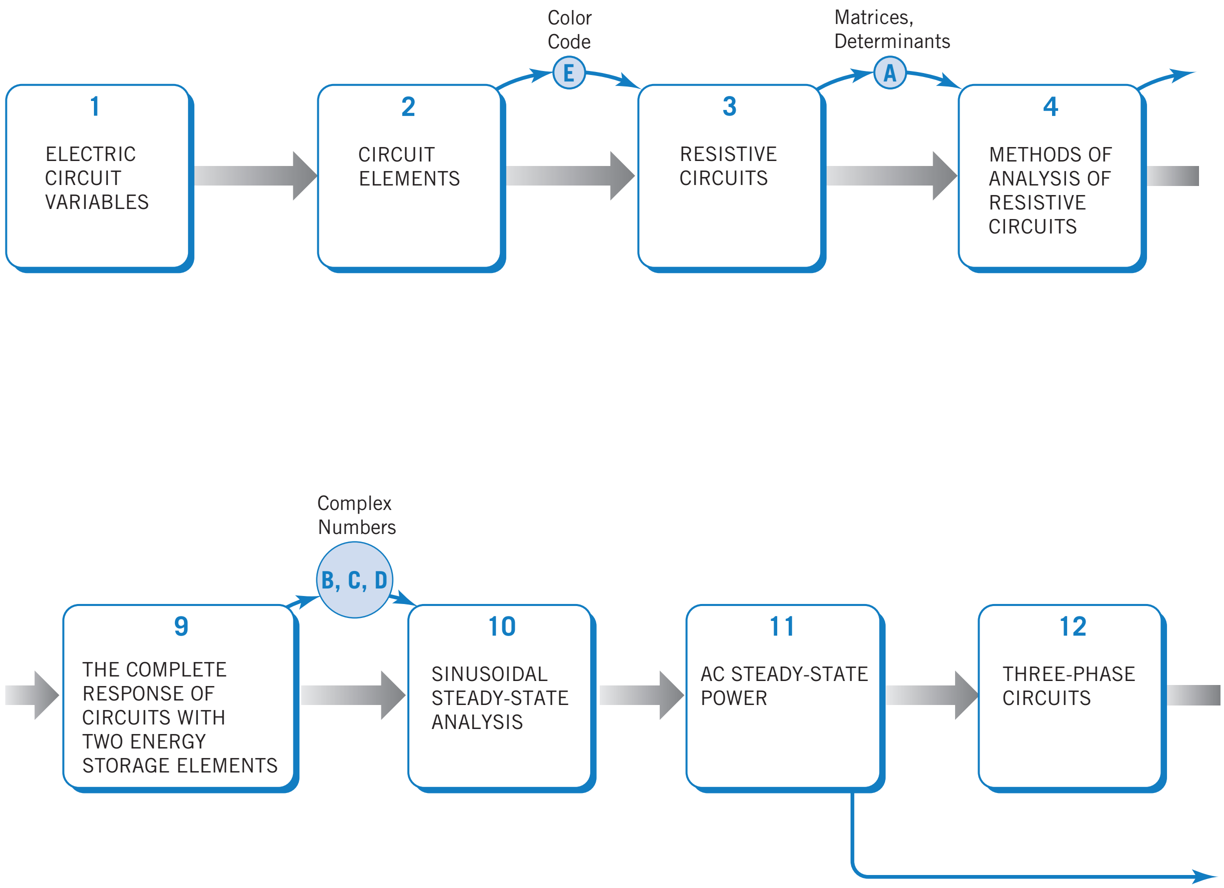



**FIGURE 1** Flow chart showing alternative paths through the topics in this textbook.

## *Features Retained from Previous Editions*

### Introduction

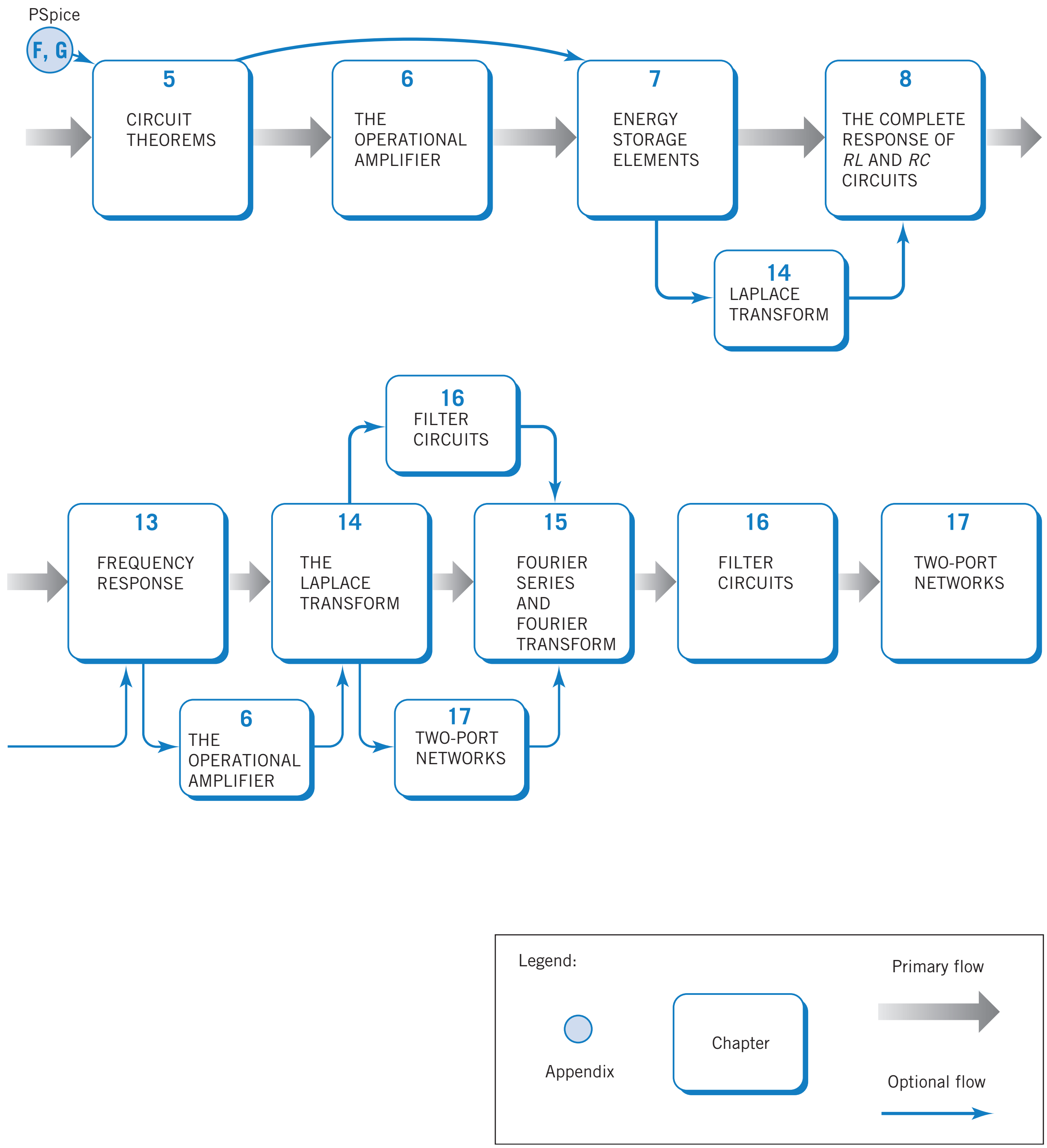
Each chapter begins with an introduction that motivates consideration of the material of that chapter.

### Examples

Because this book is oriented toward providing expertise in problem solving, we have included more than 260 illustrative examples. Also, each example has a title that directs the student to exactly what is being illustrated in that particular example.

Various methods of solving problems are incorporated into select examples. These cases show students that multiple methods can be used to derive similar solutions or, in some cases, that multiple solutions can be correct. This helps students build the critical thinking skills necessary to discern the best choice between multiple outcomes.

Much attention has been given to using PSpice and MATLAB to solve circuits problems. Two appendices, one introducing PSpice and the other introducing MATLAB, briefly describe the capabilities of the programs and illustrate the steps needed to get started using them. Next, PSpice




and MATLAB are used throughout the text to solve various circuit analysis and design problems. For example, PSpice is used in Chapter 5 to find a Thévenin equivalent circuit and in Chapter 15 to represent circuit inputs and outputs as Fourier series. MATLAB is frequently used to obtain plots of circuit inputs and outputs that help us to see what our equations are telling us. MALAB also helps us with some long and tedious arithmetic. For example, in Chapter 10, MATLAB helps us do the complex arithmetic that we must do in order to analyze ac circuits, and in Chapter 14, MATLAB helps with the partial fraction required to find inverse Laplace transforms.

Of course, there's more to using PSpice and MATLAB than simply running the programs. We pay particular attention to interpreting the output of these computer programs and checking it to make sure that it is correct. Frequently, this is done in the section called "How Can We Check . . . ?" that is included in every chapter. For example, Section 8.9 shows how to interpret and check a PSpice "Transient Response," and Section 13.7 shows how to interpret and check a frequency response produced using MATLAB or PSpice.

## Design Examples, a Problem-Solving Method, and "How Can We Check . . . ?" Sections

Each chapter concludes with a design example that uses the methods of that chapter to solve a design problem. A formal five-step problem-solving method is introduced in Chapter 1 and then used in each of the design examples. An important step in the problem-solving method requires you to check your results to verify that they are correct. Each chapter includes a section entitled "How Can We Check . . . ?" that illustrates how the kind of results obtained in that chapter can be checked to ensure correctness.

## Key Equations and Formulas

You will find that key equations, formulas, and important notes have been called out in a shaded box to help you pinpoint critical information.

## Summarizing Tables and Figures

The procedures and methods developed in this text have been summarized in certain key tables and figures. Students will find these to be an important problem-solving resource.

- Table 1.5-1. The passive convention.
- Figure 2.7-1 and Table 2.7-1. Dependent sources.
- Table 3.10-1. Series and parallel sources.
- Table 3.10-1. Series and parallel elements. Voltage and current division.
- Figure 4.2-3. Node voltages versus element currents and voltages.
- Figure 4.5-4. Mesh currents versus element currents and voltages.
- Figures 5.4-3 and 5.4-4. Thévenin equivalent circuits.
- Figure 6.3-1. The ideal op amp.
- Figure 6.5-1. A catalog of popular op amp circuits.
- Table 7.8-1. Capacitors and inductors.
- Table 7.13-2. Series and parallel capacitors and inductors.
- Table 8.11-1. First-order circuits.
- Tables 9.13-1, 2, and 3. Second-order circuits.
- Table 10.5-1. Voltage and current division for AC circuits.
- Table 10.16-1. AC circuits in the frequency domain (phasors and impedances).
- Table 11.5-1. Power formulas for AC circuits.
- Tables 11.13-1 and 11.13-2. Coupled inductors and ideal transformers.
- Table 13.4-1. Resonant circuits.
- Tables 14.2-1 and 14.2-2. Laplace transform tables.

- Table 14.7-1. s-domain models of circuit elements.
- Table 15.4-1. Fourier series of selected periodic waveforms.

### Introduction to Signal Processing

Signal processing is an important application of electric circuits. This book introduces signal processing in two ways. First, two sections (Sections 6.6 and 7.9) describe methods to design electric circuits that implement algebraic and differential equations. Second, numerous examples and problems throughout this book illustrate signal processing. The input and output signals of an electric circuit are explicitly identified in each of these examples and problems. These examples and problems investigate the relationship between the input and output signals that is imposed by the circuit.

### Interactive Examples and Exercises

Numerous examples throughout this book are labeled as interactive examples. This label indicates that computerized versions of that example are available at the textbook's companion site, www.wiley.com/svoboda. Figure 2 illustrates the relationship between the textbook example and the computerized example available on the Web site. Figure 2*a* shows an example from Chapter 3. The problem presented by the interactive example shown in Figure 2*b* is similar to the textbook example but different in several ways:

- The values of the circuit parameters have been randomized.
- The independent and dependent sources may be reversed.
- The reference direction of the measured voltage may be reversed.
- A different question is asked. Here, the student is asked to work the textbook problem backward, using the measured voltage to determine the value of a circuit parameter.

The interactive example poses a problem and then accepts and checks the user's answer. Students are provided with immediate feedback regarding the correctness of their work. The interactive example chooses parameter values somewhat randomly, providing a seemingly endless supply of problems. This pairing of a solution to a particular problem with an endless supply of similar problems is an effective aid for learning about electric circuits.

The interactive exercise shown in Figure 2*c* considers a similar, but different, circuit. Like the interactive example, the interactive exercise poses a problem and then accepts and checks the user's answer. Student learning is further supported by extensive help in the form of worked example problems, available from within the interactive exercise, using the Worked Example button.

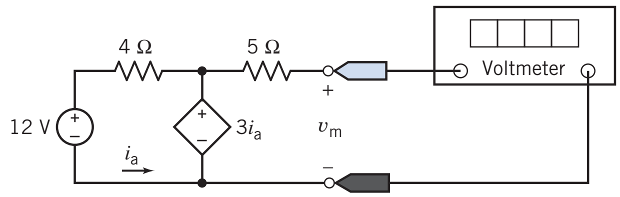
Variations of this problem are obtained using the New Problem button. We can peek at the answer, using the Show Answer button. The interactive examples and exercises provide hundreds of additional practice problems with countless variations, all with answers that are checked immediately by the computer.

## Supplements and Web Site Material

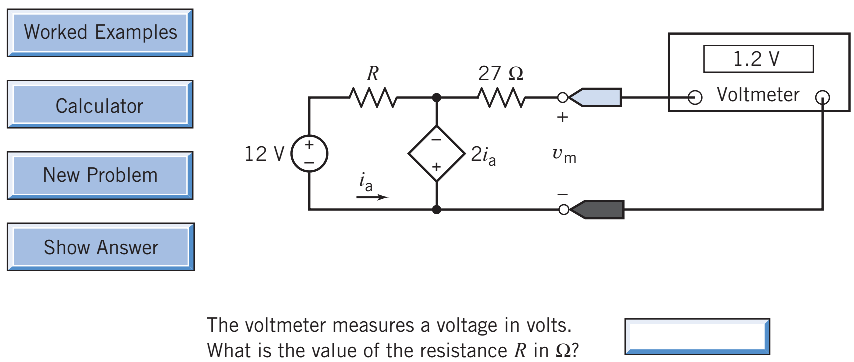
The almost ubiquitous use of computers and the Web have provided an exciting opportunity to rethink supplementary material. The supplements available have been greatly enhanced.

### *Book Companion Site*

Additional student and instructor resources can be found on the John Wiley & Sons textbook companion site at www.wiley.com/college/svoboda.




*(a)*

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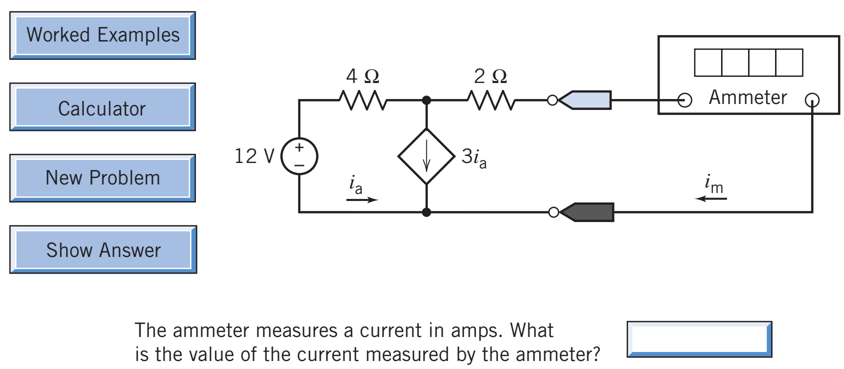


*(c)*

**FIGURE 2** (*a*) The circuit considered Example 3.2-5. (*b*) A corresponding interactive example. (*c*) A corresponding interactive exercise.

## Student

- **Interactive Examples** The interactive examples and exercises are powerful support resources for students. They were created as tools to assist students in mastering skills and building their confidence. The examples selected from the text and included on the Web give students options for navigating through the problem. They can immediately request to see the solution or select a more gradual approach to help. Then they can try their hand at a similar problem by simply electing to change the values in the problem. By the time students attempt the homework, they have built the confidence and skills to complete their assignments successfully. It's a virtual homework helper.

- *PSpice for Linear Circuits*, available for purchase.
- *WileyPLUS* option.

### Instructor

- Solutions manual.
- PowerPoint slides.
- *WileyPLUS* option.

### *WileyPLUS*

***Pspice for Linear Circuits*** is a student supplement available for purchase. The *PSpice for Linear Circuits* manual describes in careful detail how to incorporate this valuable tool in solving problems. This manual emphasizes the need to verify the correctness of computer output. No example is finished until the simulation results have been checked to ensure that they are correct.

## *Acknowledgments and Commitment to Accuracy*

We are grateful to many people whose efforts have gone into the making of this textbook. We are especially grateful to our Executive Editor Daniel Sayre, Executive Marketing Manager Chris Ruel and Marketing Assistant Marissa Carroll for their support and enthusiasm. We are grateful to Tim Lindner and Kevin Holm of Wiley and Bruce Hobart of Laserwords Maine for their efforts in producing this textbook. We wish to thank Senior Product Designer Jenny Welter, Content Editor Wendy Ashenberg, and Editorial Assistant Jess Knecht for their significant contributions to this project.

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# Contents

# CHAPTER 1 Electric Circuit Variables

## IN THIS CHAPTER



## 1.1 *Introduction*

A circuit consists of electrical elements connected together. Engineers use electric circuits to solve problems that are important to modern society. In particular:

1. Electric circuits are used in the generation, transmission, and consumption of electric power and energy.
2. Electric circuits are used in the encoding, decoding, storage, retrieval, transmission, and processing of information.

In this chapter, we will do the following:

- Represent the current and voltage of an electric circuit element, paying particular attention to the reference direction of the current and to the reference direction or polarity of the voltage.
- Calculate the power and energy supplied or received by a circuit element.
- Use the passive convention to determine whether the product of the current and voltage of a circuit element is the power supplied by that element or the power received by the element.
- Use scientific notation to represent electrical quantities with a wide range of magnitudes.

## 1.2 *Electric Circuits and Current*

The outstanding characteristics of electricity when compared with other power sources are its mobility and flexibility. Electrical energy can be moved to any point along a couple of wires and, depending on the user's requirements, converted to light, heat, or motion.

An **electric circuit** or electric network is an interconnection of electrical elements linked together in a closed path so that an electric current may flow continuously.

Consider a simple circuit consisting of two well-known electrical elements, a battery and a resistor, as shown in Figure 1.2-1. Each element is represented by the two-terminal element shown in Figure 1.2-2. Elements are sometimes called devices, and terminals are sometimes called nodes.

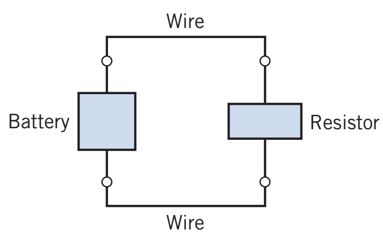



**FIGURE 1.2-1** A simple circuit.


**FIGURE 1.2-2** A general two-terminal electrical element with terminals a and b.

Charge may flow in an electric circuit. *Current is the time rate of change of charge past a given point*. Charge is the intrinsic property of matter responsible for electric phenomena. The quantity of charge $q$ can be expressed in terms of the charge on one electron, which is $-1.602 \times 10^{-19}$ coulombs. Thus, $-1$ coulomb is the charge on $6.24 \times 10^{18}$ electrons. The current through a specified area is defined by the electric charge passing through the area per unit of time. Thus, $q$ is defined as the charge expressed in coulombs (C).

**Charge** is the quantity of electricity responsible for electric phenomena.

Then we can express current as

$$i = \frac{dq}{dt} \tag{1.2-1}$$

The unit of current is the ampere (A); an ampere is 1 coulomb per second.

**Current** is the time rate of flow of electric charge past a given point.

Note that throughout this chapter we use a lowercase letter, such as $q$, to denote a variable that is a function of time, $q(t)$. We use an uppercase letter, such as $Q$, to represent a constant.

The flow of current is conventionally represented as a flow of positive charges. This convention was initiated by Benjamin Franklin, the first great American electrical scientist. Of course, we now know that charge flow in metal conductors results from electrons with a negative charge. Nevertheless, we will conceive of current as the flow of positive charge, according to accepted convention.

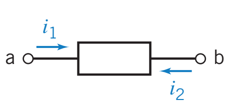



**FIGURE 1.2-3** Current in a circuit element.

Figure 1.2-3 shows the notation that we use to describe a current. There are two parts to this notation: a value (perhaps represented by a variable name) and an assigned direction. As a matter of vocabulary, we say that a current exists *in* or *through* an element. Figure 1.2-3 shows that there are two ways to assign the direction of the current through an element. The current $i_1$ is the rate of flow of electric charge from terminal a to terminal b. On the other hand, the current $i_2$ is the flow of electric charge from terminal b to terminal a. The currents $i_1$ and $i_2$ are

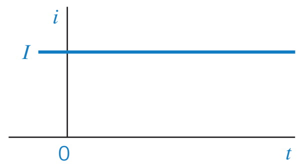



**FIGURE 1.2-4** A direct current of magnitude $I$.

similar but different. They are the same size but have different directions. Therefore, $i_2$ is the negative of $i_1$ and

$$i_1 = -i_2$$

We always associate an arrow with a current to denote its direction. A complete description of current requires both a value (which can be positive or negative) and a direction (indicated by an arrow).

If the current flowing through an element is constant, we represent it by the constant $I$, as shown in Figure 1.2-4. A constant current is called a *direct current* (dc).

A **direct current** (dc) is a current of constant magnitude.

A time-varying current $i(t)$ can take many forms, such as a ramp, a sinusoid, or an exponential, as shown in Figure 1.2-5. The sinusoidal current is called an *alternating current* (ac).

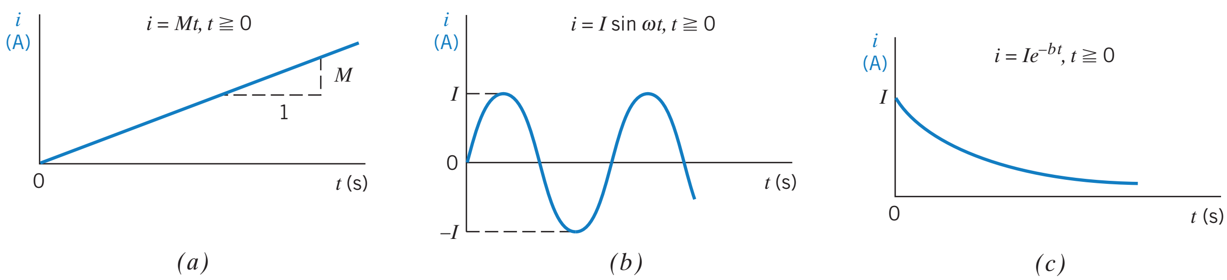



**FIGURE 1.2-5** (*a*) A ramp with a slope $M$. (*b*) A sinusoid. (*c*) An exponential. $I$ is a constant. The current $i$ is zero for $t < 0$.

If the charge $q$ is known, the current $i$ is readily found using Eq. 1.2-1. Alternatively, if the current $i$ is known, the charge $q$ is readily calculated. Note that from Eq. 1.2-1, we obtain

$$q = \int_{-\infty}^{t} i \, d\tau = \int_{0}^{t} i \, d\tau + q(0) \tag{1.2-2}$$

where $q(0)$ is the charge at $t = 0$.

## Example 1.2-1 Current from Charge

Find the current in an element when the charge entering the element is

$$q = 12t \text{ C}$$

where $t$ is the time in seconds.

## Solution

Recall that the unit of charge is coulombs, C. Then the current, from Eq. 1.2-1, is

$$i = \frac{dq}{dt} = 12\text{ A}$$

where the unit of current is amperes, A.

Try it yourself in WileyPLUS

### EXAMPLE 1.2-2 Charge from Current

Find the charge that has entered the terminal of an element from $t = 0$ s to $t = 3$ s when the current entering the element is as shown in Figure 1.2-6.

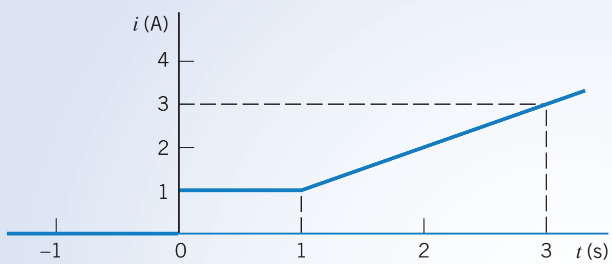



**FIGURE 1.2-6** Current waveform for Example 1.2-2.

## Solution

From Figure 1.2-6, we can describe $i(t)$ as

$$i(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t \le 1 \\ t & t > 1 \end{cases}$$

Using Eq. 1.2-2, we have

$$\begin{aligned} q(3) - q(0) &= \int_0^3 i(t)dt = \int_0^1 1\,dt + \int_1^3 t\,dt \\ &= t\Big|_0^1 + \frac{t^2}{2}\Big|_1^3 = 1 + \frac{1}{2}(9-1) = 5\text{ C} \end{aligned}$$

Alternatively, we note that integration of $i(t)$ from $t = 0$ to $t = 3$ s simply requires the calculation of the area under the curve shown in Figure 1.2-6. Then, we have

$$q = 1 + 2 \times 2 = 5\text{ C}$$

**EXERCISE 1.2-1** Find the charge that has entered an element by time $t$ when $i = 8t^2 - 4t$ A, $t \ge 0$. Assume $q(t) = 0$ for $t < 0$.

*Answer:* $q(t) = \frac{8}{3}t^3 - 2t^2$ C

**EXERCISE 1.2-2** The total charge that has entered a circuit element is $q(t) = 4 \sin 3t$ C when $t \ge 0$, and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t > 0$.

*Answer:* $i(t) = \frac{d}{dt} 4 \sin 3t = 12 \cos 3t$ A

## 1.3 *Systems of Units*

In representing a circuit and its elements, we must define a consistent system of units for the quantities occurring in the circuit. At the 1960 meeting of the General Conference of Weights and Measures, the representatives modernized the metric system and created the Système International d'Unites, commonly called SI units.

**SI** is *Système International d'Unités* or the International System of Units.

The fundamental, or base, units of SI are shown in Table 1.3-1. Symbols for units that represent proper (persons') names are capitalized; the others are not. Periods are not used after the symbols, and the symbols do not take on plural forms. The derived units for other physical quantities are obtained by combining the fundamental units. Table 1.3-2 shows the more common derived units along with their formulas in terms of the fundamental units or preceding derived units. Symbols are shown for the units that have them.

**Table 1.3-1 SI Base Units**

| | SI UNIT | |
|---|---|---|
| QUANTITY | NAME | SYMBOL |
| Length | meter | m |
| Mass | kilogram | kg |
| Time | second | s |
| Electric current | ampere | A |
| Thermodynamic temperature | kelvin | K |
| Amount of substance | mole | mol |
| Luminous intensity | candela | cd |

**Table 1.3-2 Derived Units in SI**

| QUANTITY | UNIT NAME | FORMULA | SYMBOL |
|---|---|---|---|
| Acceleration — linear | meter per second per second | $m/s^2$ | |
| Velocity — linear | meter per second | m/s | |
| Frequency | hertz | $s^{-1}$ | Hz |
| Force | newton | $kg \cdot m/s^2$ | N |
| Pressure or stress | pascal | $N/m^2$ | Pa |
| Density | kilogram per cubic meter | $kg/m^3$ | |
| Energy or work | joule | $N \cdot m$ | J |
| Power | watt | J/s | W |
| Electric charge | coulomb | $A \cdot s$ | C |
| Electric potential | volt | W/A | V |
| Electric resistance | ohm | V/A | Ω |
| Electric conductance | siemens | A/V | S |
| Electric capacitance | farad | C/V | F |
| Magnetic flux | weber | $V \cdot s$ | Wb |
| Inductance | henry | Wb/A | H |

**Table 1.3-3 SI Prefixes**

| MULTIPLE | PREFIX | SYMBOL |
|---|---|---|
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |

The basic units such as length in meters (m), time in seconds (s), and current in amperes (A) can be used to obtain the derived units. Then, for example, we have the unit for charge (C) derived from the product of current and time (A · s). The fundamental unit for energy is the joule (J), which is force times distance or N · m.

The great advantage of the SI system is that it incorporates a decimal system for relating larger or smaller quantities to the basic unit. The powers of 10 are represented by standard prefixes given in Table 1.3-3. An example of the common use of a prefix is the centimeter (cm), which is 0.01 meter.

The decimal multiplier must always accompany the appropriate units and is never written by itself. Thus, we may write 2500 W as 2.5 kW. Similarly, we write 0.012 A as 12 mA.

## EXAMPLE 1.3-1 SI Units

A mass of 150 grams experiences a force of 100 newtons. Find the energy or work expended if the mass moves 10 centimeters. Also, find the power if the mass completes its move in 1 millisecond.

### Solution

The energy is found as

$$\text{energy} = \text{force} \times \text{distance} = 100 \times 0.1 = 10\ \text{J}$$

Note that we used the distance in units of meters. The power is found from

$$\text{power} = \frac{\text{energy}}{\text{time period}}$$

where the time period is $10^{-3}$ s. Thus,

$$\text{power} = \frac{10}{10^{-3}} = 10^4\ \text{W} = 10\ \text{kW}$$

**EXERCISE 1.3-1** Which of the three currents, $i_1 = 45\ \mu\text{A}$, $i_2 = 0.03$ mA, and $i_3 = 25 \times 10^{-4}$ A, is largest?

***Answer:*** $i_3$ is largest.

## 1.4 Voltage

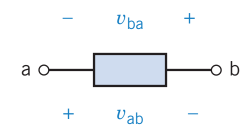



**FIGURE 1.4-1** Voltage across a circuit element.

The basic variables in an electrical circuit are current and voltage. These variables describe the flow of charge through the elements of a circuit and the energy required to cause charge to flow. Figure 1.4-1 shows the notation we use to describe a voltage. There are two parts to this notation: a value (perhaps represented by a variable name) and an assigned direction. The value of a voltage may be positive or negative. The direction of a voltage is given by its polarities (+, −). As a matter of vocabulary, we say that a voltage exists *across* an element. Figure 1.4-1 shows that there are two ways to label the voltage across an element. The voltage $v_{ba}$ is proportional to the work required to move a positive charge from terminal a to terminal b. On the other hand, the voltage $v_{ab}$ is proportional to the work required to move a positive charge from terminal b to terminal a. We sometimes read $v_{ba}$ as "the voltage at terminal b with respect to terminal a." Similarly, $v_{ab}$ can be read as "the voltage at terminal a with respect to terminal b." Alternatively, we sometimes say that $v_{ba}$ is the voltage drop from terminal a to terminal b. The voltages $v_{ab}$ and $v_{ba}$ are similar but different. They have the same magnitude but different polarities. This means that

$$v_{ab} = -v_{ba}$$

When considering $v_{ba}$, terminal b is called the "+ terminal" and terminal a is called the "− terminal." On the other hand, when talking about $v_{ab}$, terminal a is called the "+ terminal" and terminal b is called the "− terminal."

The **voltage** across an element is the work (energy) required to move a unit positive charge from the − terminal to the + terminal. The unit of voltage is the volt, V.

The equation for the voltage across the element is

$$v = \frac{dw}{dq} \tag{1.4-1}$$

where $v$ is voltage, $w$ is energy (or work), and $q$ is charge. A charge of 1 coulomb delivers an energy of 1 joule as it moves through a voltage of 1 volt.

## 1.5 Power and Energy

The power and energy delivered to an element are of great importance. For example, the useful output of an electric lightbulb can be expressed in terms of power. We know that a 300-watt bulb delivers more light than a 100-watt bulb.

**Power** is the time rate of supplying or receiving power.

Thus, we have the equation

$$p = \frac{dw}{dt} \tag{1.5-1}$$

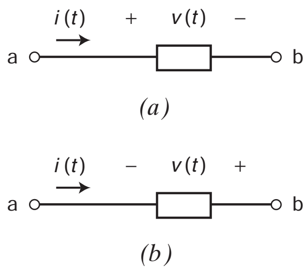



**FIGURE 1.5-1** (*a*) The element voltage and current **adhere** to the passive convention. (*b*) The element voltage and current **do not adhere** to the passive convention.

where $p$ is power in watts, $w$ is energy in joules, and $t$ is time in seconds. The power associated with the current through an element is

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \tag{1.5-2}$$

From Eq. 1.5-2, we see that the power is simply the product of the voltage across an element times the current through the element. The power has units of watts.

Two circuit variables are assigned to each element of a circuit: a voltage and a current. Figure 1.5-1 shows that there are two different ways to arrange the direction of the current and the polarity of the voltage. In Figure 1.5-1*a*, the current is directed from the + toward the − of the voltage polarity. In contrast, in Figure 1.5-1*b*, the current is directed from the − toward the + of the voltage polarity.

First, consider Figure 1.5-1*a*. When the current enters the circuit element at the + terminal of the voltage and exits at the − terminal, the voltage and current are said to "adhere to the passive convention." In the passive convention, the voltage pushes a positive charge in the direction indicated by the current. Accordingly, the power calculated by multiplying the element voltage by the element current

$$p = vi$$

is the power **received** by the element. (This power is sometimes called "the power absorbed by the element" or "the power dissipated by the element.") The power received by an element can be either positive or negative. This will depend on the values of the element voltage and current.

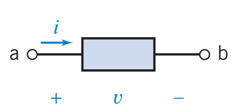
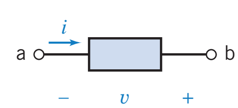
Next, consider Figure 1.5-1*b*. Here the passive convention has not been used. Instead, the current enters the circuit element at the − terminal of the voltage and exits at the + terminal. In this case, the voltage pushes a positive charge in the direction opposite to the direction indicated by the current. Accordingly, when the element voltage and current do not adhere to the passive convention, the power calculated by multiplying the element voltage by the element current is the power **supplied** by the element. The power supplied by an element can be either positive or negative, depending on the values of the element voltage and current.

The power received by an element and the power supplied by that same element are related by

$$\text{power received} = -\text{power supplied}$$

The rules for the passive convention are summarized in Table 1.5-1. When the element voltage and current adhere to the passive convention, the energy received by an element can be determined

**Table 1.5-1 Power Received or Supplied by an Element**

| POWER RECEIVED BY AN ELEMENT | POWER SUPPLIED BY AN ELEMENT |
|---|---|
|   |   |
| Because the reference directions of $v$ and $i$ adhere to the passive convention, the power $p = vi$ is the power received by the element. | Because the reference directions of $v$ and $i$ do not adhere to the passive convention, the power $p = vi$ is the power supplied by the element. |

from Eq. 1.5-1 by rewriting it as

$$dw = p\,dt \tag{1.5-3}$$

On integrating, we have

$$w = \int_{-\infty}^{t} p\,d\tau \tag{1.5-4}$$

If the element only receives power for $t \geq t_0$ and we let $t_0 = 0$, then we have

$$w = \int_{0}^{t} p\,d\tau \tag{1.5-5}$$

## EXAMPLE 1.5-1 Electrical Power and Energy

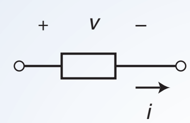



**FIGURE 1.5-2** The element considered in Example 1.5-1.

Let us consider the element shown in Figure 1.5-2 when $v = 8$ V and $i = 25$ mA. Find the power received by the element and the energy received during a 10-ms interval.

### Solution

In Figure 1.5-2 the current $i$ and voltage $v$ adhere to the passive convention. Consequently the power

$$p = vi = 8\,(0.025) = 0.2 \text{ W} = 200 \text{ mW}$$

is the power *received* by the circuit element. Next, the energy received by the element is

$$w = \int_0^t p\,dt = \int_0^{0.010} 0.2\,dt = 0.2(0.010) = 0.002 \text{ J} = 2 \text{ mJ}$$

## EXAMPLE 1.5-2 Electrical Power and the Passive Convention

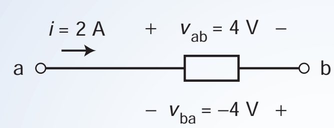



**FIGURE 1.5-3** The element considered in Example 1.5-2.

Consider the element shown in Figure 1.5-3. The current $i$ and voltage $v_{ab}$ adhere to the passive convention, so

$$i \cdot v_{ab} = 2 \cdot (-4) = -8 \text{ W}$$

is the power *received* by this element. The current $i$ and voltage $v_{ba}$ do not adhere to the passive convention, so

$$i \cdot v_{ba} = 2 \cdot (4) = 8 \text{ W}$$

is the power *supplied* by this element. As expected

$$\text{power received} = -\text{power supplied}$$

Try it yourself in WileyPLUS

## EXAMPLE 1.5-3 Power, Energy, and the Passive Convention

Consider the circuit shown in Figure 1.5-4 with $v(t) = 12e^{-8t}$ V and $i(t) = 5e^{-8t}$ A for $t \geq 0$. Both $v(t)$ and $i(t)$ are zero for $t < 0$. Find the power supplied by this element and the energy supplied by the element over the first 100 ms of operation.

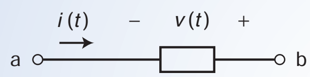



**FIGURE 1.5-4** The element considered in Example 1.5-3.

### Solution

The power

$$p(t) = v(i)\,i(t) = \left(12e^{-8t}\right)\left(5e^{-8t}\right) = 60e^{-16t}\ \text{W}$$

is the power *supplied* by the element because $v(t)$ and $i(t)$ do not adhere to the passive convention. This element is supplying power to the charge flowing through it.

The energy supplied during the first 100 ms = 0.1 seconds is

$$w(0.1) = \int_0^{0.1} p\,dt = \int_0^{0.1} \left(60e^{-16t}\right)dt$$

$$= 60\frac{e^{-16t}}{-16}\bigg|_0^{0.1} = -\frac{60}{16}\left(e^{-1.6} - 1\right) = 3.75\left(1 - e^{-1.6}\right) = 2.99\ \text{J}$$

## EXAMPLE 1.5-4 Energy in a Thunderbolt

The average current in a typical lightning thunderbolt is $2 \times 10^4$ A, and its typical duration is 0.1 s (Williams, 1988). The voltage between the clouds and the ground is $5 \times 10^8$ V. Determine the total charge transmitted to the earth and the energy released.

### Solution

The total charge is

$$Q = \int_0^{0.1} i(t)\,dt = \int_0^{0.1} 2 \times 10^4\,dt = 2 \times 10^3\ \text{C}$$

The total energy released is

$$w = \int_0^{0.1} i(t) \times v(t)\,dt = \int_0^{0.1} \left(2 \times 10^4\right)\left(5 \times 10^8\right)dt = 10^{12}\ \text{J} = 1\ \text{TJ}$$

**EXERCISE 1.5-1** Figure E 1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.

**(a)** Which of the devices supply 12 W?

**(b)** Which of the devices absorb 12 W?

**(c)** What is the value of the power received by device *B*?

**(d)** What is the value of the power delivered by device *B*?

**(e)** What is the value of the power delivered by device *D*?

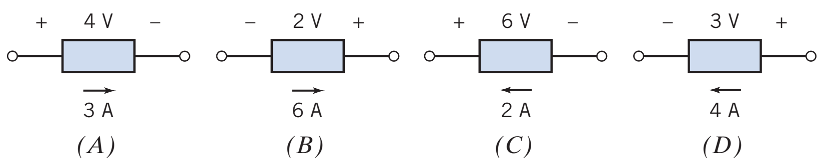



**FIGURE E 1.5-1**

***Answers:*** **(a)** *B* and *C*, **(b)** *A* and *D*, **(c)** −12 W, **(d)** 12 W, **(e)** −12 W

## 1.6 *Circuit Analysis and Design*

The analysis and design of electric circuits are the primary activities described in this book and are key skills for an electrical engineer. The *analysis* of a circuit is concerned with the methodical study of a given circuit designed to obtain the magnitude and direction of one or more circuit variables, such as a current or voltage.

The analysis process begins with a statement of the problem and usually includes a given circuit model. The goal is to determine the magnitude and direction of one or more circuit variables, and the final task is to verify that the proposed solution is indeed correct. Usually, the engineer first identifies what is known and the principles that will be used to determine the unknown variable.

The problem-solving method that will be used throughout this book is shown in Figure 1.6-1. Generally, the problem statement is given. The analysis process then moves sequentially through the five steps shown in Figure 1.6-1. First, we describe the situation and the assumptions. We also record or review the circuit model that is provided. Second, we state the goals and requirements, and we

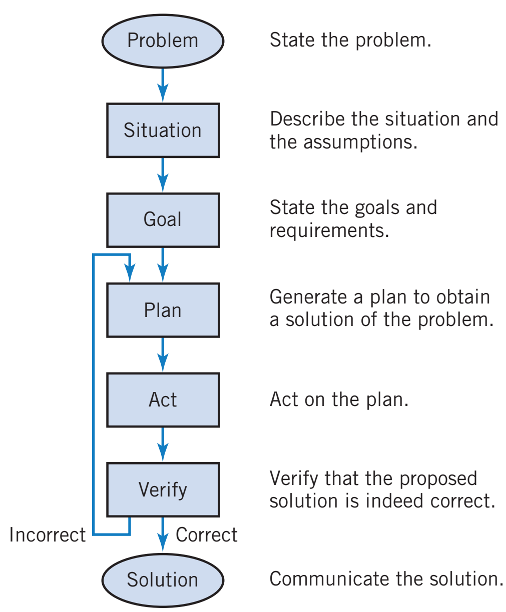



**FIGURE 1.6-1** The problem-solving method.

normally record the required circuit variable to be determined. The third step is to create a plan that will help obtain the solution of the problem. Typically, we record the principles and techniques that pertain to this problem. The fourth step is to act on the plan and carry out the steps described in the plan. The final step is to verify that the proposed solution is indeed correct. If it is correct, we communicate this solution by recording it in writing or by presenting it verbally. If the verification step indicates that the proposed solution is incorrect or inadequate, then we return to the plan steps, reformulate an improved plan, and repeat steps 4 and 5.

To illustrate this analytical method, we will consider an example. In Example 1.6-1, we use the steps described in the problem-solving method of Figure 1.6-1.

## EXAMPLE 1.6-1 The Formal Problem-Solving Method

An experimenter in a lab assumes that an element is absorbing power and uses a voltmeter and ammeter to measure the voltage and current as shown in Figure 1.6-2. The measurements indicate that the voltage is $v = +12$ V and the current is $i = -2$ A. Determine whether the experimenter's assumption is correct.

**Describe the Situation and the Assumptions:** Strictly speaking, the element *is* absorbing power. The value of the power absorbed by the element may be positive or zero or negative. When we say that someone "assumes that an element is absorbing power," we mean that someone assumes that the power absorbed by the element is positive.

The meters are ideal. These meters have been connected to the element in such a way as to measure the voltage labeled $v$ and the current labeled $i$. The values of the voltage and current are given by the meter readings.

**State the Goals:** Calculate the power absorbed by the element to determine whether the value of the power absorbed is positive.

**Generate a Plan:** Verify that the element voltage and current adhere to the passive convention. If so, the power absorbed by the device is $p = vi$. If not, the power absorbed by the device is $p = -vi$.

**Act on the Plan:** Referring to Table 1.5-1, we see that the element voltage and current do adhere to the passive convention. Therefore, power absorbed by the element is

$$p = vi = 12 \cdot (-2) = -24 \text{ W}$$

The value of the power absorbed is not positive.

**Verify the Proposed Solution:** Let's reverse the ammeter probes as shown in Figure 1.6-3. Now the ammeter measures the current $i_1$ rather than the current $i$, so $i_1 = 2$ A and $v = 12$ V. Because $i_1$ and $v$ do not adhere to the passive convention, $p = i_1 \cdot v = 24$ W is the power supplied by the element. Supplying 24 W is equivalent to absorbing $-24$ W, thus verifying the proposed solution.

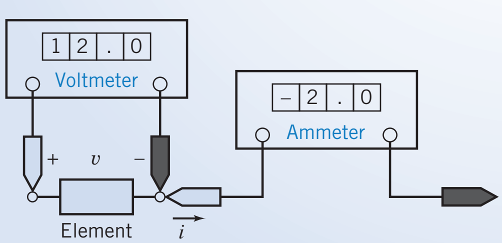



**FIGURE 1.6-2** An element with a voltmeter and ammeter.

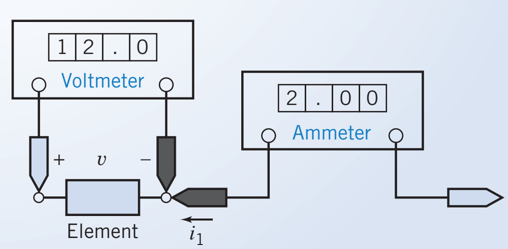



**FIGURE 1.6-3** The circuit from Figure 1.6-2 with the ammeter probes reversed.

*Design* is a purposeful activity in which a designer visualizes a desired outcome. It is the process of originating circuits and predicting how these circuits will fulfill objectives. Engineering design is the process of producing a set of descriptions of a circuit that satisfy a set of performance requirements and constraints.

The design process may incorporate three phases: analysis, synthesis, and evaluation. The first task is to diagnose, define, and prepare—that is, to understand the problem and produce an explicit statement of goals; the second task involves finding plausible solutions; the third concerns judging the validity of solutions relative to the goals and selecting among alternatives. A cycle is implied in which the solution is revised and improved by reexamining the analysis. These three phases are part of a framework for planning, organizing, and evolving design projects.

**Design** is the process of creating a circuit to satisfy a set of goals.

The problem-solving process shown in Figure 1.6-1 is used in Design Examples included in each chapter.

## 1.7 How Can We Check . . . ?

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able quickly to identify those solutions that need more work.

This text includes some examples that illustrate techniques useful for checking the solutions of the particular problems discussed in that chapter. At the end of each chapter, some problems are presented that provide an opportunity to practice these techniques.

### EXAMPLE 1.7-1 How Can We Check Power and the Passive Convention?

A laboratory report states that the measured values of $v$ and $i$ for the circuit element shown in Figure 1.7-1 are $-5$ V and 2 A, respectively. The report also states that the power absorbed by the element is 10 W. **How can we check** the reported value of the power absorbed by this element?

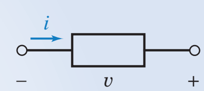



**FIGURE 1.7-1** A circuit element with measured voltage and current.

### Solution

Does the circuit element absorb $-10$ W or $+10$ W? The voltage and current shown in Figure 1.7-1 do not adhere to the passive sign convention. Referring to Table 1.5-1, we see that the product of this voltage and current is the power supplied by the element rather than the power absorbed by the element.

Then the power supplied by the element is

$$p = vi = (-5)(2) = -10 \text{ W}$$

The power absorbed and the power supplied by an element have the same magnitude but the opposite sign. Thus, we have verified that the circuit element is indeed absorbing 10 W.

## 1.8 Design Example Jet Valve Controller

A small, experimental space rocket uses a two-element circuit, as shown in Figure 1.8-1, to control a jet valve from point of liftoff at $t = 0$ until expiration of the rocket after one minute. The energy that must be supplied by element 1 for the one-minute period is 40 mJ. Element 1 is a battery to be selected.

It is known that $i(t) = De^{-t/60}$ mA for $t \geq 0$, and the voltage across the second element is $v_2(t) = Be^{-t/60}$ V for $t \geq 0$. The maximum magnitude of the current, $D$, is limited to 1 mA. Determine the required constants $D$ and $B$ and describe the required battery.

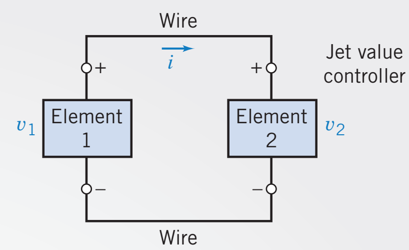



**FIGURE 1.8-1** The circuit to control a jet valve for a space rocket.

### Describe the Situation and the Assumptions

1. The current enters the plus terminal of the second element.
2. The current leaves the plus terminal of the first element.
3. The wires are perfect and have no effect on the circuit (they do not absorb energy).
4. The model of the circuit, as shown in Figure 1.8-1, assumes that the voltage across the two elements is equal; that is, $v_1 = v_2$.
5. The battery voltage $v_1$ is $v_1 = Be^{-t/60}$ V where $B$ is the initial voltage of the battery that will discharge exponentially as it supplies energy to the valve.
6. The circuit operates from $t = 0$ to $t = 60$ s.
7. The current is limited, so $D \leq 1$ mA.

### State the Goal

Determine the energy supplied by the first element for the one-minute period and then select the constants $D$ and $B$. Describe the battery selected.

### Generate a Plan

First, find $v_1(t)$ and $i(t)$ and then obtain the power, $p_1(t)$, supplied by the first element. Next, using $p_1(t)$, find the energy supplied for the first 60 s.

| GOAL | EQUATION | NEED | INFORMATION |
|---|---|---|---|
| The energy $w_1$ for the first 60 s | $w_1 = \int_0^{60} p_1(t)\, dt$ | $p_1(t)$ | $v_1$ and $i$ known except for constants $D$ and $B$ |

### Act on the Plan

First, we need $p_1(t)$, so we first calculate

$$\begin{aligned} p_1(t) &= iv_1 = \left(De^{-t/60} \times 10^{-3}\text{ A}\right)\left(Be^{-t/60}\text{ V}\right) \\ &= DBe^{-t/30} \times 10^{-3}\text{ W } = DBe^{-t/30}\text{ mW} \end{aligned}$$

Second, we need to find $w_1$ for the first 60 s as

$$w_1 = \int_0^{60} \left(DBe^{-t/30} \times 10^{-3}\right) dt = \left.\frac{DB \times 10^{-3} e^{-t/30}}{-1/30}\right|_0^{60}$$
$$= -30DB \times 10^{-3}(e^{-2} - 1) = 25.9DB \times 10^{-3} \text{ J}$$

Because we require $w_1 \geq 40$ mJ,

$$40 \leq 25.9DB$$

Next, select the limiting value, $D = 1$, to get

$$B \geq \frac{40}{(25,.9)(1)} = 1.54 \text{ V}$$

Thus, we select a 2-V battery so that the magnitude of the current is less than 1 mA.

### Verify the Proposed Solution

We must verify that at least 40 mJ is supplied using the 2-V battery. Because $i = e^{-t/60}$ mA and $v_2 = 2e^{-t/60}$ V, the energy supplied by the battery is

$$w = \int_0^{60} \left(2e^{-t/60}\right)\left(e^{-t/60} \times 10^{-3}\right) dt = \int_0^{60} 2e^{-t/30} \times 10^{-3}\, dt = 51.8 \text{ mJ}$$

Thus, we have verified the solution, and we communicate it by recording the requirement for a 2-V battery.

## 1.9 SUMMARY

- Charge is the intrinsic property of matter responsible for electric phenomena. The current in a circuit element is the rate of movement of charge through the element. The voltage across an element indicates the energy available to cause charge to move through the element.
- Given the current, $i$, and voltage, $v$, of a circuit element, the power, $p$, and energy, $w$, are given by

$$p = v \cdot i \quad \text{and} \quad w = \int_0^t p d\tau$$

- Table 1.5-1 summarizes the use of the passive convention when calculating the power supplied or received by a circuit element.
- The SI units (Table 1.3-1) are used by today's engineers and scientists. Using decimal prefixes (Table 1.3-3), we may simply express electrical quantities with a wide range of magnitudes.

## PROBLEMS

⊕ Problem available in WileyPLUS at instructor's discretion.

### Section 1.2 Electric Circuits and Current

**P 1.2-1** ⊕ The total charge that has entered a circuit element is $q(t) = 1.25(1-e^{-5t})$ when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t \geq 0$.

*Answer:* $i(t) = 6.25e^{-5t}$ A

**P 1.2-2** ⊕ The current in a circuit element is $i(t) = 4(1-e^{-5t})$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

*Hint:* $q(0) = \int_{-\infty}^{0} i(\tau)\, d\tau = \int_{-\infty}^{0} 0\, d\tau = 0$

*Answer:* $q(t) = 4t + 0.8e^{-5t} - 0.8$ C for $t \geq 0$

**P 1.2-3** ⊕ The current in a circuit element is $i(t) = 4 \sin 5t$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

*Hint:* $q(0) = \int_{-\infty}^{0} i(\tau)\, d\tau = \int_{-\infty}^{0} 0\, d\tau = 0$

**P 1.2-4** The current in a circuit element is

$$i(t) = \begin{cases} 0 & t < 2 \\ 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & 8 < t \end{cases}$$

where the units of current are A and the units of time are s. Determine the total charge that has entered a circuit element for $t \geq 0$.

*Answer:*

$$q(t) = \begin{cases} 0 & t < 2 \\ 2t - 4 & 2 < t < 4 \\ 8 - t & 4 < t < 8 \\ 0 & 8 < t \end{cases}$$ where the units of charge are C.

**P 1.2-5** ⊕ The total charge $q(t)$, in coulombs, that enters the terminal of an element is

$$q(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \leq t \leq 2 \\ 3 + e^{-2(t-2)} & t > 2 \end{cases}$$

Find the current $i(t)$ and sketch its waveform for $t \geq 0$.

**P 1.2-6** ⊕ An electroplating bath, as shown in Figure P 1.2-6, is used to plate silver uniformly onto objects such as kitchenware and plates. A current of 450 A flows for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams?

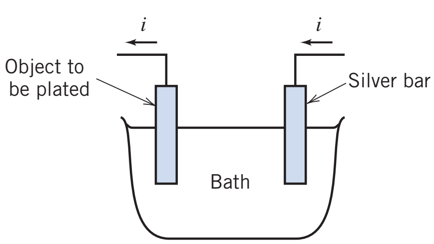



**Figure P 1.2-6** An electroplating bath.

**P 1.2-7** Find the charge $q(t)$ and sketch its waveform when the current entering a terminal of an element is as shown in Figure P 1.2-7. Assume that $q(t) = 0$ for $t < 0$.

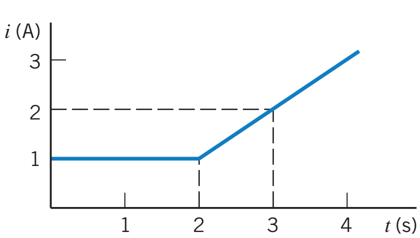



**Figure P 1.2-7**

### Section 1.3 Systems of Units

**P 1.3-1** ⊕ A constant current of 3.2 $\mu$A flows through an element. What is the charge that has passed through the element in the first millisecond?

*Answer:* 3.2 nC

**P 1.3-2** ⊕ A charge of 45 nC passes through a circuit element during a particular interval of time that is 5 ms in duration. Determine the average current in this circuit element during that interval of time.

*Answer:* $i = 9\ \mu$A

**P 1.3-3** ⊕ Ten billion electrons per second pass through a particular circuit element. What is the average current in that circuit element?

*Answer:* $i = 1.602$ nA

**P 1.3-4** The charge flowing in a wire is plotted in Figure P 1.3-4. Sketch the corresponding current.

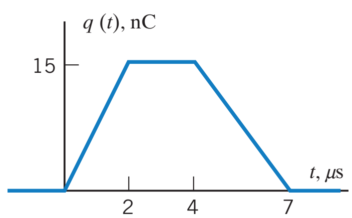



**Figure P 1.3-4**

**P 1.3-5** The current in a circuit element is plotted in Figure P 1.3-5. Sketch the corresponding charge flowing through the element for $t > 0$.

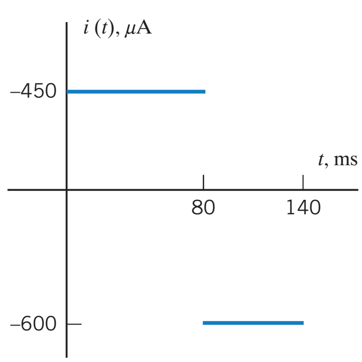



**Figure P 1.3-5**

**P 1.3-6** The current in a circuit element is plotted in Figure P 1.3-6. Determine the total charge that flows through the circuit element between 300 and 1200 $\mu$s.

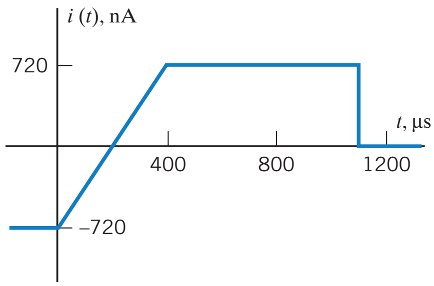



Figure P 1.3-6

## Section 1.5 Power and Energy

**P 1.5-1** ⊕ Figure P 1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.

**(a)** Which of the devices supply 30 mW?
**(b)** Which of the devices absorb 0.03 W?
**(c)** What is the value of the power received by device *B*?
**(d)** What is the value of the power delivered by device *B*?
**(e)** What is the value of the power delivered by device *C*?

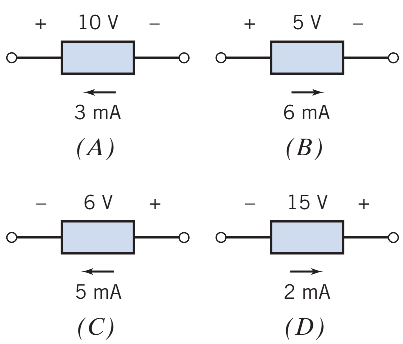



Figure P 1.5-1

**P 1.5-2** ⊕ An electric range has a constant current of 10 A entering the positive voltage terminal with a voltage of 110 V. The range is operated for two hours. (a) Find the charge in coulombs that passes through the range. (b) Find the power absorbed by the range. (c) If electric energy costs 12 cents per kilowatt-hour, determine the cost of operating the range for two hours.

**P 1.5-3** A walker's cassette tape player uses four AA batteries in series to provide 6 V to the player circuit. The four alkaline battery cells store a total of 200 watt-seconds of energy. If the cassette player is drawing a constant 10 mA from the battery pack, how long will the cassette operate at normal power?

**P 1.5-4** The current through and voltage across an element vary with time as shown in Figure P 1.5-4. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.

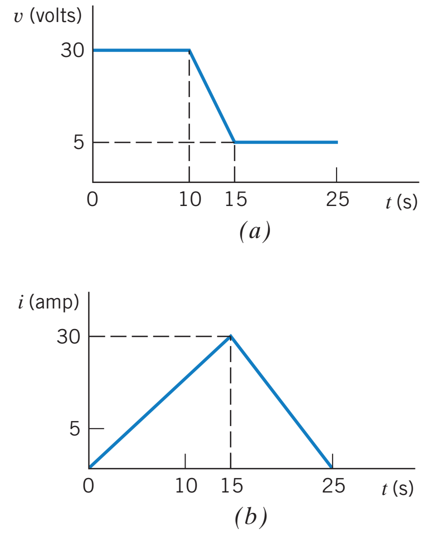



Figure P 1.5-4 (*a*) Voltage $v(t)$ and (*b*) current $i(t)$ for an element.

**P 1.5-5** ⊕ An automobile battery is charged with a constant current of 2 A for five hours. The terminal voltage of the battery is $v = 11 + 0.5t$ V for $t > 0$, where $t$ is in hours. (a) Find the energy delivered to the battery during the five hours. (b) If electric energy costs 15 cents/kWh, find the cost of charging the battery for five hours.

*Answer:* (b) 1.84 cents

**P 1.5-6** ⊕ Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 4 \cos 3t$ V and $i(t) = \dfrac{\sin 3t}{12}$ A. Evaluate $p(t)$ at $t = 0.5$ s and at $t = 1$ s. Observe that the power supplied by this element has a positive value at some times and a negative value at other times.

*Hint:* $(\sin at)(\cos bt) = \frac{1}{2}(\sin(a+b)t + \sin(a-b)t)$

*Answer:*

$$p(t) = \frac{1}{6}\sin 6t \text{ W}, \quad p(0.5) = 0.0235 \text{ W}, \quad p(1) = -0.0466 \text{ W}$$

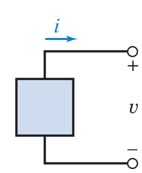



Figure P 1.5-6 An element.

**P 1.5-7** ⊕ Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 8 \sin 3t$ V and $i(t) = 2 \sin 3t$ A.

*Hint:* $(\sin at)(\sin bt) = \frac{1}{2}(\cos(a-b)t - \cos(a+b)t)$

*Answer:* $p(t) = 8 - 8\cos 6t$ W

**P 1.5-8** ⊕ Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6. The element voltage is represented as $v(t) = 4(1-e^{-2t})$V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The element current is represented as $i(t) = 2e^{-2t}$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$.

***Answer:*** $p(t) = 8(1 - e^{-2t})e^{-2t}$ W

**P 1.5-9** ⊕ The battery of a flashlight develops 3 V, and the current through the bulb is 200 mA. What power is absorbed by the bulb? Find the energy absorbed by the bulb in a five-minute period.

**P 1.5-10** Medical researchers studying hypertension often use a technique called "2D gel electrophoresis" to analyze the protein content of a tissue sample. An image of a typical "gel" is shown in Figure P1.5-10*a*.

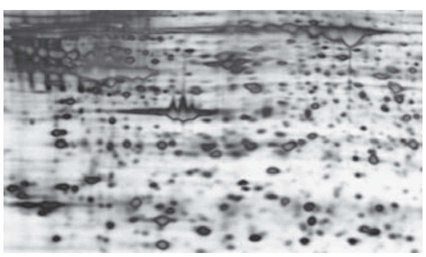
The procedure for preparing the gel uses the electric circuit illustrated in Figure 1.5-10*b*. The sample consists of a gel and a filter paper containing ionized proteins. A voltage source causes a large, constant voltage, 500 V, across the sample. The large, constant voltage moves the ionized proteins from the filter paper to the gel. The current in the sample is given by

$$i(t) = 2 + 30e^{-at} \text{ mA}$$

where $t$ is the time elapsed since the beginning of the procedure and the value of the constant $a$ is

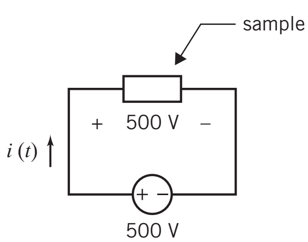
$$a = 0.85 \frac{1}{\text{hr}}$$

Determine the energy supplied by the voltage source when the gel preparation procedure lasts 3 hours.



Devon Svoboda, Queen's University

(*a*)




(*b*)

**Figure P 1.5-10** (*a*) An image of a gel and (*b*) the electric circuit used to prepare gel.

## Section 1.7 How Can We Check . . . ?

**P 1.7-1** ⊕ Conservation of energy requires that the sum of the power received by all of the elements in a circuit be zero. Figure P 1.7-1 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.

***Hint:*** Calculate the power received by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.

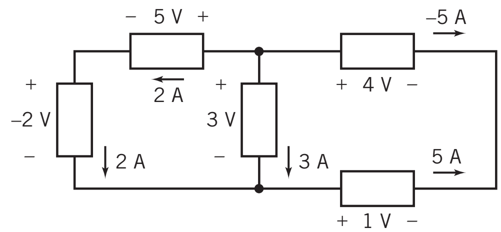



**Figure P 1.7-1**

**P 1.7-2** ⊕ Conservation of energy requires that the sum of the power received by all of the elements in a circuit be zero. Figure P 1.7-2 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.

***Hint:*** Calculate the power received by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.

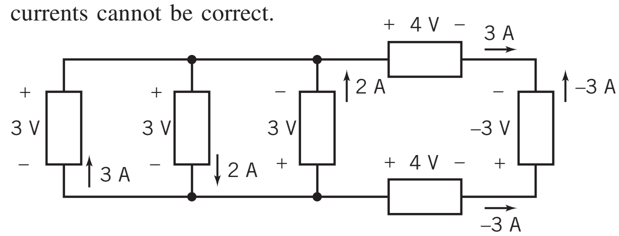



**Figure P 1.7-2**

**P 1.7-3** ⊕ The element currents and voltages shown in Figure P 1.7-3 are correct with one exception: the reference direction of exactly one of the element currents is reversed. Determine which reference direction has been reversed.

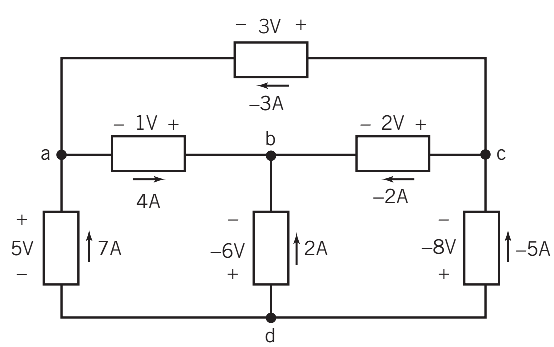



**Figure P 1.7-3**

## Design Problems

**DP 1-1** A particular circuit element is available in three grades. Grade A guarantees that the element can safely absorb 1/2 W continuously. Similarly, Grade B guarantees that 1/4 W can be absorbed safely, and Grade C guarantees that 1/8 W can be absorbed safely. As a rule, elements that can safely absorb more power are also more expensive and bulkier.

The voltage across an element is expected to be about 20 V, and the current in the element is expected to be about 8 mA. Both estimates are accurate to within 25 percent. The voltage and current reference adhere to the passive convention.

Specify the grade of this element. Safety is the most important consideration, but don't specify an element that is more expensive than necessary.

**DP 1-2** The voltage across a circuit element is $v(t) = 20\,(1 - e^{-8t})$ V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The current in this element is $i(t) = 30e^{-8t}$ mA when $t \geq 0$ and $i(t) = 0$ when $t < 0$. The element current and voltage adhere to the passive convention. Specify the power that this device must be able to absorb safely.

***Hint:*** Use MATLAB, or a similar program, to plot the power.

# CHAPTER 2 Circuit Elements

## IN THIS CHAPTER



## 2.1 Introduction

Not surprisingly, the behavior of an electric circuit depends on the behaviors of the individual circuit elements that comprise the circuit. Of course, different types of circuit elements behave differently. The equations that describe the behaviors of the various types of circuit elements are called the constitutive equations. Frequently, the constitutive equations describe a relationship between the current and voltage of the element. Ohm's law is a well-known example of a constitutive equation.

In this chapter, we will investigate the behavior of several common types of circuit element:

- Resistors.
- Independent voltage and current sources.
- Open circuits and short circuits.
- Voltmeters and ammeters.
- Dependent sources.
- Transducers.
- Switches.

## 2.2 Engineering and Linear Models

The art of engineering is to take a bright idea and, using money, materials, knowledgeable people, and a regard for the environment, produce something the buyer wants at an affordable price.

Engineers use *models* to represent the elements of an electric circuit. A model is a description of those properties of a device that we think are important. Frequently, the model will consist of an equation relating the element voltage and current. Though the model is different from the electric device, the model can be used in pencil-and-paper calculations that will predict how a circuit composed of actual devices will operate. Engineers frequently face a trade-off when selecting a model for a device. Simple models are easy to work with but may not be accurate. Accurate models are usually more complicated and harder to use. The conventional wisdom suggests that simple models be used first. The results obtained using the models must be checked to verify that use of these simple models is appropriate. More accurate models are used when necessary.

The idealized models of electric devices are precisely defined. It is important to distinguish between actual devices and their idealized models, which we call circuit elements. The goal of circuit analysis is to predict the quantitative electrical behavior of physical circuits. Its aim is to predict and to explain the terminal voltages and terminal currents of the circuit elements and thus the overall operation of the circuit.

Models of circuit elements can be categorized in a variety of ways. For example, it is important to distinguish linear models from nonlinear models because circuits that consist entirely of linear circuit elements are easier to analyze than circuits that contain some nonlinear elements.

An element or circuit is *linear* if the element's excitation and response satisfy certain properties. Consider the element shown in Figure 2.2-1. Suppose that the excitation is the current $i$ and the response is the voltage $v$. When the element is subjected to a current $i_1$, it provides a response $v_1$. Furthermore, when the element is subjected to a current $i_2$, it provides a response $v_2$. For a linear element, it is necessary that the excitation $i_1 + i_2$ result in a response $v_1 + v_2$. This is usually called the *principle of superposition.*

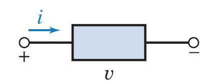


**FIGURE 2.2-1** An element with an excitation current $i$ and a response $v$.

Also, multiplying the input of a linear device by a constant must have the consequence of multiplying the output by the same constant. For example, doubling the size of the input causes the size of the output to double. This is called the *property of homogeneity.* An element is linear if, and only if, the properties of superposition and homogeneity are satisfied for all excitations and responses.

A **linear element** satisfies the properties of both superposition and homogeneity.

Let us restate mathematically the two required properties of a linear circuit, using the arrow notation to imply the transition from excitation to response:

$$i \to v$$

Then we may state the two properties required as follows.
*Superposition:*

$$i_1 \to v_1$$
$$i_2 \to v_2$$

then $$i_1 + i_2 \to v_1 + v_2 \tag{2.2-1}$$

*Homogeneity:*

$$i \to v$$

then $$ki \to kv \tag{2.2-2}$$

A device that does not satisfy either the superposition or the homogeneity principle is said to be nonlinear.

## EXAMPLE 2.2-1 A Linear Device

Consider the element represented by the relationship between current and voltage as

$$v = Ri$$

Determine whether this device is linear.

## Solution

The response to a current $i_1$ is $$v_1 = Ri_1$$

The response to a current $i_2$ is $$v_2 = Ri_2$$

The sum of these responses is

$$v_1 + v_2 = Ri_1 + Ri_2 = R(i_1 + i_2)$$

Because the sum of the responses to $i_1$ and $i_2$ is equal to the response to $i_1 + i_2$, the principle of superposition is satisfied. Next, consider the principle of homogeneity. Because

$$v_1 = Ri_1$$

we have for an excitation $i_2 = ki_1$

$$v_2 = Ri_2 = Rki_1$$

Therefore, $$v_2 = kv_1$$

satisfies the principle of homogeneity. Because the element satisfies the properties of both superposition and homogeneity, it is linear.

## Example 2.2-2 A Nonlinear Device

Now let us consider an element represented by the relationship between current and voltage:

$$v = i^2$$

Determine whether this device is linear.

## Solution

The response to a current $i_1$ is $$v_1 = i_1^2$$

The response to a current $i_2$ is $$v_2 = i_1^2$$

The sum of these responses is

$$v_1 + v_2 = i_1^2 + i_1^2$$

The response to $i_1 + i_2$ is

$$(i_1 + i_2)^2 = i_1^2 + 2i_1 i_2 + i_1^2$$

Because

$$i_1^2 + i_1^2 \neq (i_1 + i_2)^2$$

the principle of superposition is not satisfied. Therefore, the device is nonlinear.

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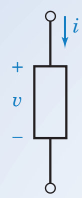
## EXAMPLE 2.2-3 A Model of a Linear Device

A linear element has voltage $v$ and current $i$ as shown in Figure 2.2-2$a$. Values of the current $i$ and corresponding voltage $v$ have been tabulated as shown in Figure 2.2-2$b$. Represent the element by an equation that expresses $v$ as a function of $i$. This equation is a model of the element. Use the model to predict the value of $v$ corresponding to a current of $i = 100\,\text{mA}$ and the value of $i$ corresponding to a voltage of $v = 18\,\text{V}$.




(a)

| $v$, V | $i$, mA |
|---|---|
| 4.5 | 10 |
| 11.25 | 25 |
| 22.5 | 50 |

(b)

**FIGURE 2.2-2** ($a$) A linear circuit element and ($b$) a tabulation of corresponding values of its voltage and current.

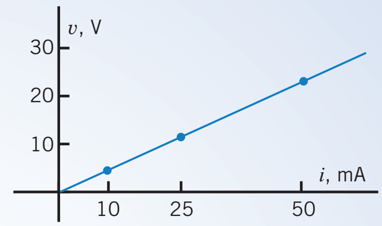



**FIGURE 2.2-3** A plot of voltage versus current for the linear element from Figure 2.2-2.

### Solution

Figure 2.2-3 is a plot of the voltage $v$ versus the current $i$. The points marked by dots represent corresponding values of $v$ and $i$ from the rows of the table in Figure 2.2-2$b$. Because the circuit element is linear, we expect these points to lie on a straight line, and indeed they do. We can represent the straight line by the equation

$$v = mi + b$$

where $m$ is the slope and $b$ is the $v$-intercept. Noticing that the straight line passes through the origin, $v = 0$ when $i = 0$, we see that $b = 0$. We are left with

$$v = mi$$

The slope $m$ can be calculated from the data in any two rows of the table in Figure 2.2-2$b$. For example:

$$\frac{11.25 - 4.5}{25 - 10} = 0.45\ \frac{\text{V}}{\text{mA}},\ \frac{22.5 - 11.25}{50 - 25} = 0.45\ \frac{\text{V}}{\text{mA}},\ \text{and}\ \frac{22.5 - 4.5}{50 - 10} = 0.45\ \frac{\text{V}}{\text{mA}}$$

Consequently,

$$m = 0.45\ \frac{\text{V}}{\text{mA}} = 450\ \frac{\text{V}}{\text{A}}$$

and

$$v = 450i$$

This equation is a model of the linear element. It predicts that the voltage $v = 450(0.1) = 45\,\text{V}$ corresponds to the current $i = 100\,\text{mA} = 0.1\,\text{A}$ and that the current $i = 18/450 = 0.04\,\text{A} = 40\,\text{mA}$ corresponds to the voltage $v = 18\,\text{V}$.

## 2.3 *Active and Passive Circuit Elements*

We may classify circuit elements in two categories, *passive* and *active*, by determining whether they absorb energy or supply energy. An element is said to be passive if the total energy delivered to it from the rest of the circuit is always nonnegative (zero or positive). Then for a passive element, with the current flowing into the $+$ terminal as shown in Figure 2.3-1$a$, this means that

$$w = \int_{-\infty}^{t} vi \, d\tau \geq 0 \tag{2.3-1}$$

for all values of $t$.

A **passive element** absorbs energy.

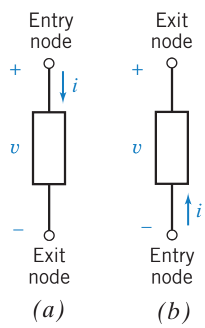



**FIGURE 2.3-1** (*a*) The entry node of the current $i$ is the positive node of the voltage $v$; (*b*) the entry node of the current $i$ is the negative node of the voltage $v$. The current flows from the entry node to the exit node.

An element is said to be *active* if it is capable of delivering energy. Thus, an active element violates Eq. 2.3-1 when it is represented by Figure 2.3-1*a*. In other words, an active element is one that is capable of generating energy. Active elements are potential sources of energy, whereas passive elements are sinks or absorbers of energy. Examples of active elements include batteries and generators. Consider the element shown in Figure 2.3-1*b*. Note that the current flows into the negative terminal and out of the positive terminal. This element is said to be active if

$$w = \int_{-\infty}^{t} vi \, d\tau \geq 0 \tag{2.3-2}$$

for at least one value of $t$.

An **active element** is capable of supplying energy.

## Example 2.3-1 An Active Circuit Element

A circuit has an element represented by Figure 2.3-1*b* where the current is a constant 5 A and the voltage is a constant 6 V. Find the energy supplied over the time interval 0 to $T$.

### Solution

Because the current enters the negative terminal, the energy *supplied* by the element is given by

$$w = \int_{0}^{T} (6)(5) d\tau = 30T \text{ J}$$

Thus, the device is a generator or an active element, in this case a dc battery.

## 2.4 Resistors

The ability of a material to resist the flow of charge is called its *resistivity*, $\rho$. Materials that are good electrical insulators have a high value of resistivity. Materials that are good conductors of electric current have low values of resistivity. Resistivity values for selected materials are given in Table 2.4-1. Copper is commonly used for wires because it permits current to flow relatively unimpeded. Silicon is commonly used to provide resistance in semiconductor electric circuits. Polystyrene is used as an insulator.

**Table 2.4-1 Resistivities of Selected Materials**

| MATERIAL | RESISTIVITY $\rho$ (OHM.CM) |
|---|---|
| Polystyrene | $1 \times 10^{18}$ |
| Silicon | $2.3 \times 10^{5}$ |
| Carbon | $4 \times 10^{-3}$ |
| Aluminum | $2.7 \times 10^{-6}$ |
| Copper | $1.7 \times 10^{-6}$ |

**Resistance** is the physical property of an element or device that impedes the flow of current; it is represented by the symbol *R*.

Georg Simon Ohm was able to show that the current in a circuit composed of a battery and a conducting wire of uniform cross-section could be expressed as

$$i = \frac{Av}{\rho L} \tag{2.4-1}$$

where $A$ is the cross-sectional area, $\rho$ the resistivity, $L$ the length, and $v$ the voltage across the wire element. Ohm, who is shown in Figure 2.4-1, defined the constant resistance $R$ as

$$R = \frac{\rho L}{A} \tag{2.4-2}$$

Ohm's law, which related the voltage and current, was published in 1827 as

$$v = Ri \tag{2.4-3}$$

The unit of resistance $R$ was named the ohm in honor of Ohm and is usually abbreviated by the $\Omega$ (capital omega) symbol, where $1\ \Omega = 1$ V/A. The resistance of a 10-m length of common TV cable is 2 m$\Omega$.

An element that has a resistance $R$ is called a *resistor*. A resistor is represented by the two-terminal symbol shown in Figure 2.4-2. Ohm's law, Eq. 2.4-3, requires that the $i$-versus-$v$ relationship be linear. As shown in Figure 2.4-3, a resistor may become nonlinear outside its normal rated range of operation. We will assume that a resistor is linear unless stated otherwise. Thus, we will use a linear model of the resistor as represented by Ohm's law.

In Figure 2.4-4, the element current and element voltage of a resistor are labeled. The relationship between the directions of this current and voltage is important. The voltage direction marks one resistor terminal + and the other −. The current $i_a$ flows from the terminal marked + to the terminal marked −. This relationship between the current and voltage reference directions is a convention called the passive convention. Ohm's law states that when the element voltage and the element current adhere to the passive convention, then

$$v = Ri_a \tag{2.4-4}$$

Photo by Hulton Archive/ Getty Images

**FIGURE 2.4-1** Georg Simon Ohm (1787–1854), who determined Ohm's law in 1827. The ohm was chosen as the unit of electrical resistance in his honor.


**FIGURE 2.4-2** Symbol for a resistor having a resistance of *R* ohms.

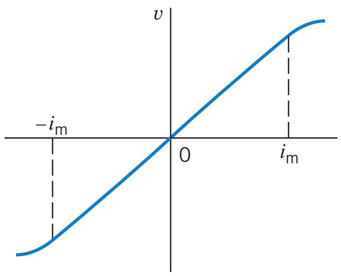



**FIGURE 2.4-3** A resistor operating within its specified current range, $\pm i_m$, can be modeled by Ohm's law.

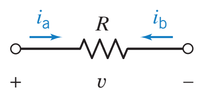



**FIGURE 2.4-4** A resistor with element current and element voltage.


Courtesy of Vishay Intertechnology, Inc.

**FIGURE 2.4-5** (*a*) Wirewound resistor with an adjustable center tap. (*b*) Wirewound resistor with a fixed tap.

Consider Figure 2.4-4. The element currents $i_a$ and $i_b$ are the same except for the assigned direction, so

$$i_a = -i_b$$

The element current $i_a$ and the element voltage $v$ adhere to the passive convention,

$$v = Ri_a$$

Replacing $i_a$ by $-i_b$ gives

$$v = -Ri_b$$

There is a minus sign in this equation because the element current $i_b$ and the element voltage $v$ do not adhere to the passive convention. We must pay attention to the current direction so that we don't overlook this minus sign.

Ohm's law, Eq. 2.4-3, can also be written as

$$i = Gv \tag{2.4-5}$$

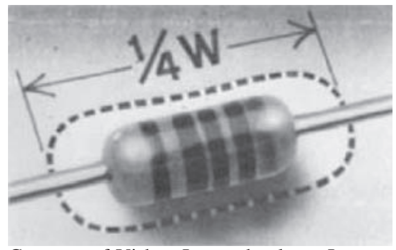
where $G$ denotes the *conductance* in siemens (S) and is the reciprocal of $R$; that is, $G = 1/R$. Many engineers denote the units of conductance as mhos with the ℧ symbol, which is an inverted omega (mho is *ohm* spelled backward). However, we will use SI units and retain siemens as the units for conductance.

Most discrete resistors fall into one of four basic categories: carbon composition, carbon film, metal film, or wirewound. Carbon composition resistors have been in use for nearly 100 years and are still popular. Carbon film resistors have supplanted carbon composition resistors for many general-purpose uses because of their lower cost and better tolerances. Two wirewound resistors are shown in Figure 2.4-5.

Carbon composition resistors, as shown in Figure 2.4-6, are used in circuits because of their low cost and small size. General-purpose resistors are available in standard values for tolerances of 2, 5, 10, and 20 percent. Carbon composition resistors and some wirewounds have a color code with three to five bands. A color code is a system of standard colors adopted for identification of the resistance of resistors. Figure 2.4-7 shows a metal film resistor with its color bands. This is a 1/4-watt resistor, implying that it should be operated at or below 1/4 watt of power delivered to it. The normal range of resistors is from less than 1 ohm to 10 megohms. Typical values of some commercially available resistors are given in Appendix D.

Courtesy of Hifi Collective.

**FIGURE 2.4-6** Carbon composition resistors.




Courtesy of Vishay Intertechnology, Inc.

**FIGURE 2.4-7** A 1/4-watt metal film resistor. The body of the resistor is 6 mm long.

The power delivered to a resistor (when the passive convention is used) is

$$p = vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{2.4-6}$$

Alternatively, because $v = iR$, we can write the equation for power as

$$p = vi = (iR)i = i^2R \tag{2.4-7}$$

Thus, the power is expressed as a nonlinear function of the current $i$ through the resistor or of the voltage $v$ across it.

## EXAMPLE 2.4-1 Power Dissipated by a Resistor

Let us devise a model for a car battery when the lights are left on and the engine is off. We have all experienced or seen a car parked with its lights on. If we leave the car for a period, the battery will run down or go dead. An auto battery is a 12-V constant-voltage source, and the lightbulb can be modeled by a resistor of 6 ohms. The circuit is shown in Figure 2.4-8. Let us find the current $i$, the power $p$, and the energy supplied by the battery for a four-hour period.

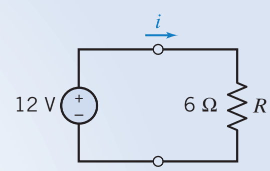



**FIGURE 2.4-8** Model of a car battery and the headlight lamp.

### Solution

According to Ohm's law, Eq. 2.4-3, we have

$$v = Ri$$

Because $v = 12$ V and $R = 6\ \Omega$, we have $i = 2$ A.

To find the power delivered by the battery, we use

$$p = vi = 12(2) = 24 \text{ W}$$

Finally, the energy delivered in the four-hour period is

$$w = \int_0^t p\,d\tau = 24t = 24(60 \times 60 \times 4) = 3.46 \times 10^5 \text{ J}$$

Because the battery has a finite amount of stored energy, it will deliver this energy and eventually be unable to deliver further energy without recharging. We then say the battery is run down or dead until recharged. A typical auto battery may store $10^6$ J in a fully charged condition.

**EXERCISE 2.4-1** Find the power absorbed by a 100-ohm resistor when it is connected directly across a constant 10-V source.

***Answer:*** 1-W

**EXERCISE 2.4-2** A voltage source $v = 10 \cos t$ V is connected across a resistor of 10 ohms. Find the power delivered to the resistor.

***Answer:*** $10 \cos^2 t$ W

## 2.5 Independent Sources

Some devices are intended to supply energy to a circuit. These devices are called sources. Sources are categorized as being one of two types: voltage sources and current sources. Figure 2.5-1*a* shows the symbol that is used to represent a voltage source. The voltage of a voltage source is specified, but the current is determined by the rest of the circuit. A voltage source is described by specifying the function $v(t)$, for example,

$$v(t) = 12 \cos 1000t \quad \text{or} \quad v(t) = 9 \quad \text{or} \quad v(t) = 12 - 2t$$

An active two-terminal element that supplies energy to a circuit is a *source* of energy. An *independent voltage source* provides a specified voltage independent of the current through it and is independent of any other circuit variable.

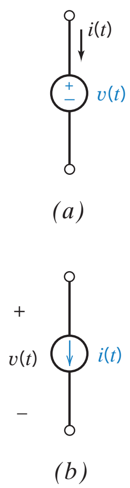



**FIGURE 2.5-1** (*a*) Voltage source. (*b*) Current source.

A **source** is a voltage or current generator capable of supplying energy to a circuit.

An *independent current source* provides a current independent of the voltage across the source element and is independent of any other circuit variable. Thus, when we say a source is independent, we mean it is independent of any other voltage or current in the circuit.

An **independent source** is a voltage or current generator not dependent on other circuit variables.

Suppose the voltage source is a battery and

$$v(t) = 9 \text{ volts}$$

The voltage of this battery is known to be 9 volts regardless of the circuit in which the battery is used. In contrast, the current of the voltage source is not known and depends on the circuit in which the source is used. The current could be 6 amps when the voltage source is connected to one circuit and 6 milliamps when the voltage source is connected to another circuit.

Figure 2.5-1*b* shows the symbol that is used to represent a current source. The current of a current source is specified, but the voltage is determined by the rest of the circuit. A current source is described by specifying the function $i(t)$, for example,

$$i(t) = 6 \sin 500t \quad \text{or} \quad i(t) = -0.25 \quad \text{or} \quad i(t) = t + 8$$

A current source specified by $i(t) = -0.25$ milliamps will have a current of $-0.25$ milliamps in any circuit in which it is used. The voltage across this current source will depend on the particular circuit.

The preceding paragraphs have ignored some complexities to give a simple description of the way sources work. The voltage across a 9-volt battery may not actually be 9 volts. This voltage depends on the age of the battery, the temperature, variations in manufacturing, and the battery current. It is useful to make a distinction between real sources, such as batteries, and the simple voltage and current sources described in the preceding paragraphs. It would be *ideal* if the real sources worked like these simple sources. Indeed, the word *ideal* is used to make this distinction. The simple sources described in the previous paragraph are called the *ideal voltage source* and the *ideal current source*.

The voltage of an **ideal voltage source** is given to be a specified function, say $v(t)$. The current is determined by the rest of the circuit.

The current of an **ideal current source** is given to be a specified function, say $i(t)$. The voltage is determined by the rest of the circuit.

An **ideal source** is a voltage or a current generator independent of the current through the voltage source or the voltage across the current source.

Engineers frequently face a trade-off when selecting a model for a device. Simple models are easy to work with but may not be accurate. Accurate models are usually more complicated and harder to use. The conventional wisdom suggests that simple models be used first. The results obtained using the models must be checked to verify that use of these simple models is appropriate. More accurate models are used when necessary.

## EXAMPLE 2.5-1 A Battery Modeled as a Voltage Source

Consider the plight of the engineer who needs to analyze a circuit containing a 9-volt battery. Is it really necessary for this engineer to include the dependence of battery voltage on the age of the battery, the temperature, variations in manufacturing, and the battery current in this analysis? Hopefully not. We expect the battery to act enough like an ideal 9-volt voltage source that the differences can be ignored. In this case, it is said that the battery is *modeled* as an ideal voltage source.

To be specific, consider a battery specified by the plot of voltage versus current shown in Figure 2.5-2*a*. This plot indicates that the battery voltage will be $v = 9$ volts when $i \leq 10$ milliamps. As the current increases above 10 milliamps, the voltage decreases from 9 volts. When $i \leq 10$ milliamps, the dependence of the battery voltage on the battery current can be ignored and the battery can be modeled as an ideal voltage source.

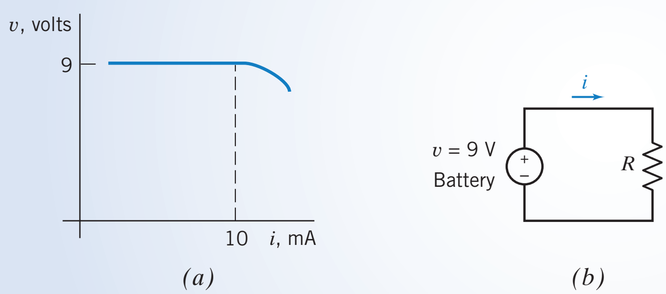



**FIGURE 2.5-2** (*a*) A plot of battery voltage versus battery current. (*b*) The battery is modeled as an independent voltage source.

Suppose a resistor is connected across the terminals of the battery as shown in Figure 2.5-2*b*. The battery current will be

$$i = \frac{v}{R} \tag{2.5-1}$$

The relationship between $v$ and $i$ shown in Figure 2.5-2*a* complicates this equation. This complication can be safely ignored when $i \leq 10$ milliamps. When the battery is modeled as an ideal 9-volt voltage source, the voltage source current is given by

$$i = \frac{9}{R} \tag{2.5-2}$$

The distinction between these two equations is important. Eq. 2.5-1, involving the $v-i$ relationship shown in Figure 2.5-2*a*, is more accurate but also more complicated. Equation 2.5-2 is simpler but may be inaccurate.

Suppose that $R = 1000$ ohms. Equation 2.5-2 gives the current of the ideal voltage source:

$$i = \frac{9}{1000} = 9 \text{ mA} \tag{2.5-3}$$

Because this current is less than 10 milliamps, the ideal voltage source is a good model for the battery, and it is reasonable to expect that the battery current is 9 milliamps.

Suppose, instead, that $R = 600$ ohms. Once again, Eq. 2.5-2 gives the current of the ideal voltage source:

$$i = \frac{9}{600} = 15 \text{ mA} \tag{2.5-4}$$

Because this current is greater than 10 milliamps, the ideal voltage source is not a good model for the battery. In this case, it is reasonable to expect that the battery current is different from the current for the ideal voltage source.

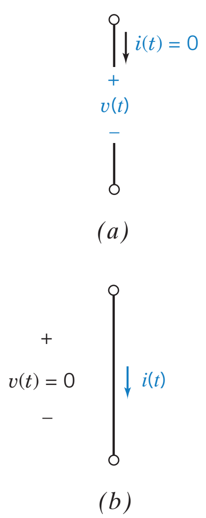



**FIGURE 2.5-3**
(*a*) Open circuit.
(*b*) Short circuit.

The short circuit and open circuit are special cases of ideal sources. A *short circuit* is an ideal voltage source having $v(t) = 0$. The current in a short circuit is determined by the rest of the circuit. An *open circuit* is an ideal current source having $i(t) = 0$. The voltage across an open circuit is determined by the rest of the circuit. Figure 2.5-3 shows the symbols used to represent the short circuit and the open circuit. Notice that the power absorbed by each of these devices is zero.

Open and short circuits can be added to a circuit without disturbing the branch currents and voltages of all the other devices in the circuit. Figure 2.6-3 shows how this can be done. Figure 2.6-3*a* shows an example circuit. In Figure 2.6-3*b* an open circuit and a short circuit have been added to this example circuit. The open circuit was connected between two nodes of the original circuit. In contrast, the short circuit was added by cutting a wire and inserting the short circuit. Adding open circuits and short circuits to a network in this way does not change the network.

Open circuits and short circuits can also be described as special cases of resistors. A resistor with resistance $R = 0$ ($G = \infty$) is a short circuit. A resistor with conductance $G = 0$ ($R = \infty$) is an open circuit.

## 2.6 Voltmeters and Ammeters

Measurements of dc current and voltage are made with direct-reading (analog) or digital meters, as shown in Figure 2.6-1. A direct-reading meter has an indicating pointer whose angular deflection depends on the magnitude of the variable it is measuring. A digital meter displays a set of digits indicating the measured variable value.

To measure a voltage or current, a meter is connected to a circuit, using terminals called probes. These probes are color coded to indicate the reference direction of the variable being measured. Frequently, meter probes are colored red and black. An ideal voltmeter measures the voltage from the red to the black probe. The red terminal is the positive terminal, and the black terminal is the negative terminal (see Figure 2.6-2*b*).

An ideal ammeter measures the current flowing through its terminals, as shown in Figure 2.6-2*a* and has zero voltage, $v_m$, across its terminals. An ideal voltmeter measures the voltage across its terminals, as shown in Figure 2.6-2*b*, and has terminal current, $i_m$, equal to zero. Practical measuring

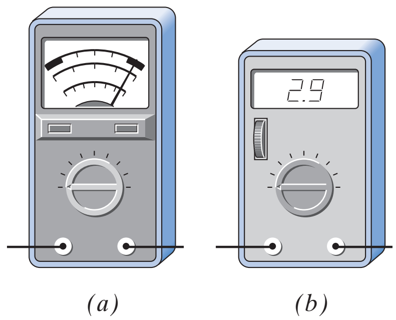



**FIGURE 2.6-1** (*a*) A direct-reading (analog) meter. (*b*) A digital meter.

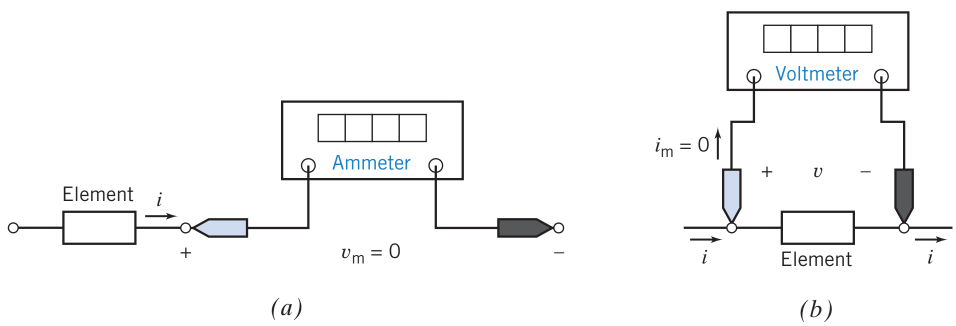



**FIGURE 2.6-2** (*a*) Ideal ammeter. (*b*) Ideal voltmeter.

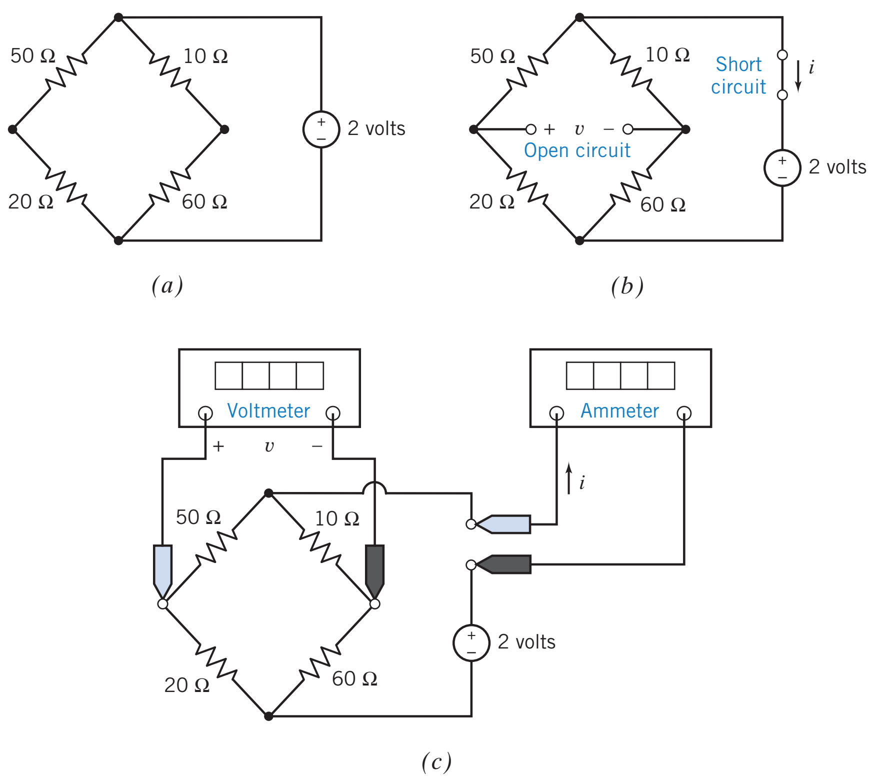



**FIGURE 2.6-3** (*a*) An example circuit, (*b*) plus an open circuit and a short circuit. (*c*) The open circuit is replaced by a voltmeter, and the short circuit is replaced by an ammeter.

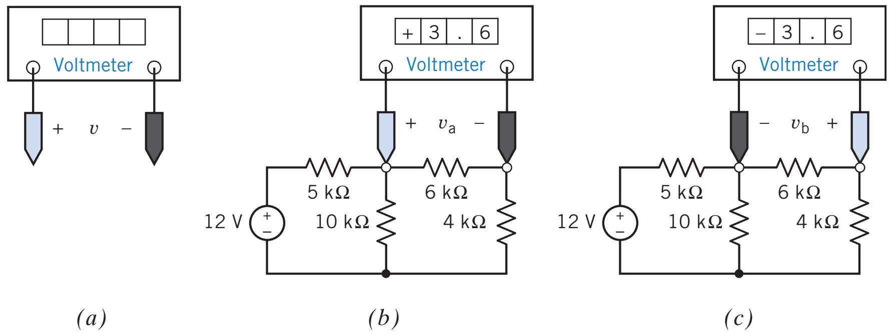



**FIGURE 2.6-4** (*a*) The correspondence between the color-coded probes of the voltmeter and the reference direction of the measured voltage. In (*b*), the + sign of $v_a$ is on the left, whereas in (*c*), the + sign of $v_b$ is on the right. The colored probe is shown here in blue. In the laboratory this probe will be red. We will refer to the colored probe as the "red probe."

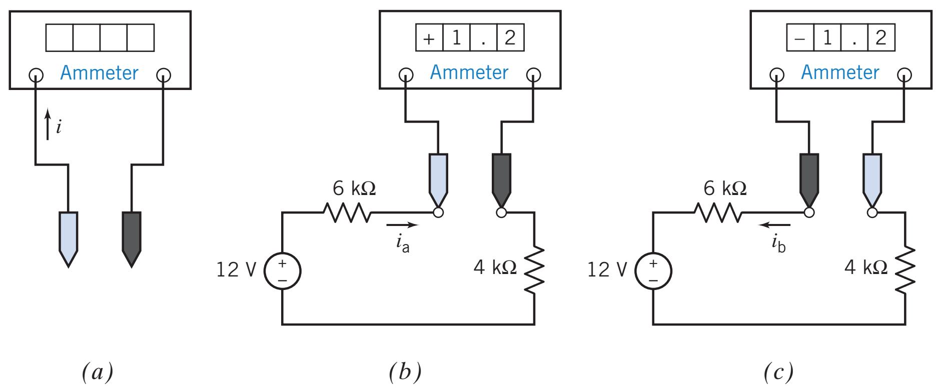



**FIGURE 2.6-5** (*a*) The correspondence between the color-coded probes of the ammeter and the reference direction of the measured current. In (*b*) the current $i_a$ is directed to the right, while in (*c*) the current $i_b$ is directed to the left. The colored probe is shown here in blue. In the laboratory this probe will be red. We will refer to the colored probe as the "red probe."

instruments only approximate the ideal conditions. For a practical ammeter, the voltage across its terminals is usually negligibly small. Similarly, the current into a voltmeter is usually negligible.

Ideal voltmeters act like open circuits, and ideal ammeters act like short circuits. In other words, the model of an ideal voltmeter is an open circuit, and the model of an ideal ammeter is a short circuit. Consider the circuit of Figure 2.6-3*a* and then add an open circuit with a voltage $v$ and a short circuit with a current $i$ as shown in Figure 2.6-3*b*. In Figure 2.6-3*c*, the open circuit has been replaced by a voltmeter, and the short circuit has been replaced by an ammeter. The voltmeter will measure the voltage labeled $v$ in Figure 2.6-3*b* whereas the ammeter will measure the current labeled $i$. Notice that Figure 2.6-3*c* could be obtained from Figure 2.6-3*a* by adding a voltmeter and an ammeter. Ideally, adding the voltmeter and ammeter in this way does not disturb the circuit. One more interpretation of Figure 2.6-3 is useful. Figure 2.6-3*b* could be formed from Figure 2.6-3*c* by replacing the voltmeter and the ammeter by their (ideal) models.

The reference direction is an important part of an element voltage or element current. Figures 2.6-4 and 2.6-5 show that attention must be paid to reference directions when measuring an element voltage or element current. Figure 2.6-4*a* shows a voltmeter. Voltmeters have two color-coded probes. This color coding indicates the reference direction of the voltage being measured. In Figures 2.6-4*b* and Figure 2.6-4*c* the voltmeter is used to measure the voltage across the 6-kΩ resistor. When the voltmeter is connected to the circuit as shown in Figure 2.6-4*b*, the voltmeter measures $v_a$, with + on the left, at the red probe. When the voltmeter probes are interchanged as shown in Figure 2.6-4*c*, the voltmeter measures $v_b$, with + on the right, again at the red probe. Note $v_b = -v_a$.

Figure 2.6-5*a* shows an ammeter. Ammeters have two color-coded probes. This color coding indicates the reference direction of the current being measured. In Figures 2.6-5*b* and *c*, the ammeter is used to measure the current in the 6-kΩ resistor. When the ammeter is connected to the circuit as shown in Figure 2.6-5*b*, the ammeter measures $i_a$, directed from the red probe toward the black probe. When the ammeter probes are interchanged as shown in Figure 2.6-5*c*, the ammeter measures $i_b$, again directed from the red probe toward the black probe. Note $i_b = -i_a$.

## 2.7 Dependent Sources

Dependent sources model the situation in which the voltage or current of one circuit element is proportional to the voltage or current of the second circuit element. (In contrast, a resistor is a circuit element in which the voltage of the element is proportional to the current in the *same* element.) Dependent sources are used to model electronic devices such as transistors and amplifiers. For example, the output voltage of an amplifier is proportional to the input voltage of that amplifier, so an amplifier can be modeled as a dependent source.

Figure 2.7-1*a* shows a circuit that includes a dependent source. The diamond symbol represents a dependent source. The plus and minus signs inside the diamond identify the dependent source as a voltage source and indicate the reference polarity of the element voltage. The label "5*i*" represents the voltage of this dependent source. This voltage is a product of two factors, 5 and *i*. The second factor, *i*, indicates that the voltage of this dependent source is controlled by the current, *i*, in the 18-Ω resistor. The first factor, 5, is the gain of this dependent source. The gain of this dependent source is the ratio of the controlled voltage, 5*i*, to the controlling current, *i*. This gain has units of V/A or Ω. Because this dependent source is a voltage source and because a current controls the voltage, the dependent source is called a current-controlled voltage source (CCVS).

Figure 2.7-1*b* shows the circuit from 2.7-1*a*, using a different point of view. In Figure 2.7-1*b*, a short circuit has been inserted in series with the 18-Ω resistor. Now we think of the controlling current *i* as the current in a short circuit rather than the current in the 18-Ω resistor itself. In this way, we can

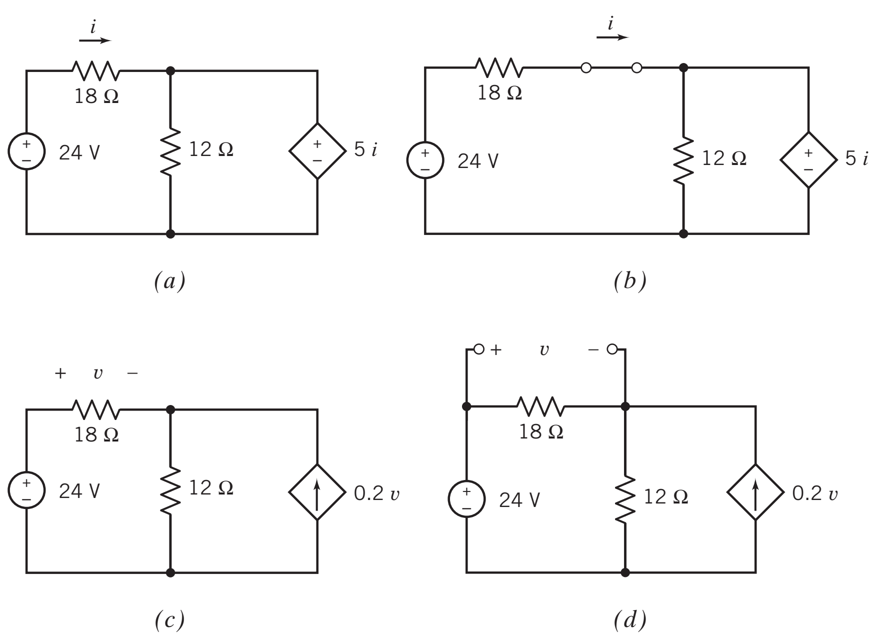
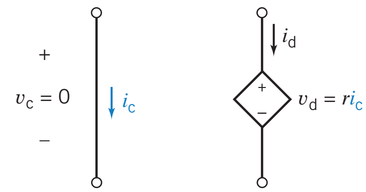
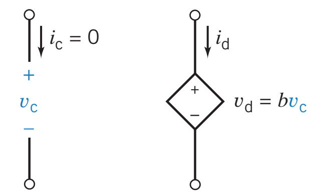
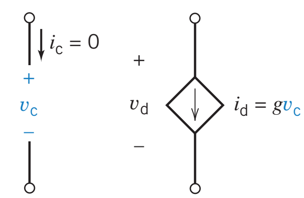
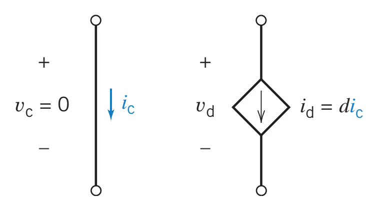



**FIGURE 2.7-1** The controlling current of a dependent source shown as (*a*) the current in an element and as (*b*) the current in a short circuit in series with that element. The controlling voltage of a dependent source shown as (*c*) the voltage across an element and as (*d*) the voltage across an open circuit in parallel with that element.

**Table 2.7-1 Dependent Sources**

| DESCRIPTION | SYMBOL |
|---|---|
| Current-Controlled Voltage Source (CCVS)<br>*r* is the gain of the CCVS.<br>*r* has units of volts/ampere. |  |
| Voltage-Controlled Voltage Source (VCVS)<br>*b* is the gain of the VCVS.<br>*b* has units of volts/volt. |  |
| Voltage-Controlled Current Source (VCCS)<br>*g* is the gain of the VCCS.<br>*g* has units of amperes/volt. |  |
| Current-Controlled Current Source (CCCS)<br>*d* is the gain of the CCCS.<br>*d* has units of amperes/ampere. |  |

always treat the controlling current of a dependent source as the current in a short circuit. We will use this second point of view to categorize dependent sources in this section.

Figure 2.7-1*c* shows a circuit that includes a dependent source, represented by the diamond symbol. The arrow inside the diamond identifies the dependent source as a current source and indicates the reference direction of the element current. The label "0.2*v*" represents the current of this dependent source. This current is a product of two factors, 0.2 and *v*. The second factor, *v*, indicates that the current of this dependent source is controlled by the voltage, *v*, across the 18-Ω resistor. The first factor, 0.2, is the gain of this dependent source. The gain of this dependent source is the ratio of the controlled current, 0.2*v*, to the controlling voltage, *v*. This gain has units of A/V. Because this dependent source is a current source and because a voltage controls the current, the dependent source is called a voltage-controlled current source (VCCS).

Figure 2.7-1*d* shows the circuit from Figure 2.7-1*c*, using a different point of view. In Figure 2.7-1*d*, an open circuit has been added in parallel with the 18-Ω resistor. Now we think of the controlling voltage *v* as the voltage across an open circuit Figure 2.7-1, rather than the voltage across the 18-Ω resistor itself. In this way, we can always treat the controlling voltage of a dependent source as the voltage across an open circuit.

We are now ready to categorize dependent source. Each dependent source consists of two parts: the controlling part and the controlled part. The controlling part is either an open circuit or a short circuit. The controlled part is either a voltage source or a current source. There are four types of dependent source that correspond to the four ways of choosing a controlling part and a controlled part. These four dependent sources are called the voltage-controlled voltage source (VCVS), current-controlled voltage source (CCVS), voltage-controlled current source (VCCS), and current-controlled current source (CCCS). The symbols that represent dependent sources are shown in Table 2.7-1.

Consider the CCVS shown in Table 2.7-1. The controlling element is a short circuit. The element current and voltage of the controlling element are denoted as $i_c$ and $v_c$. The voltage across a short circuit is zero, so $v_c = 0$. The short-circuit current, $i_c$, is the controlling signal of this dependent source. The controlled element is a voltage source. The element current and voltage of the controlled element are denoted as $i_d$ and $v_d$. The voltage $v_d$ is controlled by $i_c$:

$$v_d = r i_c$$

The constant $r$ is called the gain of the CCVS. The current $i_d$, like the current in any voltage source, is determined by the rest of the circuit.

Next, consider the VCVS shown in Table 2.7-1. The controlling element is an open circuit. The current in an open circuit is zero, so $i_c = 0$. The open-circuit voltage, $v_c$, is the controlling signal of this dependent source. The controlled element is a voltage source. The voltage $v_d$ is controlled by $v_c$:

$$v_d = b v_c$$

The constant $b$ is called the gain of the VCVS. The current $i_d$ is determined by the rest of the circuit.

The controlling element of the VCCS shown in Table 2.7-1 is an open circuit. The current in this open circuit is $i_c = 0$. The open-circuit voltage, $v_c$, is the controlling signal of this dependent source. The controlled element is a current source. The current $i_d$ is controlled by $v_c$:

$$i_d = g v_c$$

The constant $g$ is called the gain of the VCCS. The voltage $v_d$, like the voltage across any current source, is determined by the rest of the circuit.

The controlling element of the CCCS shown in Table 2.7-1 is a short circuit. The voltage across this short circuit is $v_c = 0$. The short-circuit current, $i_c$, is the controlling signal of this dependent source. The controlled element is a current source. The current $i_d$ is controlled by $i_c$:

$$i_d = d i_c$$

The constant $d$ is called the gain of the CCCS. The voltage $v_d$, like the voltage across any current source, is determined by the rest of the circuit.

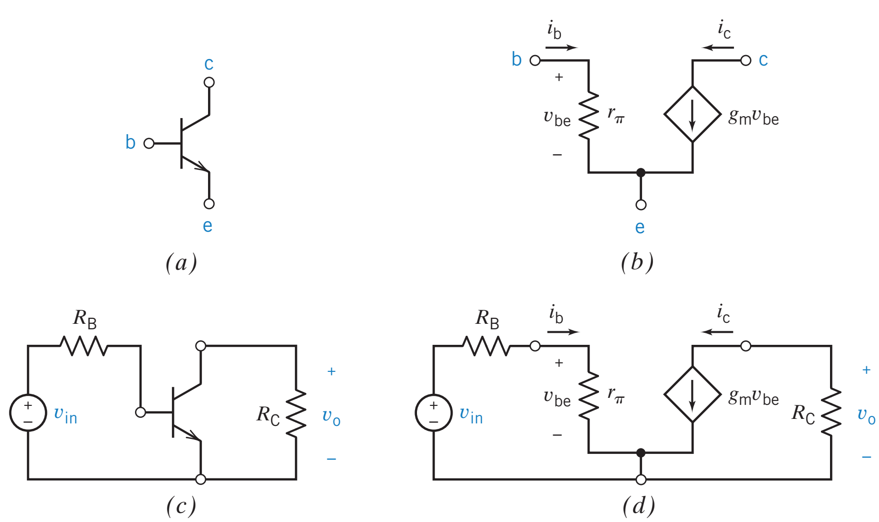



**FIGURE 2.7-2** (*a*) A symbol for a transistor. (*b*) A model of the transistor. (*c*) A transistor amplifier. (*d*) A model of the transistor amplifier.

Figure 2.7-2 illustrates the use of dependent sources to model electronic devices. In certain circumstances, the behavior of the transistor shown in Figure 2.7-2*a* can be represented using the model shown in Figure 2.7-2*b*. This model consists of a dependent source and a resistor. The controlling element of the dependent source is an open circuit connected across the resistor. The controlling voltage is $v_{be}$. The gain of the dependent source is $g_m$. The dependent source is used in this model to represent a property of the transistor, namely, that the current $i_c$ is proportional to the voltage $v_{be}$, that is,

$$i_c = g_m v_{be}$$

where $g_m$ has units of amperes/volt. Figures 2.7-2*c* and *d* illustrate the utility of this model. Figure 2.7-2*d* is obtained from Figure 2.7-2*c* by replacing the transistor by the transistor model.

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## EXAMPLE 2.7-1 Power and Dependent Sources

Determine the power absorbed by the VCVS in Figure 2.7-3.

### Solution

The VCVS consists of an open circuit and a controlled-voltage source. There is no current in the open circuit, so no power is absorbed by the open circuit.

The voltage $v_c$ across the open circuit is the controlling signal of the VCVS. The voltmeter measures $v_c$ to be

$$v_c = 2 \text{ V}$$

The voltage of the controlled voltage source is

$$v_d = 2\, v_c = 4 \text{ V}$$

The ammeter measures the current in the controlled voltage source to be

$$i_d = 1.5 \text{ A}$$

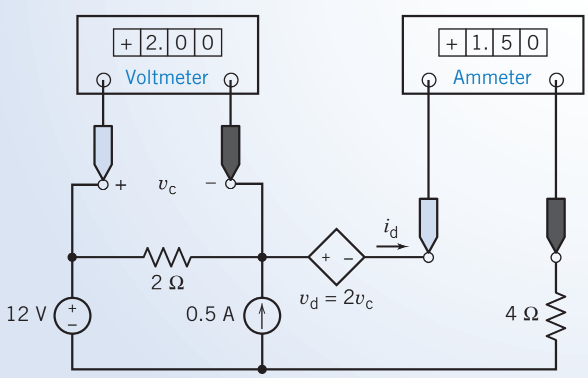


**FIGURE 2.7-3** A circuit containing a VCVS. The meters indicate that the voltage of the controlling element is $v_c = 2.0$ volts and that the current of the controlled element is $i_d = 1.5$ amperes.

The element current $i_d$ and voltage $v_d$ adhere to the passive convention. Therefore,

$$p = i_d v_d = (1.5)(4) = 6 \text{ W}$$

is the power absorbed by the VCVS.

**EXERCISE 2.7-1** Find the power absorbed by the CCCS in Figure E 2.7-1.

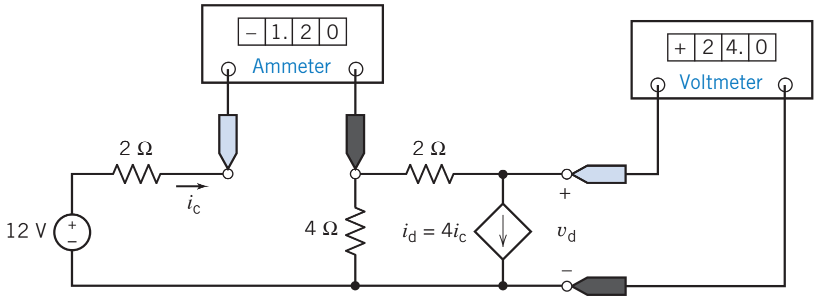


**FIGURE E 2.7-1** A circuit containing a CCCS. The meters indicate that the current of the controlling element is $i_c = -1.2$ amperes and that the voltage of the controlled element is $v_d = 24$ volts.

***Hint:*** The controlling element of this dependent source is a short circuit. The voltage across a short circuit is zero. Hence, the power absorbed by the controlling element is zero. How much power is absorbed by the controlled element?

***Answer:*** −115.2 watts are received by the CCCS. (The CCCS supplies +115.2 watts to the rest of the circuit.)

## 2.8 Transducers

Transducers are devices that convert physical quantities to electrical quantities. This section describes two transducers: potentiometers and temperature sensors. Potentiometers convert position to resistance, and temperature sensors convert temperature to current.

Figure 2.8-1*a* shows the symbol for the potentiometer. The potentiometer is a resistor having a third contact, called the wiper, that slides along the resistor. Two parameters, $R_p$ and $a$, are needed to describe the potentiometer. The parameter $R_p$ specifies the potentiometer resistance ($R_p > 0$). The parameter $a$ represents the wiper position and takes values in the range $0 \le a \le 1$. The values $a = 0$ and $a = 1$ correspond to the extreme positions of the wiper.

Figure 2.8-1*b* shows a model for the potentiometer that consists of two resistors. The resistances of these resistors depend on the potentiometer parameters $R_p$ and $a$.

Frequently, the position of the wiper corresponds to the angular position of a shaft connected to the potentiometer. Suppose $\theta$ is the angle in degrees and $0 \le \theta \le 360$. Then,

$$a = \frac{\theta}{360}$$

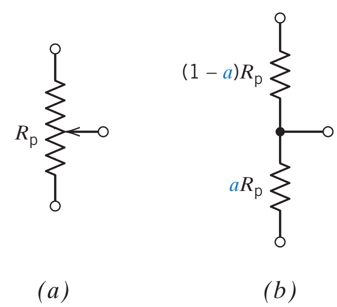


**FIGURE 2.8-1** (*a*) The symbol and (*b*) a model for the potentiometer.

Temperature sensors, such as the AD590 manufactured by Analog Devices, are current sources having current proportional to absolute temperature. Figure 2.8-3*a* shows the symbol used to represent the temperature sensor. Figure 2.8-3*b* shows the circuit model of the temperature sensor. For the temperature sensor to operate properly, the branch voltage $v$ must satisfy the condition

$$4 \text{ volts} \le v \le 30 \text{ volts}$$

When this condition is satisfied, the current, $i$, in microamps, is numerically equal to the temperature $T$, in degrees Kelvin. The phrase *numerically equal* indicates that the current and temperature have the same value but different units. This relationship can be expressed as

$$i = k \cdot T$$

where $k = 1\dfrac{\mu\text{A}}{°\text{K}}$, a constant associated with the sensor.

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## EXAMPLE 2.8-1 Potentiometer Circuit

Figure 2.8-2*a* shows a circuit in which the voltage measured by the meter gives an indication of the angular position of the shaft. In Figure 2.8-2*b*, the current source, the potentiometer, and the voltmeter have been replaced by models of these devices. Analysis of Figure 2.8-2*b* yields

$$v_m = R_p I a = \frac{R_p I}{360}\theta$$

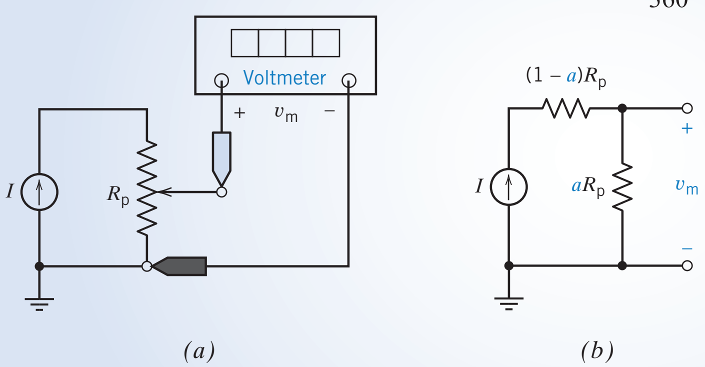


**FIGURE 2.8-2** (*a*) A circuit containing a potentiometer. (*b*) An equivalent circuit containing a model of the potentiometer.

Solving for the angle gives

$$\theta = \frac{360}{R_p I} v_m$$

Suppose $R_p = 10$ kΩ and $I = 1$ mA. An angle of 163° would cause an output of $v_m = 4.53$ V. A meter reading of 7.83 V would indicate that $\theta = 282°$.

**EXERCISE 2.8-1** For the potentiometer circuit of Figure 2.8-2, calculate the meter voltage, $v_m$, when $\theta = 45°$, $R_p = 20$ kΩ, and $I = 2$ mA.

***Answer:*** $v_m = 5$ V

**EXERCISE 2.8-2** The voltage and current of an AD590 temperature sensor of Figure 2.8-3 are 10 V and 280 μA, respectively. Determine the measured temperature.

***Answer:*** $T = 280°$K, or approximately 6.85°C

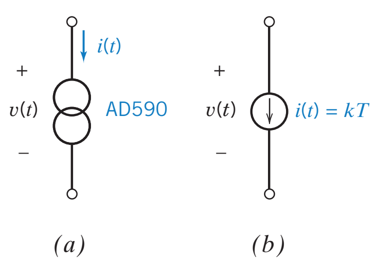


**FIGURE 2.8-3** (*a*) The symbol and (*b*) a model for the temperature sensor.

## 2.9 Switches

Switches have two distinct states: open and closed. Ideally, a switch acts as a short circuit when it is closed and as an open circuit when it is open.

Figures 2.9-1 and 2.9-2 show several types of switches. In each case, the time when the switch changes state is indicated. Consider first the single-pole, single-throw (SPST) switches shown in Figure 2.9-1. The switch in Figure 2.9-1*a* is initially open. This switch changes state, becoming closed, at time $t = 0$ s. When this switch is modeled as an ideal switch, it is treated like an open circuit when $t < 0$ s and like a short circuit when $t > 0$ s. The ideal switch changes state instantaneously. The switch in Figure 2.9-1*b* is initially closed. This switch changes state, becoming open, at time $t = 0$ s.

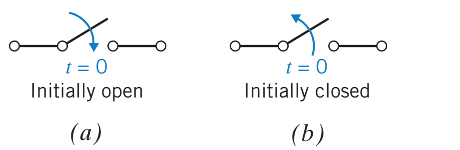


**FIGURE 2.9-1** SPST switches. (*a*) Initially open and (*b*) initially closed.

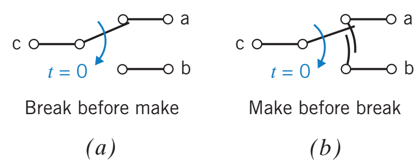


**FIGURE 2.9-2** SPDT switches. (*a*) Break before make and (*b*) make before break.

Next, consider the single-pole, double-throw (SPDT) switch shown in Figure 2.9-1*a*. This SPDT switch acts like two SPST switches, one between terminals c and a, another between terminals c and b. Before $t = 0$ s, the switch between c and a is closed and the switch between c and b is open. At $t = 0$ s, both switches change state; that is, the switch between a and c opens, and the switch between c and b closes. Once again, the ideal switches are modeled as open circuits when they are open and as short circuits when they are closed.

In some applications, it makes a difference whether the switch between c and b closes before, or after, the switch between c and a opens. Different symbols are used to represent these two types of single-pole, double-throw switch. The break-before-make switch is manufactured so that the switch between c and b closes after the switch between c and a opens. The symbol for the break-before-make switch is shown in Figure 2.9-2*a*. The make-before-break switch is manufactured so that the switch between c and b closes before the switch between c and a opens. The symbol for the make-before-break switch is shown in Figure 2.9-2*b*. Remember: the switch transition from terminal a to terminal b is assumed to take place instantaneously. This instantaneous transition is an accurate model when the actual make-before-break transition is very fast compared to the circuit time response.

### EXAMPLE 2.9-1 Switches

Figure 2.9-3 illustrates the use of open and short circuits for modeling ideal switches. In Figure 2.9-3*a*, a circuit containing three switches is shown. In Figure 2.9-3*b*, the circuit is shown as it would be modeled before $t = 0$ s. The two single-pole, single-throw switches change state at time $t = 0$ s. Figure 2.9-3*c* shows the circuit as it would be modeled when the time is between 0 s and 2 s. The single-pole, double-throw switch changes state at time $t = 2$ s. Figure 2.9-3*d* shows the circuit as it would be modeled after 2 s.

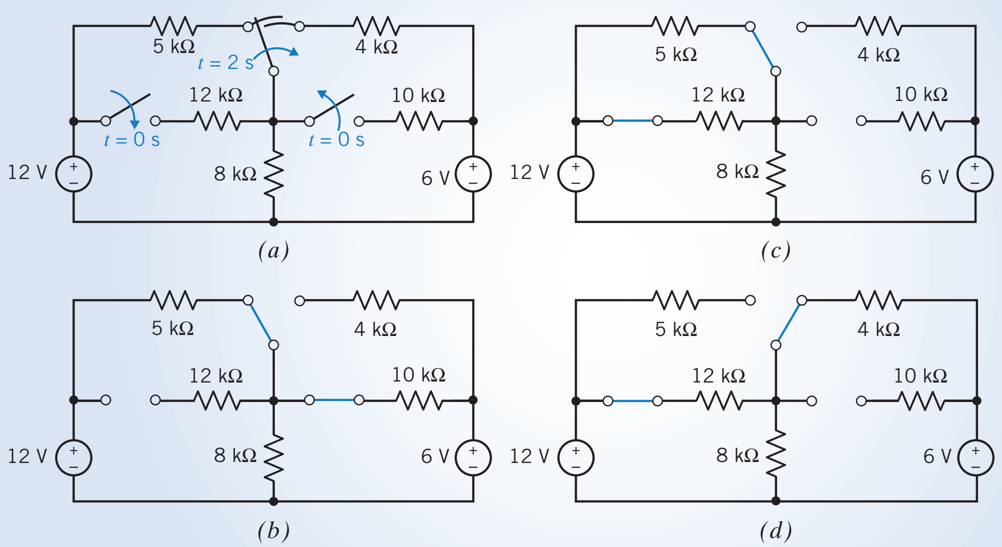


**FIGURE 2.9-3** (*a*) A circuit containing several switches. (*b*) The equivalent circuit for $t \leq 0$ s. (*c*) The equivalent circuit for $0 < t < 2$ s. (*d*) The equivalent circuit for $t > 2$ s.

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**EXERCISE 2.9-1** What is the value of the current $i$ in Figure E 2.9-1 at time $t = 4$ s?

*Answer:* $i = 0$ amperes at $t = 4$ s (both switches are open).

**EXERCISE 2.9-2** What is the value of the voltage $v$ in Figure E 2.9-2 at time $t = 4$ s? At $t = 6$ s?

*Answer:* $v = 6$ volts at $t = 4$ s, and $v = 0$ volts at $t = 6$ s.

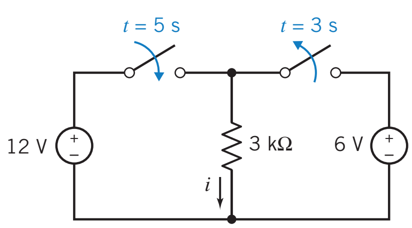



**FIGURE E 2.9-1** A circuit with two SPST switches.

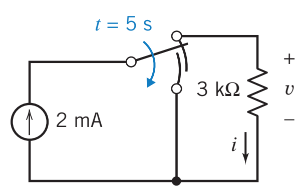



**FIGURE E 2.9-2** A circuit with a make-before-break SPDT switch.

## 2.10 *How Can We Check . . . ?*

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able quickly to identify those solutions that need more work.

The following example illustrates techniques useful for checking the solutions of the sort of problem discussed in this chapter.

## EXAMPLE 2.10-1 How Can We Check Voltage and Current Values?

The meters in the circuit of Figure 2.10-1 indicate that $v_1 = -4$ V, $v_2 = 8$ V and that $i = 1$ A. **How can we check** that the values of $v_1$, $v_2$, and $i$ have been measured correctly? Let's check the values of $v_1$, $v_2$, and $i$ in two ways:

**(a)** Verify that the given values satisfy Ohm's law for both resistors.

**(b)** Verify that the power supplied by the voltage source is equal to the power absorbed by the resistors.

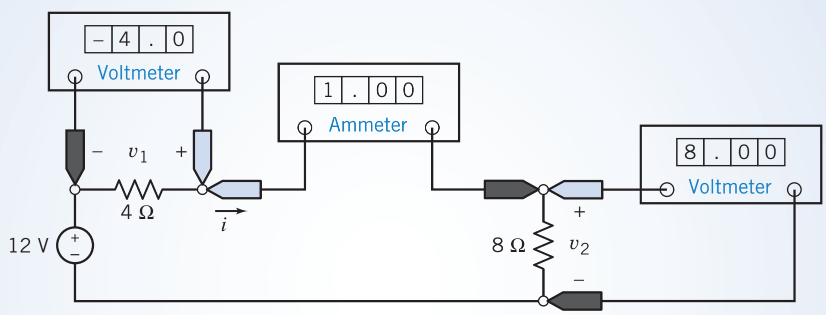



**FIGURE 2.10-1** A circuit with meters.

## Solution

**(a)** Consider the 8-Ω resistor. The current $i$ flows through this resistor from top to bottom. Thus, the current $i$ and the voltage $v_2$ adhere to the passive convention. Therefore, Ohm's law requires that $v_2 = 8i$. The values $v_2 = 8$ V and $i = 1$ A satisfy this equation.

Next, consider the 4-Ω resistor. The current $i$ flows through this resistor from left to right. Thus, the current $i$ and the voltage $v_1$ do not adhere to the passive convention. Therefore, Ohm's law requires that $v_1 = 4(-i)$. The values $v_1 = -4$ V and $i = 1$ A satisfy this equation.

Thus, Ohm's law is satisfied.

**(b)** The current $i$ flows through the voltage source from bottom to top. Thus the current $i$ and the voltage 12 V do not adhere to the passive convention. Therefore, $12i = 12(1) = 12$ W is the power supplied by the voltage source. The power absorbed by the 4-Ω resistor is $4i^2 = 4(1^2) = 4$ W, and the power absorbed by the 8-Ω resistor is $8i^2 = 8(1^2) = 8$ W. The power supplied by the voltage source is indeed equal to the power absorbed by the resistors.

## 2.11 Design Example Temperature Sensor

Currents can be measured easily, using ammeters. A temperature sensor, such as Analog Devices' AD590, can be used to measure temperature by converting temperature to current. Figure 2.11-1 shows a symbol used to represent a temperature sensor. For this sensor to operate properly, the voltage $v$ must satisfy the condition

$$4 \text{ volts } \leq v \leq 30 \text{ volts}$$

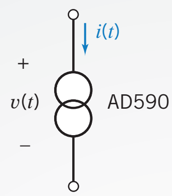



**FIGURE 2.11-1**
A temperature sensor.

When this condition is satisfied, the current $i$, in $\mu$A, is numerically equal to the temperature $T$, in °K. The phrase *numerically equal* indicates that the two variables have the same value but different units.

$$i = k \cdot T \quad \text{where} \quad k = 1 \frac{\mu\text{A}}{^\circ\text{K}}$$

The goal is to design a circuit using the AD590 to measure the temperature of a container of water. In addition to the AD590 and an ammeter, several power supplies and an assortment of standard 2 percent resistors are available. The power supplies are voltage sources. Power supplies having voltages of 10, 12, 15, 18, or 24 volts are available.

### Describe the Situation and the Assumptions

For the temperature transducer to operate properly, its element voltage must be between 4 volts and 30 volts. The power supplies and resistors will be used to establish this voltage. An ammeter will be used to measure the current in the temperature transducer.

The circuit must be able to measure temperatures in the range from 0°C to 100°C because water is a liquid at these temperatures. Recall that the temperature in °C is equal to the temperature in °K minus 273°.

### State the Goal

Use the power supplies and resistors to cause the voltage $v$ of the temperature transducer to be between 4 volts and 30 volts.

Use an ammeter to measure the current, $i$, in the temperature transducer.

### Generate a Plan

Model the power supply as an ideal voltage source and the temperature transducer as an ideal current source. The circuit shown in Figure 2.11-2*a* causes the voltage across the temperature transducer to be equal to the power supply voltage. Because all of the available power supplies have voltages between 4 volts and 30 volts, any one of the power supplies can be used. Notice that the resistors are not needed.

In Figure 2.11-2*b*, a short circuit has been added in a way that does not disturb the network. In Figure 2.11-2*c*, this short circuit has been replaced with an (ideal) ammeter. Because the ammeter will measure the current in the temperature transducer, the ammeter reading will be numerically equal to the temperature in °K.

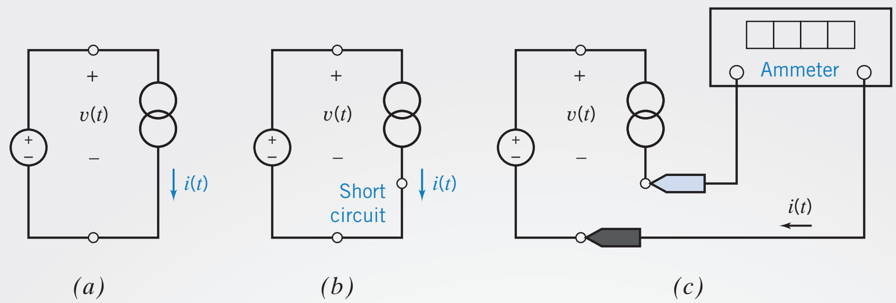



**FIGURE 2.11-2** (*a*) Measuring temperature with a temperature sensor. (*b*) Adding a short circuit. (*c*) Replacing the short circuit by an ammeter.

Although any of the available power supplies is adequate to meet the specifications, there may still be an advantage to choosing a particular power supply. For example, it is reasonable to choose the power supply that causes the transducer to absorb as little power as possible.

## Act on the Plan

The power absorbed by the transducer is

$$p = v \cdot i$$

where $v$ is the power supply voltage. Choosing $v$ as small as possible, 10 volts in this case, makes the power absorbed by the temperature transducer as small as possible. Figure 2.11-3*a* shows the final design. Figure 2.11-3*b* shows a graph that can be used to find the temperature corresponding to any ammeter current.

## Verify the Proposed Solution

Let's try an example. Suppose the temperature of the water is 80.6°F. This temperature is equal to 27°C or 300°K. The current in the temperature sensor will be

$$i = \left(1 \frac{\mu\text{A}}{^\circ\text{K}}\right) 300^\circ\text{K} = 300\ \mu\text{A}$$

Next, suppose that the ammeter in Figure 2.11-3*a* reads 300 $\mu$A. A sensor current of 300 $\mu$A corresponds to a temperature of

$$T = \frac{300\ \mu\text{A}}{1 \dfrac{\mu\text{A}}{^\circ\text{K}}} = 300^\circ\text{K} = 27^\circ\text{C} = 80.6^\circ\text{F}$$

The graph in Figure 2.11-3*b* indicates that a sensor current of 300 $\mu$A does correspond to a temperature of 27°C.

This example shows that the circuit is working properly.

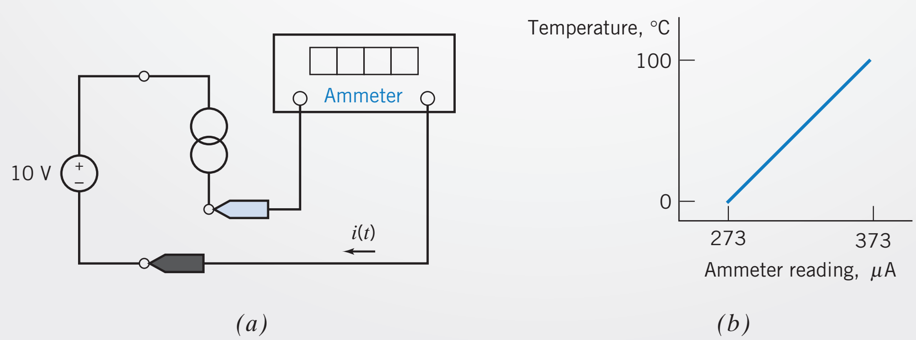



**FIGURE 2.11-3** (*a*) Final design of a circuit that measures temperature with a temperature sensor. (*b*) Graph of temperature versus ammeter current.

## 2.12 SUMMARY

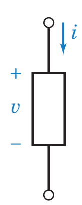
- The engineer uses models, called circuit elements, to represent the devices that make up a circuit. In this book, we consider only linear elements or linear models of devices. A device is linear if it satisfies the properties of both superposition and homogeneity.
- The relationship between the reference directions of the current and voltage of a circuit element is important. The voltage polarity marks one terminal + and the other −. The element voltage and current adhere to the passive convention if the current is directed from the terminal marked + to the terminal marked −.
- Resistors are widely used as circuit elements. When the resistor voltage and current adhere to the passive convention, resistors obey Ohm's law; the voltage across the terminals of the resistor is related to the current into the positive terminal as $v = Ri$. The power delivered to a resistance is $p = i^2R = v^2/R$ watts.
- An independent source provides a current or a voltage independent of other circuit variables. The voltage of an independent voltage source is specified, but the current is not. Conversely, the current of an independent current source is specified whereas the voltage is not. The voltages of independent voltage sources and currents of independent current sources are frequently used as the inputs to electric circuits.
- A dependent source provides a current (or a voltage) that is dependent on another variable elsewhere in the circuit. The constitutive equations of dependent sources are summarized in Table 2.7-1.
- The **short circuit** and **open circuit** are special cases of independent sources. A **short circuit** is an ideal voltage source having $v(t) = 0$. The current in a short circuit is determined by the rest of the circuit. An **open circuit** is an ideal current source having $i(t) = 0$. The voltage across an open circuit is determined by the rest of the circuit. Open circuits and short circuits can also be described as special cases of resistors. A resistor with resistance $R = 0$ ($G = \infty$) is a short circuit. A resistor with conductance $G = 0$ ($R = \infty$) is an open circuit.
- An ideal ammeter measures the current flowing through its terminals and has zero voltage across its terminals. An ideal voltmeter measures the voltage across its terminals and has terminal current equal to zero. Ideal voltmeters act like open circuits, and ideal ammeters act like short circuits.
- Transducers are devices that convert physical quantities, such as rotational position, to an electrical quantity such as voltage. In this chapter, we describe two transducers: potentiometers and temperature sensors.
- Switches are widely used in circuits to connect and disconnect elements and circuits. An open switch is modeled as an open circuit and a closed switch is modeled as a short circuit.

## PROBLEMS

⊕ Problem available in WileyPLUS at instructor's discretion.

### Section 2.2 Engineering and Linear Models

**P 2.2-1** An element has voltage $v$ and current $i$ as shown in Figure P 2.2-1*a*. Values of the current $i$ and corresponding voltage $v$ have been tabulated as shown in Figure P 2.2-1*b*. Determine whether the element is linear.




(*a*)

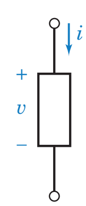
| $v$, V | $i$, A |
|---|---|
| −3 | −3 |
| −4 | −2 |
| 0 | 0 |
| 12 | 2 |
| 32 | 4 |
| 60 | 6 |

(*b*)

**Figure P 2.2-1**

**P 2.2-2** ⊕ A linear element has voltage $v$ and current $i$ as shown in Figure P 2.2-2*a*. Values of the current $i$ and corresponding voltage $v$ have been tabulated as shown in Figure P 2.2-2*b*. Represent the element by an equation that expresses $v$ as a function of $i$. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of $v$ corresponding to a current of $i = 40$ mA. (c) Use the model to predict the value of $i$ corresponding to a voltage of $v = 3$ V.

***Hint:*** Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.




(*a*)

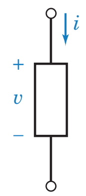
| $v$, V | $i$, A |
|---|---|
| −3.6 | −30 |
| 2.4 | 20 |
| 6.0 | 50 |

(*b*)

**Figure P 2.2-2**

**P 2.2-3** A linear element has voltage $v$ and current $i$ as shown in Figure P 2.2-3*a*. Values of the current $i$ and corresponding voltage $v$ have been tabulated as shown in Figure P 2.2-3*b*. Represent the element by an equation that expresses $v$ as a function of $i$. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of $v$ corresponding to a current of $i = 6$ mA. (c) Use the model to predict the value of $i$ corresponding to a voltage of $v = 12$ V.

***Hint:*** Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.




| $v$, V | $i$, mA |
|---|---|
| 3.078 | 12 |
| 5.13 | 20 |
| 12.825 | 50 |

(a) (b)

**Figure P 2.2-3**

**P 2.2-4** An element is represented by the relation between current and voltage as

$$v = 3i + 5$$

Determine whether the element is linear.

**P 2.2-5** The circuit shown in Figure P 2.2-5 consists of a current source, a resistor, and element A. Consider three cases.

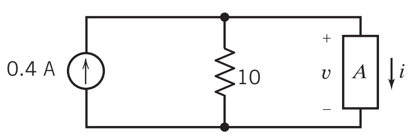



**Figure P 2.2-5**

**(a)** When element A is a 40-Ω resistor, described by $i = v/40$, then the circuit is represented by

$$0.4 = \frac{v}{10} + \frac{v}{40}$$

Determine the values of $v$ and $i$. Notice that the above equation has a unique solution.

**(b)** When element A is a nonlinear resistor described by $i = v^2/2$, then the circuit is represented by

$$0.4 = \frac{v}{10} + \frac{v^2}{2}$$

Determine the values of $v$ and $i$. In this case, there are two solutions of the above equation. Nonlinear circuits exhibit more complicated behavior than linear circuits.

**(c)** When element A is a nonlinear resistor described by $i = 0.8 + \frac{v^2}{2}$, then the circuit is described by

$$0.4 = \frac{v}{10} + 0.8 + \frac{v^2}{2}$$

Show that this equation has no solution. This result usually indicates a modeling problem. At least one of the three elements in the circuit has not been modeled accurately.

## Section 2.4 Resistors

**P 2.4-1** ⊕ A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that $i_s = 3$ A and $R = 7$ Ω. Calculate the voltage $v$ across the resistor and the power absorbed by the resistor.

***Answer:*** $v = 21$ V and the resistor absorbs 63 W.

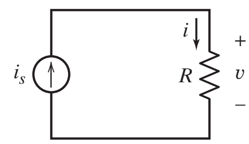



**Figure P 2.4-1**

**P 2.4-2** ⊕ A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that $i = 3$ mA and $v = 48$ V. Calculate the resistance $R$ and the power absorbed by the resistor.

**P 2.4-3** ⊕ A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that $v_s = 10$ V and $R = 5$ Ω. Calculate the current $i$ in the resistor and the power absorbed by the resistor.

***Answer:*** $i = 2$ A and the resistor absorbs 20 W.

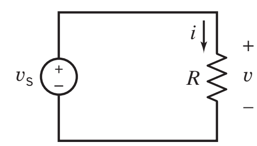



**Figure P 2.4-3**

**P 2.4-4** ⊕ A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that $v_s = 24$ V and $i = 3$ A. Calculate the resistance $R$ and the power absorbed by the resistor.

**P 2.4-5** ⊕ A voltage source and two resistors are connected in parallel in the circuit shown in Figure P 2.4-5. Elements connected in parallel have the same voltage, so $v_1 = v_s$ and $v_2 = v_s$ in this circuit. Suppose that $v_s = 150$ V, $R_1 = 50$ Ω, and $R_2 = 25$ Ω. Calculate the current in each resistor and the power absorbed by each resistor.

***Hint:*** Notice the reference directions of the resistor currents.

***Answer:*** $i_1 = 3$ A and $i_2 = -6$ A. $R_1$ absorbs 450 W and $R_2$ absorbs 900 W.

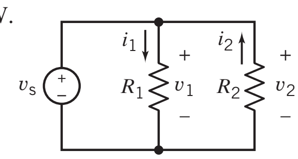



**Figure P 2.4-5**

**P 2.4-6** A current source and two resistors are connected in series in the circuit shown in Figure P 2.4-6. Elements connected in series have the same current, so $i_1 = i_s$ and $i_2 = i_s$ in this circuit. Suppose that $i_s = 25$ mA, $R_1 = 4\ \Omega$, and $R_2 = 8\ \Omega$. Calculate the voltage across each resistor and the power absorbed by each resistor.

*Hint:* Notice the reference directions of the resistor voltages.

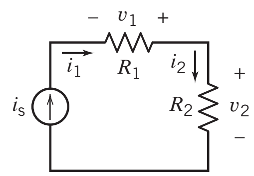



**Figure P 2.4-6**

**P 2.4-7** An electric heater is connected to a constant 250-V source and absorbs 1000 W. Subsequently, this heater is connected to a constant 220-V source. What power does it absorb from the 220-V source? What is the resistance of the heater?

*Hint:* Model the electric heater as a resistor.

**P 2.4-8** The portable lighting equipment for a mine is located 100 meters from its dc supply source. The mine lights use a total of 5 kW and operate at 120 V dc. Determine the required cross-sectional area of the copper wires used to connect the source to the mine lights if we require that the power lost in the copper wires be less than or equal to 5 percent of the power required by the mine lights.

*Hint:* Model both the lighting equipment and the wire as resistors.

**P 2.4-9** The resistance of a practical resistor depends on the nominal resistance and the resistance tolerance as follows:

$$R_{nom}\left(1 - \frac{t}{100}\right) \leq R \leq R_{nom}\left(1 + \frac{t}{100}\right)$$

where $R_{nom}$ is the nominal resistance and $t$ is the resistance tolerance expressed as a percentage. For example, a 100-Ω, 2 percent resistor will have a resistance given by

$$98\ \Omega \leq R \leq 102\ \Omega$$

The circuit shown in Figure P 2.4-9 has one input, $v_s$, and one output, $v_o$. The gain of this circuit is given by

$$\text{gain} = \frac{v_o}{v_s} = \frac{R_2}{R_1 + R_2}$$

Determine the range of possible values of the gain when $R_1$ is the resistance of a 100-Ω, 2 percent resistor and $R_2$ is the resistance of a 400-Ω, 5 percent resistor. Express the gain in terms of a nominal gain and a gain tolerance.

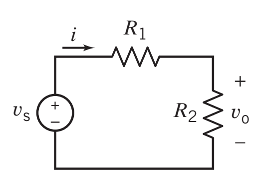
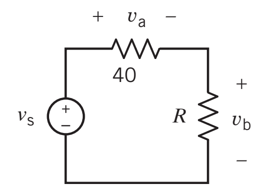



**Figure P 2.4-9**

**P 2.4-10** The voltage source shown in Figure P 2.4-10 is an adjustable dc voltage source. In other words, the voltage $v_s$ is a constant voltage, but the value of that constant can be adjusted. The tabulated data were collected as follows. The voltage, $v_s$, was set to some value, and the voltages across the resistor, $v_a$ and $v_b$, were measured and recorded. Next, the value of $v_s$ was changed, and the voltages across the resistors were measured again and recorded. This procedure was repeated several times. (The values of $v_s$ were not recorded.) Determine the value of the resistance, $R$.




| $v_a$, V | $v_b$, V |
|---|---|
| 11.75 | 7.05 |
| 7.5 | 4.5 |
| 5.625 | 3.375 |
| 10 | 6 |
| 4.375 | 2.625 |

**Figure P 2.4-10**

**P 2.4-11** Consider the circuit shown in Figure P2.4-11.

**(a)** Suppose the current source supplies 3.125 W of power. Determine the value of the resistance $R$.

**(b)** Suppose instead the resistance is $R = 12\ \Omega$. Determine the value of the power supplied by the current source.

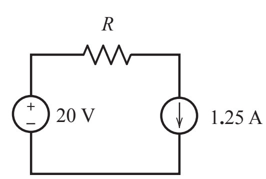



**Figure P 2.4-11**

**P 2.4-12** We will encounter "ac circuits" in Chapter 10. Frequently we analyze ac circuits using "phasors" and "impedances." Phasors are complex numbers that represent currents and voltages in an ac circuit. Impedances are complex numbers that describe ac circuit elements. (See Appendix B for a discussion of complex numbers.) Figure P 2.4-11 shows a circuit element in an ac circuit. **I** and **V** are complex numbers representing the element current and voltage. **Z** is a complex number describing the element itself. "Ohm's law for ac circuits" indicates that

$$\mathbf{V} = \mathbf{Z}\,\mathbf{I}$$

(a) Suppose $\mathbf{V} = 12\angle 45^\circ$ V, $\mathbf{I} = B\angle\theta$ A, and $\mathbf{Z} = 18 + j\,8\ \Omega$. Determine the values of $B$ and $\theta$.
(b) Suppose $\mathbf{V} = 48\angle 135^\circ$ V, $\mathbf{I} = 3\angle 15^\circ$ A, and $\mathbf{Z} = R + j\,X\ \Omega$. Determine the values of $R$ and $X$.

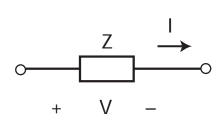



**Figure P 2.4-12**

## Section 2.5 Independent Sources

**P 2.5-1** A current source and a voltage source are connected in parallel with a resistor as shown in Figure P 2.5-1. All of the elements connected in parallel have the same voltage $v_s$ in this circuit. Suppose that $v_s = 15$ V, $i_s = 3$ A, and $R = 5\ \Omega$. (a) Calculate the current $i$ in the resistor and the power absorbed by the resistor. (b) Change the current source current to $i_s = 5$ A and recalculate the current $i$ in the resistor and the power absorbed by the resistor.

***Answer:*** $i = 3$ A and the resistor absorbs 45 W both when $i_s = 3$ A and when $i_s = 5$ A.

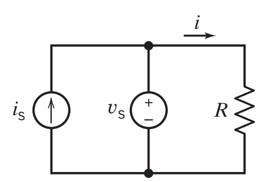



**Figure P 2.5-1**

**P 2.5-2** A current source and a voltage source are connected in series with a resistor as shown in Figure P 2.5-2. All of the elements connected in series have the same current $i_s$ in this circuit. Suppose that $v_s = 10$ V, $i_s = 3$ A, and $R = 5\ \Omega$. (a) Calculate the voltage $v$ across the resistor and the power absorbed by the resistor. (b) Change the voltage source voltage to $v_s = 5$ V and recalculate the voltage, $v$, across the resistor and the power absorbed by the resistor.

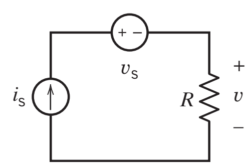



**Figure P 2.5-2**

**P 2.5-3** The current source and voltage source in the circuit shown in Figure P 2.5-3 are connected in parallel so that they both have the same voltage, $v_s$. The current source and voltage source are also connected in series so that they both have the same current, $i_s$. Suppose that $v_s = 12$ V and $i_s = 3$ A. Calculate the power supplied by each source.

***Answer:*** The voltage source supplies −36 W, and the current source supplies 36 W.

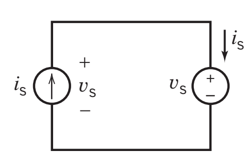



**Figure P 2.5-3**

**P 2.5-4** The current source and voltage source in the circuit shown in Figure P 2.5-4 are connected in parallel so that they both have the same voltage, $v_s$. The current source and voltage source are also connected in series so that they both have the same current, $i_s$. Suppose that $v_s = 12$ V and $i_s = 2$ A. Calculate the power supplied by each source.

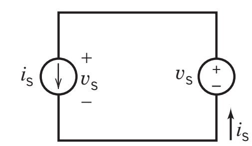



**Figure P 2.5-4**

**P 2.5-5**

**(a)** Find the power supplied by the voltage source shown in Figure P 2.5-5 when for $t \geq 0$ we have

$$v = 2\cos t \text{ V}$$

and

$$i = 10\cos t \text{ mA}$$

**(b)** Determine the energy supplied by this voltage source for the period $0 \leq t \leq 1$ s.

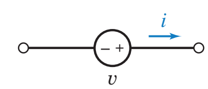



**Figure P 2.5-5**

**P 2.5-6** Figure P 2.5-6 shows a battery connected to a load. The load in Figure P 2.5-6 might represent automobile headlights, a digital camera, or a cell phone. The energy supplied by the battery to load is given by

$$w = \int_{t_1}^{t_2} vi\,dt$$

When the battery voltage is constant and the load resistance is fixed, then the battery current will be constant and

$$w = vi(t_2 - t_1)$$

The capacity of a battery is the product of the battery current and time required to discharge the battery. Consequently, the energy stored in a battery is equal to the product of the battery voltage and the battery capacity. The capacity is usually given with the units of Ampere-hours (Ah). A new 12-V battery having a capacity of 800 mAh is connected to a load that draws a current of 25 mA. (a) How long will it take for the load to discharge the battery? (b) How much energy will be supplied to the load during the time required to discharge the battery?

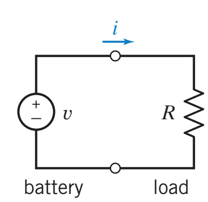



**Figure P 2.5-6**

## Section 2.6 Voltmeters and Ammeters

**P 2.6-1** For the circuit of Figure P 2.6-1:

**(a)** What is the value of the resistance $R$?
**(b)** How much power is delivered by the voltage source?

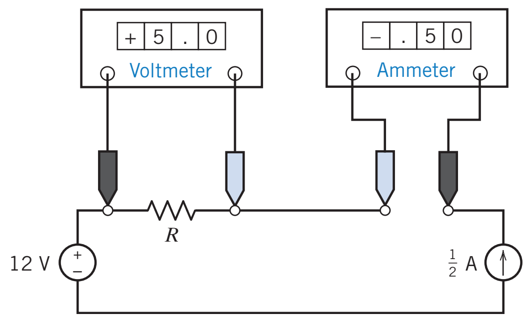



**Figure P 2.6-1**

**P 2.6-2** The current source in Figure P 2.6-2 supplies 40 W. What values do the meters in Figure P 2.6-2 read?

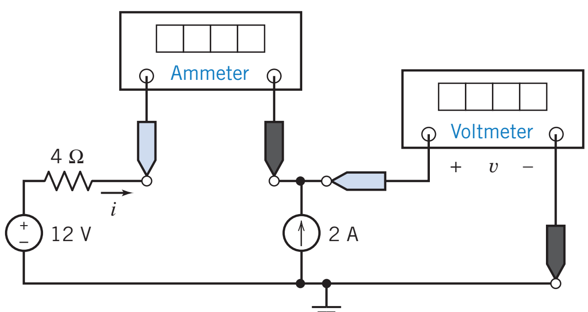



**Figure P 2.6-2**

**P 2.6-3** An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 2.6-3*a* shows a circuit with a voltmeter that measures the voltage $v_m$. In Figure P 2.6-3*b*, the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. Ideally, there is no current in the 100-Ω resistor, and the voltmeter measures $v_{mi}$ = 12 V, the ideal value of $v_m$. In Figure P 2.6-3*c*, the voltmeter is modeled by the resistance $R_m$. Now the voltage measured by the voltmeter is

$$v_m = \left(\frac{R_m}{R_m + 100}\right)12$$

Because $R_m \to \infty$, the voltmeter becomes an ideal voltmeter, and $v_m \to v_{mi} = 12$ V . When $R_m < \infty$, the voltmeter is not ideal, and $v_m < v_{mi}$. The difference between $v_m$ and $v_{mi}$ is a measurement error caused by the fact that the voltmeter is not ideal.

**(a)** Express the measurement error that occurs when $R_m = 900$ Ω as a percent of $v_{mi}$.
**(b)** Determine the minimum value of $R_m$ required to ensure that the measurement error is smaller than 2 percent of $v_{mi}$.

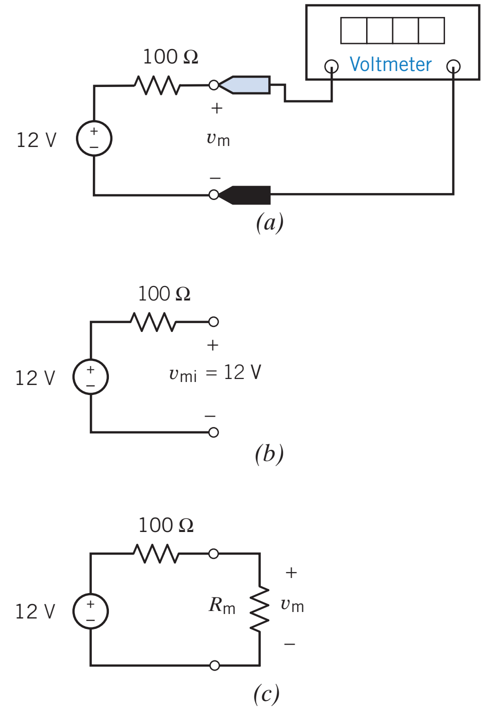


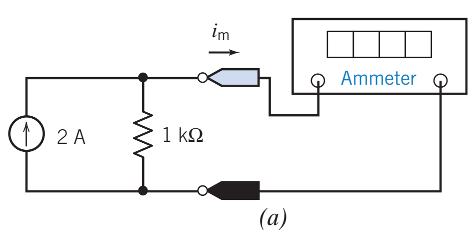

**Figure P 2.6-3**

**P 2.6-4** An ideal ammeter is modeled as a short circuit. A more realistic model of an ammeter is a small resistance. Figure P 2.6-4*a* shows a circuit with an ammeter that measures the current $i_m$. In Figure P 2.6-4*b*, the ammeter is replaced by the model of an ideal ammeter, a short circuit. Ideally, there is no voltage across the 1-kΩ resistor, and the ammeter measures $i_{mi} = 2$ A, the ideal value of $i_m$. In Figure P 2.6-4*c*, the ammeter is modeled by the resistance $R_m$. Now the current measured by the ammeter is

$$i_m = \left(\frac{1000}{1000 + R_m}\right)2$$

As $R_m \to 0$, the ammeter becomes an ideal ammeter, and $i_m \to i_{mi} = 2$ A. When $R_m > 0$, the ammeter is not ideal, and $i_m < i_{mi}$. The difference between $i_m$ and $i_{mi}$ is a measurement error caused by the fact that the ammeter is not ideal.

**(a)** Express the measurement error that occurs when $R_m$ = 10 Ω as a percent of $i_{mi}$.
**(b)** Determine the maximum value of $R_m$ required to ensure that the measurement error is smaller than 5 percent.



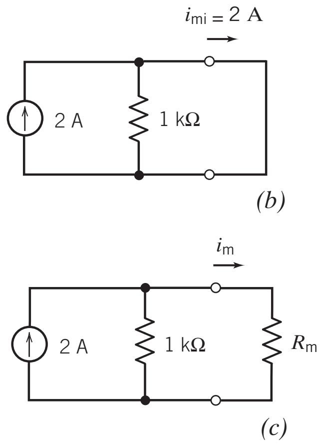



**Figure P 2.6-4**

**P 2.6-5** The voltmeter in Figure P 2.6-5*a* measures the voltage across the current source. Figure P 2.6-5*b* shows the circuit after removing the voltmeter and labeling the voltage measured by the voltmeter as $v_m$. Also, the other element voltages and currents are labeled in Figure P 2.6-5*b*.

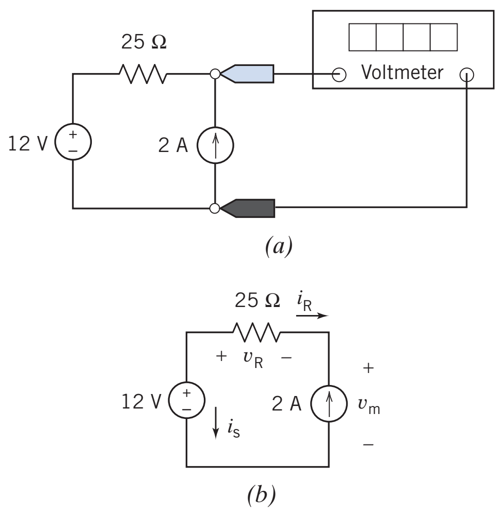



**Figure P 2.6-5**

Given that

$$12 = v_R + v_m \text{ and } -i_R = i_s = 2\text{ A}$$

and

$$v_R = 25 i_R$$

**(a)** Determine the value of the voltage measured by the meter.
**(b)** Determine the power supplied by each element.

**P 2.6-6** The ammeter in Figure P 2.6-6*a* measures the current in the voltage source. Figure P 2.6-6*b* shows the circuit after removing the ammeter and labeling the current measured by the ammeter as $i_m$. Also, the other element voltages and currents are labeled in Figure P 2.6-6*b*.

Given that

$$2 + i_m = i_R \text{ and } v_R = v_s = 12\text{ V}$$

and

$$v_R = 25 i_R$$

**(a)** Determine the value of the current measured by the meter.
**(b)** Determine the power supplied by each element.

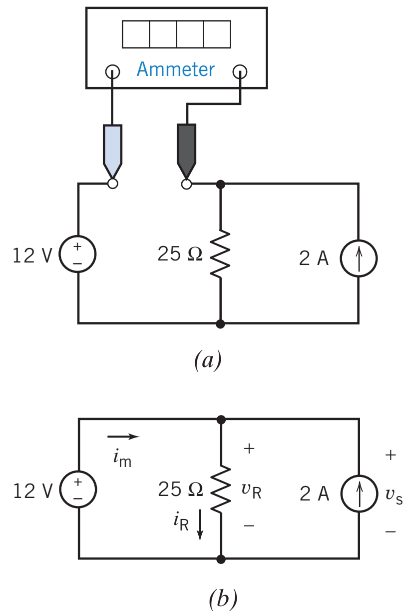



**Figure P 2.6-6**

## Section 2.7 Dependent Sources

**P 2.7-1** The ammeter in the circuit shown in Figure P 2.7-1 indicates that $i_a = 2$ A, and the voltmeter indicates that $v_b = 8$ V. Determine the value of $r$, the gain of the CCVS.

***Answer:*** $r = 4$ V/A

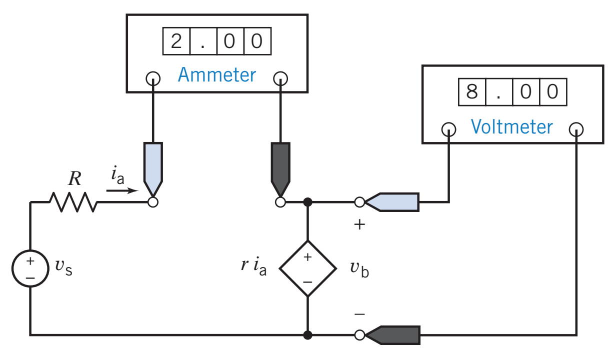



**Figure P 2.7-1**

**P 2.7-2** The ammeter in the circuit shown in Figure P 2.7-2 indicates that $i_a = 2$ A, and the voltmeter indicates that $v_b = 8$ V. Determine the value of $g$, the gain of the VCCS.

*Answer:* $g = 0.25$ A/V

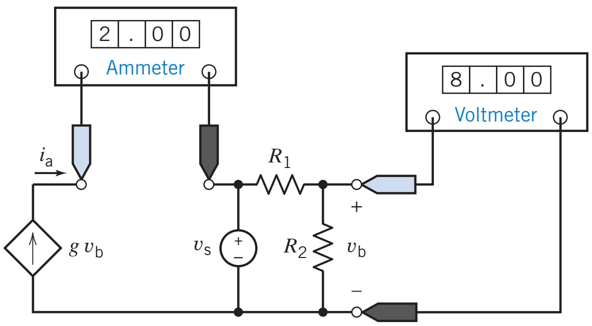



**Figure P 2.7-2**

**P 2.7-3** The ammeters in the circuit shown in Figure P 2.7-3 indicate that $i_a = 32$ A and $i_b = 8$ A. Determine the value of $d$, the gain of the CCCS.

*Answer:* $d = 4$ A/A

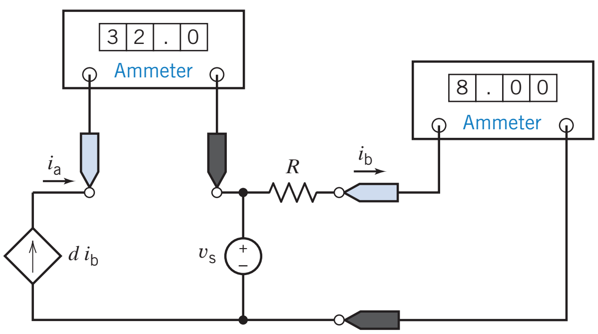



**Figure P 2.7-3**

**P 2.7-4** The voltmeters in the circuit shown in Figure P 2.7-4 indicate that $v_a = 2$ V and $v_b = 8$ V. Determine the value of $b$, the gain of the VCVS.

*Answer:* $b = 4$ V/V

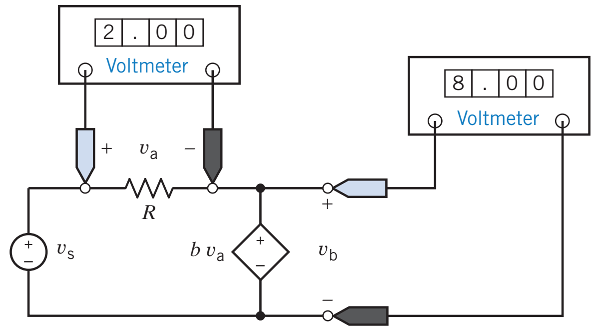



**Figure P 2.7-4**

**P 2.7-5** The values of the current and voltage of each circuit element are shown in Figure P 2.7-5.

Determine the values of the resistance $R$ and of the gain of the dependent source $A$.

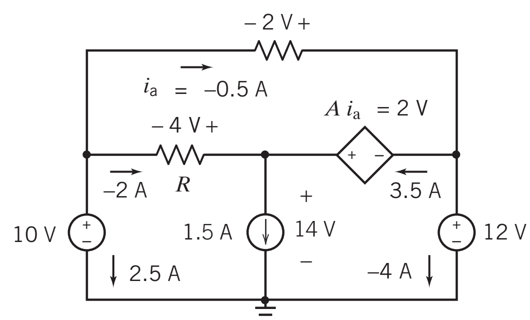



**Figure P 2.7-5**

**P 2.7-6** Find the power supplied by the VCCS in Figure P 2.7-6.

*Answer:* 17.6 watts are supplied by the VCCS. (–17.6 watts are absorbed by the VCCS.)

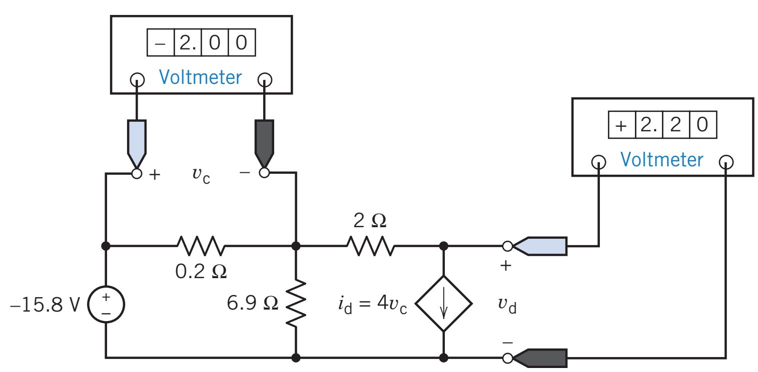



**Figure P 2.7-6**

**P 2.7-7** ⊕ The circuit shown in Figure P 2.7-7 contains a dependent source. Determine the value of the gain $k$ of that dependent source.

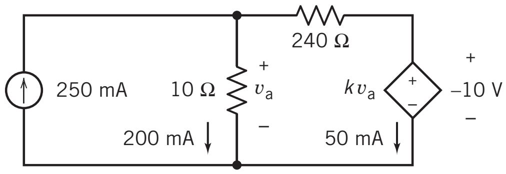



**Figure P 2.7-7**

**P 2.7-8** The circuit shown in Figure P 2.7-8 contains a dependent source. Determine the value of the gain $k$ of that dependent source.

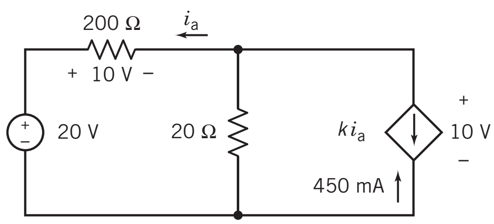



**Figure P 2.7-8**

**P 2.7-9** The circuit shown in Figure P 2.7-9 contains a dependent source. The gain of that dependent source is

$$k = 25\frac{\text{V}}{\text{A}}$$

Determine the value of the voltage $v_b$.

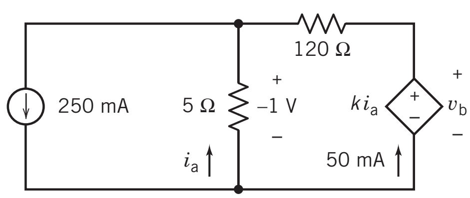



**Figure P 2.7-9**

**P 2.7-10** The circuit shown in Figure P 2.7-10 contains a dependent source. The gain of that dependent source is

$$k = 90\frac{\text{mA}}{\text{V}} = 0.09\frac{\text{A}}{\text{V}}$$

Determine the value of the current $i_b$.

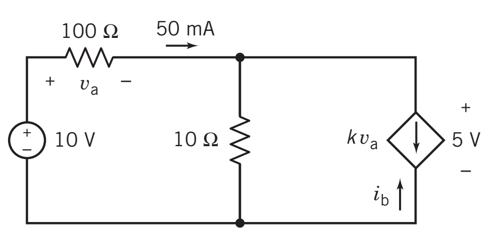



**Figure P 2.7-10**

## Section 2.8 Transducers

**P 2.8-1** For the potentiometer circuit of Figure 2.8-2, the current source current and potentiometer resistance are 1.1 mA and 100 kΩ, respectively. Calculate the required angle, $\theta$, so that the measured voltage is 23 V.

**P 2.8-2** An AD590 sensor has an associated constant $k = 1\frac{\mu A}{°K}$. The sensor has a voltage $v = 20$ V; and the measured current, $i(t)$, as shown in Figure 2.8-3, is $4\ \mu A < i < 13\ \mu A$ in a laboratory setting. Find the range of measured temperature.

## Section 2.9 Switches

**P 2.9-1** ⊕ Determine the current $i$ at $t = 1$ s and at $t = 4$ s for the circuit of Figure P 2.9-1.

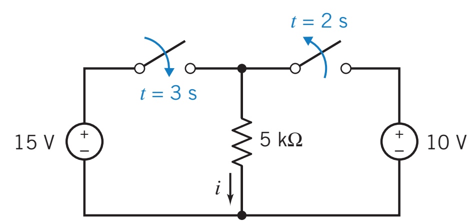



**Figure P 2.9-1**

**P 2.9-2** ⊕ Determine the voltage, $v$, at $t = 1$ s and at $t = 4$ s for the circuit shown in Figure P 2.9-2.

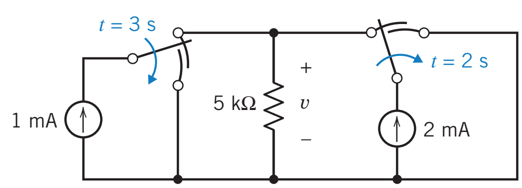



**Figure P 2.9-2**

**P 2.9-3** Ideally, an open switch is modeled as an open circuit and a closed switch is modeled as a closed circuit. More realistically, an open switch is modeled as a large resistance, and a closed switch is modeled as a small resistance.

Figure P 2.9-3*a* shows a circuit with a switch. In Figure P 2.9-3*b*, the switch has been replaced with a resistance. In Figure P 2.9-3*b*, the voltage $v$ is given by

$$v = \left(\frac{100}{R_s + 100}\right)12$$

Determine the value of $v$ for each of the following cases.

**(a)** The switch is closed and $R_s = 0$ (a short circuit).
**(b)** The switch is closed and $R_s = 5\ \Omega$.
**(c)** The switch is open and $R_s = \infty$ (an open circuit).
**(d)** The switch is open and $R_s = 10\ \text{k}\Omega$.

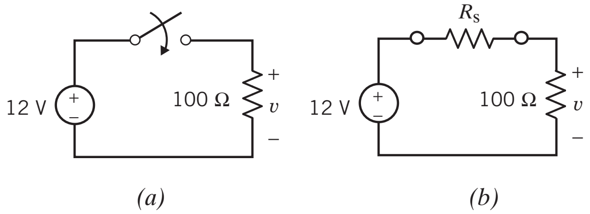



**Figure P 2.9-3**

### Section 2-10 How Can We Check . . . ?

**P 2.10-1** The circuit shown in Figure P 2.10-1 is used to test the CCVS. Your lab partner claims that this measurement shows that the gain of the CCVS is −20 V/A instead of +20 V/A. Do you agree? Justify your answer.

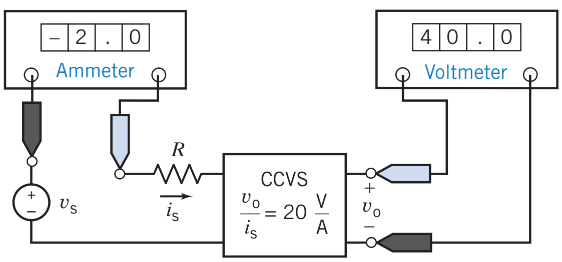



**Figure P 2.10-1**

**P 2.10-2** The circuit of Figure P 2.10-2 is used to measure the current in the resistor. Once this current is known, the resistance can be calculated as $R = \frac{v_s}{i}$. The circuit is constructed using a voltage source with $v_s$ = 12 V and a 25-Ω, 1/2-W resistor. After a puff of smoke and an unpleasant smell, the ammeter indicates that $i = 0$ A. The resistor must be bad. You have more 25-Ω, 1/2-W resistors. Should you try another resistor? Justify your answer.

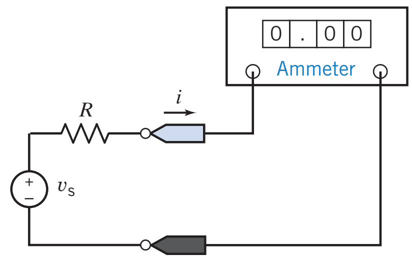



**Figure P 2.10-2**

*Hint:* 1/2-W resistors are able to safely dissipate one 1/2 W of power. These resistors may fail if required to dissipate more than 1/2 watt of power.

## Design Problems

**DP 2-1** Specify the resistance $R$ in Figure DP 2-1 so that both of the following conditions are satisfied:

1. $i > 40$ mA.
2. The power absorbed by the resistor is less than 0.5 W.

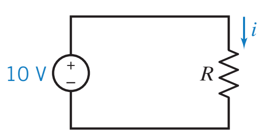



**Figure DP 2-1**

**DP 2-2** Specify the resistance $R$ in Figure DP 2-2 so that both of the following conditions are satisfied:

1. $v > 40$ V.
2. The power absorbed by the resistor is less than 15 W.

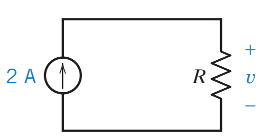



**Figure DP 2-2**

*Hint:* There is no guarantee that specifications can always be satisfied.

**DP 2-3** Resistors are given a power rating. For example, resistors are available with ratings of 1/8 W, 1/4 W, 1/2 W, and 1 W. A 1/2-W resistor is able to safely dissipate 1/2 W of power, indefinitely. Resistors with larger power ratings are more expensive and bulkier than resistors with lower power ratings. Good engineering practice requires that resistor power ratings be specified to be as large as, but not larger than, necessary.

Consider the circuit shown in Figure DP 2-3. The values of the resistances are

$$R_1 = 1000\ \Omega,\ R_2 = 2000\ \Omega,\ \text{and}\ R_3 = 4000\ \Omega$$

The value of the current source current is

$$i_s = 30\text{ mA}$$

Specify the power rating for each resistor.

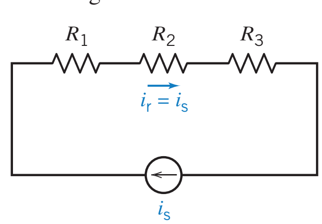



**Figure DP 2-3**

# CHAPTER 3 Resistive Circuits

## IN THIS CHAPTER



## 3.1 *Introduction*

In this chapter, we will do the following:

- Write equations using Kirchhoff's laws.

  Not surprisingly, the behavior of an electric circuit is determined both by the types of elements that comprise the circuit and by the way those elements are connected together. The constitutive equations describe the elements themselves, and Kirchhoff's laws describe the way the elements are connected to each other to form the circuit.
- Analyze simple electric circuits, using only Kirchhoff's laws and the constitutive equations of the circuit elements.
- Analyze two very common circuit configurations: series resistors and parallel resistors.

  We will see that series resistors act like a "voltage divider," and parallel resistors act like a "current divider." Also, series resistors and parallel resistors provide our first examples of an "equivalent circuit." Figure 3.1-1 illustrates this important concept. Here, a circuit has been partitioned into two parts, *A* and *B*. Replacing *B* by an equivalent circuit, $B_{eq}$, does not change the current or voltage of any circuit element in part *A*. It is in this sense that $B_{eq}$ is equivalent to *B*. We will see how to obtain an equivalent circuit when part *B* consists either of series resistors or of parallel resistors.
- Determine equivalent circuits for series voltage sources and parallel current sources.
- Determine the equivalent resistance of a resistive circuit.

Often, circuits consisting entirely of resistors can be reduced to a single equivalent resistor by repeatedly replacing series and/or parallel resistors by equivalent resistors.

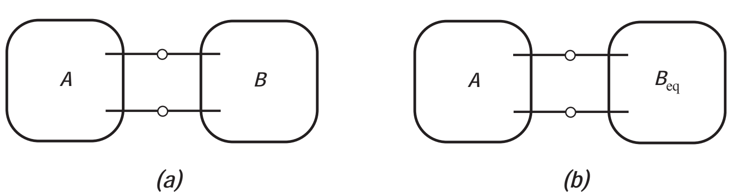



**FIGURE 3.1-1** Replacing *B* by an equivalent circuit $B_{eq}$ does not change the current or voltage of any circuit element in *A*.

## 3.2 *Kirchhoff's Laws*

An electric circuit consists of circuit elements that are connected together. The places where the elements are connected to each other are called nodes. Figure 3.2-1*a* shows an electric circuit that consists of six elements connected together at four nodes. It is common practice to draw electric circuits using straight lines and to position the elements horizontally or vertically as shown in Figure 3.2-1*b*.

The circuit is shown again in Figure 3.2-1*c*, this time emphasizing the nodes. Notice that redrawing the circuit, using straight lines and horizontal and vertical elements, has changed the way that the nodes are represented. In Figure 3.2-1*a*, nodes are represented as points. In Figures 3.2-1*b*,*c*, nodes are represented using both points and straight-line segments.

The same circuit can be drawn in several ways. One drawing of a circuit might look much different from another drawing of the same circuit. How can we tell when two circuit drawings represent the same circuit? Informally, we say that two circuit drawings represent the same circuit if

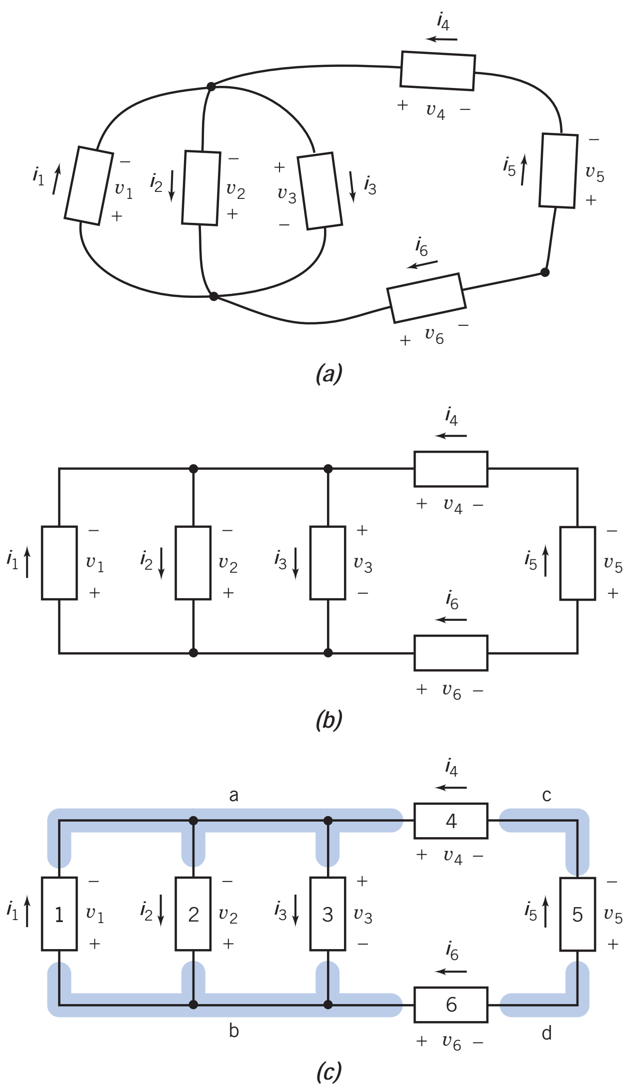



**FIGURE 3.2-1** (*a*) An electric circuit. (*b*) The same circuit, redrawn using straight lines and horizontal and vertical elements. (*c*) The circuit after labeling the nodes and elements.

corresponding elements are connected to corresponding nodes. More formally, we say that circuit drawings A and B represent the same circuit when the following three conditions are met.

1. There is a one-to-one correspondence between the nodes of drawing A and the nodes of drawing B. (A one-to-one correspondence is a matching. In this one-to-one correspondence, each node in drawing A is matched to exactly one node of drawing B and vice versa. The position of the nodes is not important.)
2. There is a one-to-one correspondence between the elements of drawing A and the elements of drawing B.
3. Corresponding elements are connected to corresponding nodes.

## EXAMPLE 3.2-1 Different Drawings of the Same Circuit

Figure 3.2-2 shows four circuit drawings. Which of these drawings, if any, represent the same circuit as the circuit drawing in Figure 3.2-1*c*?

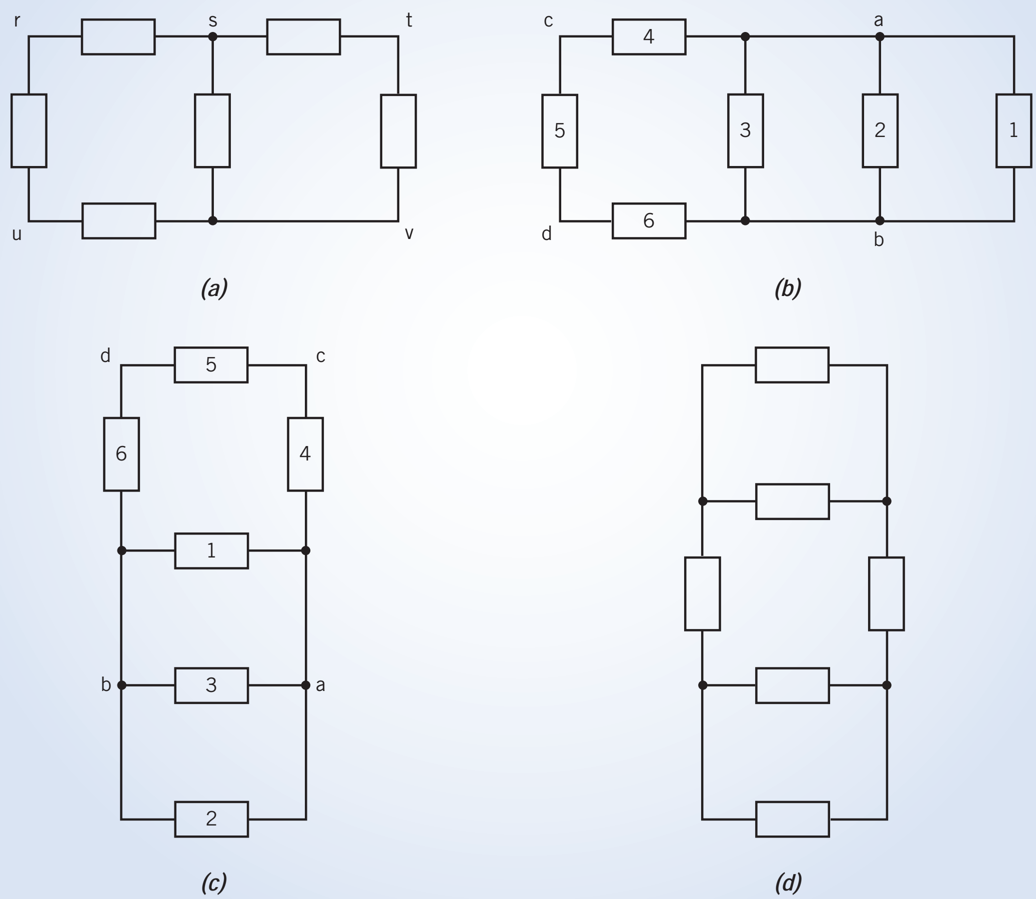



**FIGURE 3.2-2** Four circuit drawings.

## Solution

The circuit drawing shown in Figure 3.2-2*a* has five nodes, labeled r, s, t, u, and v. The circuit drawing in Figure 3.2-1*c* has four nodes. Because the two drawings have different numbers of nodes, there cannot be a one-to-one correspondence between the nodes of the two drawings. Hence, these drawings represent different circuits.

The circuit drawing shown in Figure 3.2-2*b* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.2-1*c*. The nodes in Figure 3.2-2*b* have been labeled in the same way as the corresponding nodes in Figure 3.2-1*c*. For example, node c in Figure 3.2-2*b* corresponds to node c in Figure 3.2-1*c*. The elements in Figure 3.2-2*b* have been labeled in the same way as the corresponding elements in Figure 3.2-1*c*. For example, element 5 in Figure 3.2-2*b* corresponds to element 5 in Figure 3.2-1*c*. Corresponding elements are indeed connected to corresponding nodes. For example, element 2 is connected to nodes a and b, in both Figure 3.2-2*b* and in Figure 3.2-1*c*. Consequently, Figure 3.2-2*b* and Figure 3.2-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.2-2*c* has four nodes and six elements, the same number of nodes and elements as the circuit drawing in Figure 3.2-1*c*. The nodes and elements in Figure 3.2-2*c* have been labeled in the same way as the corresponding nodes and elements in Figure 3.2-1*c*. Corresponding elements are indeed connected to corresponding nodes. Therefore, Figure 3.2-2*c* and Figure 3.2-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.2-2*d* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.2-1*c*. However, the nodes and elements of Figure 3.2-2*d* cannot be labeled so that corresponding elements of Figure 3.2-1*c* are connected to corresponding nodes. (For example, in Figure 3.2-1*c*, three elements are connected between the same pair of nodes, a and b. That does not happen in Figure 3.2-2*d*.) Consequently, Figure 3.2-2*d* and Figure 3.2-1*c* represent different circuits.



**FIGURE 3.2-3** Gustav Robert Kirchhoff (1824–1887). Kirchhoff stated two laws in 1847 regarding the current and voltage in an electrical circuit.

In 1847, Gustav Robert Kirchhoff, a professor at the University of Berlin, formulated two important laws that provide the foundation for analysis of electric circuits. These laws are referred to as *Kirchhoff's current law* (KCL) and *Kirchhoff's voltage law* (KVL) in his honor. Kirchhoff's laws are a consequence of conservation of charge and conservation of energy. Gustav Robert Kirchhoff is pictured in Figure 3.2-3.

Kirchhoff's current law states that the algebraic sum of the currents entering any node is identically zero for all instants of time.

**Kirchhoff's current law (KCL):** The algebraic sum of the currents into a node at any instant is zero.

The phrase *algebraic sum* indicates that we must take reference directions into account as we add up the currents of elements connected to a particular node. One way to take reference directions into account is to use a plus sign when the current is directed away from the node and a minus sign when the current is directed toward the node. For example, consider the circuit shown in Figure 3.2-1*c*. Four elements of this circuit—elements 1, 2, 3, and 4—are connected to node a. By Kirchhoff's current law, the algebraic sum of the element currents $i_1$, $i_2$, $i_3$, and $i_4$ must be zero. Currents $i_2$ and $i_3$ are directed away from node a, so we will use a plus sign for $i_2$ and $i_3$. In contrast, currents $i_1$ and $i_4$ are directed toward node a, so we will use a minus sign for $i_1$ and $i_4$. The KCL equation for node a of Figure 3.2-1*c* is

$$-i_1 + i_2 + i_3 - i_4 = 0 \tag{3.2-1}$$

An alternate way of obtaining the algebraic sum of the currents into a node is to set the sum of all the currents directed away from the node equal to the sum of all the currents directed toward that node. Using this technique, we find that the KCL equation for node a of Figure 3.2-1*c* is

$$i_2 + i_3 = i_1 + i_4 \tag{3.2-2}$$

Clearly, Eqs. 3.2-1 and 3.2-2 are equivalent.

Similarly, the Kirchhoff's current law equation for node b of Figure 3.2-1*c* is

$$i_1 = i_2 + i_3 + i_6$$

Before we can state Kirchhoff's voltage law, we need the definition of a loop. A *loop* is a closed path through a circuit that does not encounter any intermediate node more than once. For example, starting at node a in Figure 3.2-1*c*, we can move through element 4 to node c, then through element 5 to node d, through element 6 to node b, and finally through element 3 back to node a. We have a closed path, and we did not encounter any of the intermediate nodes—b, c, or d—more than once. Consequently, elements 3, 4, 5, and 6 comprise a loop. Similarly, elements 1, 4, 5, and 6 comprise a loop of the circuit shown in Figure 3.2-1*c*. Elements 1 and 3 comprise yet another loop of this circuit. The circuit has three other loops: elements 1 and 2, elements 2 and 3, and elements 2, 4, 5, and 6.

We are now ready to state Kirchhoff's voltage law.

**Kirchhoff's voltage law (KVL):** The algebraic sum of the voltages around any loop in a circuit is identically zero for all time.

The phrase *algebraic sum* indicates that we must take polarity into account as we add up the voltages of elements that comprise a loop. One way to take polarity into account is to move around the loop in the clockwise direction while observing the polarities of the element voltages. We write the voltage with a plus sign when we encounter the + of the voltage polarity before the −. In contrast, we write the voltage with a minus sign when we encounter the − of the voltage polarity before the +. For example, consider the circuit shown in Figure 3.2-1*c*. Elements 3, 4, 5, and 6 comprise a loop of the circuit. By Kirchhoff's voltage law, the algebraic sum of the element voltages $v_3$, $v_4$, $v_5$, and $v_6$ must be zero. As we move around the loop in the clockwise direction, we encounter the + of $v_4$ before the −, the − of $v_5$ before the +, the − of $v_6$ before the +, and the − of $v_3$ before the +. Consequently, we use a minus sign for $v_3$, $v_5$, and $v_6$ and a plus sign for $v_4$. The KCL equation for this loop of Figure 3.2-1*c* is

$$v_4 - v_5 - v_6 - v_3 = 0$$

Similarly, the Kirchhoff's voltage law equation for the loop consisting of elements 1, 4, 5, and 6 is

$$v_4 - v_5 - v_6 + v_1 = 0$$

The Kirchhoff's voltage law equation for the loop consisting of elements 1 and 2 is

$$-v_2 + v_1 = 0$$

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## EXAMPLE 3.2-2 Kirchhoff's Laws

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 3.2-4*a*. Determine the power supplied by element *C* and the power received by element *D*.

### Solution

Figure 3.2-4*a* provides a value for the current in element *C* but not for the voltage *v* across element *C*. The voltage and current of element *C* given in Figure 3.2-4*a* adhere to the passive convention, so the product of this voltage and current is the power *received* by element *C*. Figure 3.2-4*a* provides a value for the voltage across element *D* but not for the current *i* in element *D*. The voltage and current of element *D* given in Figure 3.2-4*a* do not adhere to the passive convention, so the product of this voltage and current is the power *supplied* by element *D*.

We need to determine the voltage *v* across element *C* and the current *i* in element *D*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 3.2-4*b*.

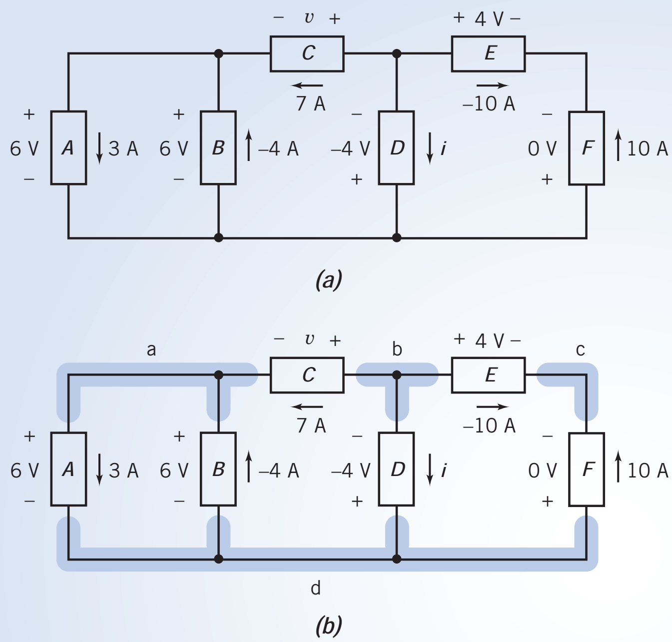


**FIGURE 3.2-4** (*a*) The circuit considered in Example 3.2-2 and (*b*) the circuit redrawn to emphasize the nodes.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *C*, *D*, and *B* to get

$$-v - (-4) - 6 = 0 \Rightarrow v = -2 \text{ V}$$

The value of the current in element *C* in Figure 3.2-4*b* is 7 A. The voltage and current of element *C* given in Figure 3.2-4*b* adhere to the passive convention, so

$$p_C = v(7) = (-2)(7) = -14 \text{ W}$$

is the power *received* by element *C*. Therefore, element *C supplies* 14 W.

Next, apply Kirchhoff's current law (KCL) at node b to get

$$7 + (-10) + i = 0 \Rightarrow i = 3 \text{ A}$$

The value of the voltage across element *D* in Figure 3.2-4*b* is −4 V. The voltage and current of element *D* given in Figure 3.2-4*b* do not adhere to the passive convention, so the power *supplied* by element *D* is given by

$$p_D = (-4)i = (-4)(3) = -12 \text{ W}$$

Therefore, element *D receives* 12 W.

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## Example 3.2-3 Ohm's and Kirchhoff's Laws

Consider the circuit shown in Figure 3.2-5. Notice that the passive convention was used to assign reference directions to the resistor voltages and currents. This anticipates using Ohm's law. Find each current and each voltage when $R_1 = 8\ \Omega$, $v_2 = -10$ V, $i_3 = 2$ A, and $R_3 = 1\ \Omega$. Also, determine the resistance $R_2$.

### Solution

The sum of the currents entering node a is

$$i_1 - i_2 - i_3 = 0$$

Using Ohm's law for $R_3$, we find that

$$v_3 = R_3 i_3 = 1(2) = 2\ \text{V}$$

Kirchhoff's voltage law for the bottom loop incorporating $v_1$, $v_3$, and the 10-V source is

$$-10 + v_1 + v_3 = 0$$

Therefore,

$$v_1 = 10 - v_3 = 8\ \text{V}$$

Ohm's law for the resistor $R_1$ is

$$v_1 = R_1 i_1$$

or

$$i_1 = v_1/R_1 = 8/8 = 1\ \text{A}$$

Next, apply Kirchhoff's current law at node a to get

$$i_2 = i_1 - i_3 = 1 - 2 = -1\ \text{A}$$

We can now find the resistance $R_2$ from

$$v_2 = R_2 i_2$$

or

$$R_2 = v_2/i_2 = -10/-1 = 10\ \Omega$$

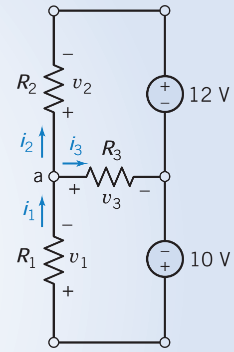


**FIGURE 3.2-5** Circuit with two constant-voltage sources.

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## EXAMPLE 3.2-4 Ohm's and Kirchhoff's Laws

INTERACTIVE EXAMPLE

Determine the value of the current, in amps, measured by the ammeter in Figure 3.2-6*a*.

### Solution

An ideal ammeter is equivalent to a short circuit. The current measured by the ammeter is the current in the short circuit. Figure 3.2-6*b* shows the circuit after replacing the ammeter by the equivalent short circuit.

The circuit has been redrawn in Figure 3.2-7 to label the nodes of the circuit. This circuit consists of a voltage source, a dependent current source, two resistors, and two short circuits. One of the short circuits is the controlling element of the CCCS, and the other short circuit is a model of the ammeter.

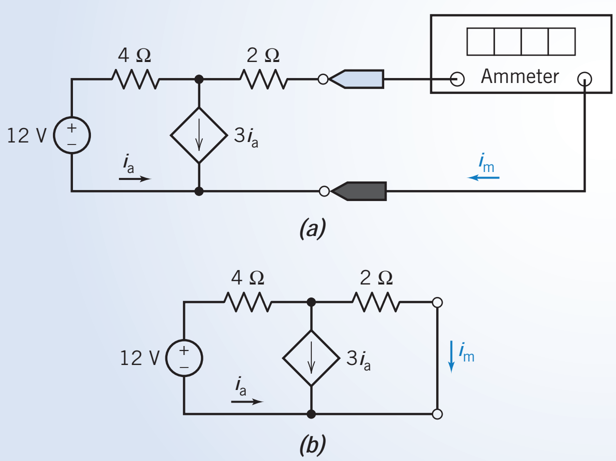


**FIGURE 3.2-6** (*a*) A circuit with dependent source and an ammeter. (*b*) The equivalent circuit after replacing the ammeter by a short circuit.

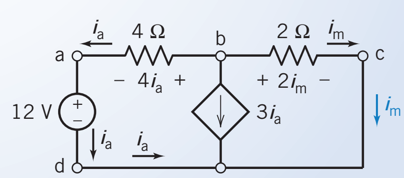


**FIGURE 3.2-7** The circuit of Figure 3.2-6 after labeling the nodes and some element currents and voltages.

Applying KCL twice, once at node d and again at node a, shows that the current in the voltage source and the current in the 4-Ω resistor are both equal to $i_a$. These currents are labeled in Figure 3.2-7. Applying KCL again, at node c, shows that the current in the 2-Ω resistor is equal to $i_m$. This current is labeled in Figure 3.2-7.

Next, Ohm's law tells us that the voltage across the 4-Ω resistor is equal to $4i_a$ and that the voltage across the 2-Ω resistor is equal to $2i_m$. Both of these voltages are labeled in Figure 3.2-7.

Applying KCL at node b gives

$$-i_a - 3i_a - i_m = 0$$

Applying KVL to closed path a-b-c-e-d-a gives

$$0 = -4i_a + 2i_m - 12 = -4\left(-\frac{1}{4}i_m\right) + 2i_m - 12 = 3i_m - 12$$

Finally, solving this equation gives

$$i_m = 4\text{ A}$$

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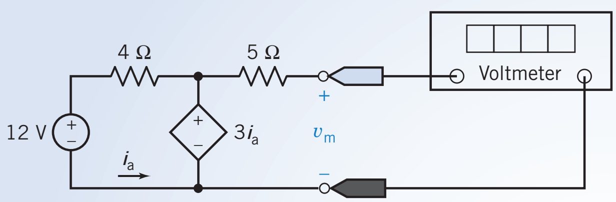
## EXAMPLE 3.2-5 Ohm's and Kirchhoff's Laws

INTERACTIVE EXAMPLE

Determine the value of the voltage, in volts, measured by the voltmeter in Figure 3.2-8*a*.



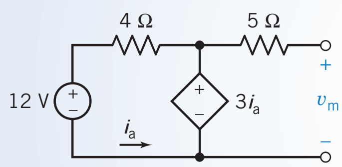




**FIGURE 3.2-8** (*a*) A circuit with dependent source and a voltmeter. (*b*) The equivalent circuit after replacing the voltmeter by an open circuit.

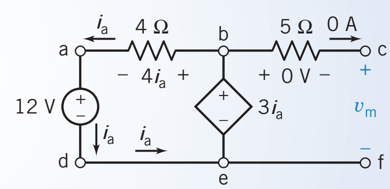



**FIGURE 3.2-9** The circuit of Figure 3.2-8*b* after labeling the nodes and some element currents and voltages.

## Solution

An ideal voltmeter is equivalent to an open circuit. The voltage measured by the voltmeter is the voltage across the open circuit. Figure 3.2-8*b* shows the circuit after replacing the voltmeter by the equivalent open circuit.

The circuit has been redrawn in Figure 3.2-9 to label the nodes of the circuit. This circuit consists of a voltage source, a dependent voltage source, two resistors, a short circuit, and an open circuit. The short circuit is the controlling element of the CCVS, and the open circuit is a model of the voltmeter.

Applying KCL twice, once at node d and again at node a, shows that the current in the voltage source and the current in the 4-Ω resistor are both equal to $i_a$. These currents are labeled in Figure 3.2-9. Applying KCL again, at node c, shows that the current in the 5-Ω resistor is equal to the current in the open circuit, that is, zero. This current is labeled in Figure 3.2-9. Ohm's law tells us that the voltage across the 5-Ω resistor is also equal to zero. Next, applying KVL to the closed path b-c-f-e-b gives $v_m = 3i_a$.

Applying KVL to the closed path a-b-e-d-a gives

$$-4i_a + 3i_a - 12 = 0$$

so

$$i_a = -12 \text{ A}$$

Finally

$$v_m = 3i_a = 3(-12) = -36 \text{ V}$$

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**EXAMPLE 3.2-6 Kirchhoff's Laws with Time-Varying Currents and Voltages**

INTERACTIVE EXAMPLE

The circuit shown in Figure 3.2-10 contains a circuit element called a capacitor. We will learn more about capacitors in Chapter 7. The only thing we will need to know about the capacitor in this example is its voltage, $v_c(t)$, and that will be given.

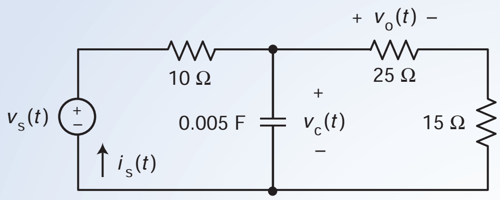


**FIGURE 3.2-10** The circuit considered in Example 3.2-6.

In this example we will determine the voltage, $v_o(t)$, across the 25-Ω resistor and the voltage source current, $i_s(t)$, for each of the following cases:

**(a)** The voltage source voltage is $v_s(t) = 50$ V and the capacitor voltage is $v_c(t) = 40 - 40\,e^{-25t}$ V.

**(b)** The voltage source voltage is $v_s(t) = 10\cos(8t)$ V and the capacitor voltage is $v_c(t) = 7.62\cos\left(8t - 17.7^\circ\right)$ V.

Notice that $v_s(t)$ and $v_c(t)$ are not constant functions of time.

## Solution

Let's label the circuit as shown in Figure 3.2-11. We've labeled the nodes of the circuit in Figure 3.2-11. Also, we've labeled the voltage and current of each circuit element. In anticipation of using Ohm's Law, we've labeled the current and voltage of each resistor to adhere to the passive convention.

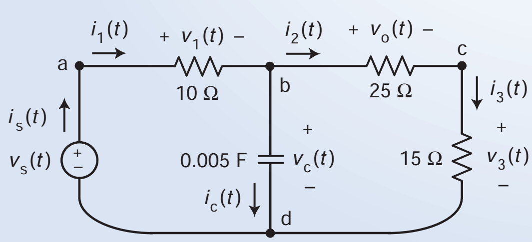


**FIGURE 3.2-11** The circuit from Figure 3.2-10 after labeling the nodes and the element voltages and currents.

## Solution

Let's see what information we can obtain using Ohm's law and Kirchhoff's laws. Applying Ohm's law to each of the resistors gives

$$v_1(t) = 10\,i_1(t),\ v_o(t) = 25\,i_2(t) \text{ and } v_3(t) = 15\,i_3(t) \tag{3.2-3}$$

Apply KCL at node a and also at node c to get

$$i_s(t) = i_1(t) \text{ and } i_2(t) = i_3(t) \tag{3.2-4}$$

Apply KVL to the loop consisting of the voltage source, 10-Ω resistor, and the capacitor to get

$$v_s(t) = v_1(t) + v_c(t) \tag{3.2-5}$$

Apply KVL to the loop consisting of the capacitor, 25-Ω resistor, and the 15-Ω resistor to get

$$v_c(t) = v_o(t) + v_3(t) \tag{3.2-6}$$

Doing a little algebra, we get

$$i_s(t) = i_1(t) = \frac{v_1(t)}{10} = \frac{v_s(t) - v_c(t)}{10} \tag{3.2-7}$$

Recalling that $i_2(t) = i_3(t)$, we do the following algebra

$$v_c(t) = v_o(t) + v_3(t) = 25\,i_2(t) + 15\,i_3(t) = 40\,i_2(t) \tag{3.2-8}$$

Combining Eqs. 3.2-8 and 3.2-3 gives

$$v_o(t) = 25\,i_2(t) = 25\,\frac{v_c(t)}{40} = \frac{5}{8}\,v_c(t) \tag{3.2-9}$$

In summary

$$v_o(t) = \frac{5}{8}\,v_c(t) \text{ and } i_s(t) = \frac{v_s(t) - v_c(t)}{10} \tag{3.2-10}$$

These equations prepare us to consider case (a) and case (b) of this example.

In case (a)

$$v_o(t) = \frac{5}{8}\left(40 - 40\,e^{-25t}\right) = 25\left(1 - e^{-25t}\right)\text{ V}$$

and

$$i_s(t) = \frac{50 - \left(40 - 40\,e^{-25t}\right)}{10} = 1 + 4\,e^{-25t}\text{ A}$$

In case (b)

$$v_o(t) = \left(\frac{5}{8}\right)7.62\cos(8t - 17.7°) = 4.76\cos(8t - 17.7°)\text{ V}$$

and

$$i_s(t) = \frac{10\cos(8t) - 7.62\cos(8t - 17.7°)}{10}\text{ A} \tag{3.2-11}$$

We can simplify this expression for $i_s(t)$ using trigonometric identities, but that process is somewhat tedious. In Chapter 10 we'll use complex arithmetic to simplify Eq. 3.2-11. The result is

$$i_s(t) = 0.349\cos(8t + 40°)\text{ A}$$

**EXERCISE 3.2-1** Determine the values of $i_3$, $i_4$, $i_6$, $v_2$, $v_4$, and $v_6$ in Figure E 3.2-1.

***Answer:*** $i_3 = -3$ A, $i_4 = 3$ A, $i_6 = 4$ A, $v_2 = -3$ V, $v_4 = -6$ V, $v_6 = 6$ V

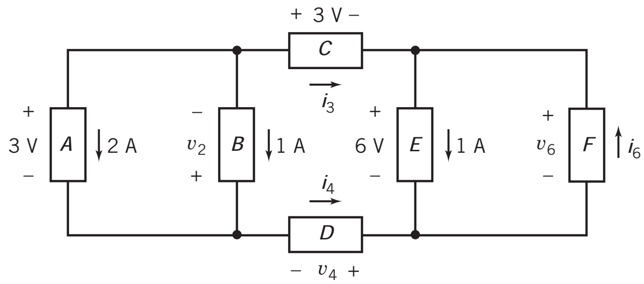


**FIGURE E 3.2-1**

## 3.3 *Series Resistors and Voltage Division*

Let us consider a single-loop circuit, as shown in Figure 3.3-1. In anticipation of using Ohm's law, the passive convention has been used to assign reference directions to resistor voltages and currents.

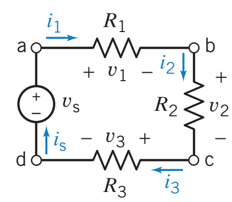


**FIGURE 3.3-1** Single-loop circuit with a voltage source $v_s$.

The connection of resistors in Figure 3.3-1 is said to be a *series* connection because all the elements carry the same current. To identify a pair of series elements, we look for two elements connected to a single node that has no other elements connected to it. Notice, for example, that resistors $R_1$ and $R_2$ are both connected to node b and that no other circuit elements are connected to node b. Consequently, $i_1 = i_2$, so both resistors have the same current. A similar argument shows that resistors $R_2$ and $R_3$ are also connected in series. Noticing that $R_2$ is connected in series with both $R_1$ and $R_3$, we say that all three resistors are connected in series. The order of series resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.3-1 will not change if we interchange the positions $R_2$ and $R_3$.

Using KCL at each node of the circuit in Figure 3.3-1, we obtain

$$\begin{aligned} \text{a}:&\ i_s = i_1 \\ \text{b}:&\ i_1 = i_2 \\ \text{c}:&\ i_2 = i_3 \\ \text{d}:&\ i_3 = i_s \end{aligned}$$

Consequently,

$$i_s = i_1 = i_2 = i_3$$

To determine $i_1$, we use KVL around the loop to obtain

$$v_1 + v_2 + v_3 - v_s = 0$$

where, for example, $v_1$ is the voltage across the resistor $R_1$. Using Ohm's law for each resistor,

$$R_1 i_1 + R_2 i_2 + R_2 i_3 - v_s = 0 \ \Rightarrow\ R_1 i_1 + R_2 i_1 + R_2 i_1 = v_s$$

Solving for $i_1$, we have

$$i_1 = \frac{v_s}{R_1 + R_2 + R_3}$$

Thus, the voltage across the $n$th resistor $R_n$ is $v_n$ and can be obtained as

$$v_n = i_1 R_n = \frac{v_s R_n}{R_1 + R_2 + R_3}$$

For example, the voltage across resistor $R_2$ is

$$v_2 = \frac{R_2}{R_1 + R_2 + R_3} v_s$$

Thus, the voltage across the series combination of resistors is divided up between the individual resistors in a predictable way. This circuit demonstrates the principle of *voltage division*, and the circuit is called a *voltage divider*.

In general, we may represent the voltage divider principle by the equation

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v_s$$

where $v_n$ is the voltage across the $n$th resistor of $N$ resistors connected in series.

We can replace series resistors by an equivalent resistor. This is illustrated in Figure 3.3-2. The series resistors $R_1$, $R_2$, and $R_3$ in Figure 3.3-2*a* are replaced by a single, equivalent resistor $R_s$ in Figure 3.3-2*b*. $R_s$ is said to be equivalent to the series resistors $R_1$, $R_2$, and $R_3$ when replacing $R_1$, $R_2$, and $R_3$ by $R_s$ does not change the current or voltage of any other element of the circuit. In this case, there is only one other element in the circuit, the voltage source. We must choose the value of the resistance $R_s$ so that replacing $R_1$, $R_2$, and $R_3$ by $R_s$ will not change the current of the voltage source. In Figure 3.3-2*a*, we have

$$i_s = \frac{v_s}{R_1 + R_2 + R_3}$$

In Figure 3.3-2*b*, we have

$$i_s = \frac{v_s}{R_s}$$

Because the voltage source current must be the same in both circuits, we require that

$$R_s = R_1 + R_2 + R_3$$

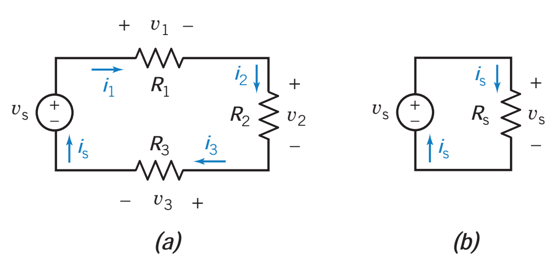


**FIGURE 3.3-2**

In general, the series connection of $N$ resistors having resistances $R_1, R_2 \ldots R_N$ is equivalent to the single resistor having resistance

$$R_s = R_1 + R_2 + \cdots + R_N$$

Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit.

Next, let's calculate the power absorbed by the series resistors in Figure 3.3-2*a*:

$$p = i_s^2 R_1 + i_s^2 R_2 + i_s^2 R_3$$

Doing a little algebra gives

$$p = i_s^2 (R_1 + R_2 + R_3) = i_s^2 R_s$$

which is equal to the power absorbed by the equivalent resistor in Figure 3.3-2*b*. We conclude that the power absorbed by series resistors is equal to the power absorbed by the equivalent resistor.

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## EXAMPLE 3.3-1 Voltage Division

Consider the two similar voltage divider circuits shown in Figure 3.3-3. Use voltage division to determine the values of the voltage $v_2$ in Figure 3.3-3*a* and the voltage $v_b$ in Figure 3.3-3*b*.

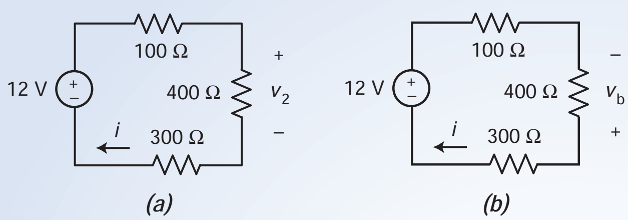


**FIGURE 3.3-3** Two similar voltage divider circuits.

### Solution

First, consider the circuit shown in Figure 3.3-3*a*. This circuit is an example of a single loop circuit like the circuit shown in Figure 3.3-1. The 100, 400, and 300-Ω resistors are connected in series. The current in the loop is given by

$$i = \frac{12}{100 + 400 + 300} = 0.015 \text{ A} = 15 \text{ mA}$$

We can calculate the value of $v_2$ using voltage division:

$$v_2 = \frac{400}{100 + 400 + 300}(12) = 6 \text{ V}$$

As a check, notice that

$$6 = v_2 = 400(i) = 400(0.015)$$

Next, consider the circuit shown in Figure 3.3-3*b*. This circuit is also an example of a single loop circuit. Again, the current in the loop is given by

$$i = \frac{12}{100 + 400 + 300} = 0.015 \text{ A} = 15 \text{ mA}$$

Notice that the voltage $v_b$ in Figure 3.3-3*b* is the same voltage as the voltage $v_2$ in Figure 3.3-3*a*, ***except for polarity***. Consequently

$$v_2 = -v_b$$

Therefore

$$v_b = \frac{400}{100 + 400 + 300}(12) = -6 \text{ V}$$

(Notice that the voltage $v_2$ in Figure 3.3-3*a* has the same polarity as the voltage $v_2$ in Figure 3.3-2*a*, but the voltage $v_b$ in Figure 3.3-3*b* has the opposite polarity from the voltage $v_2$ in Figure 3.3-2*a*)

As a check, noticing that the current $i$ and voltage $v_b$ in Figure 3.3-3*b* *do not* adhere to the passive convention, we write

$$-6 = v_b = -400(i) = -400(0.015)$$

Clearly, we will need to pay attention to voltage polarities when we use voltage division.

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## EXAMPLE 3.3-2 Series Resistors

For the circuit of Figure 3.3-4*a*, find the current measured by the ammeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

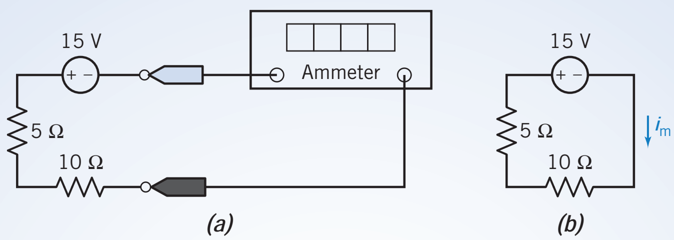


**FIGURE 3.3-4** (*a*) A circuit containing series resistors. (*b*) The circuit after the ideal ammeter has been replaced by the equivalent short circuit, and a label has been added to indicate the current measured by the ammeter $i_m$.

### Solution

Figure 3.3-4*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter $i_m$. Applying KVL gives

$$15 + 5i_m + 10i_m = 0$$

The current measured by the ammeter is

$$i_m = -\frac{15}{5+10} = -1 \text{ A}$$

(Why is $i_m$ negative? Why can't we just divide the source voltage by the equivalent resistance? Recall that when we use Ohm's law, the voltage and current must adhere to the passive convention. In this case, the current calculated by dividing the source voltage by the equivalent resistance does not have the same reference direction as $i_m$, so we need a minus sign.)

The total power absorbed by the two resistors is

$$p_R = 5{i_m}^2 + 10{i_m}^2 = 15\left(1^2\right) = 15 \text{ W}$$

The power supplied by the source is

$$p_s = -v_s\, i_m = -15(-1) = 15 \text{ W}$$

Thus, the power supplied by the source is equal to that absorbed by the series connection of resistors.

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## EXAMPLE 3.3-3 Voltage Divider Design

The input to the voltage divider in Figure 3.3-5 is the voltage $v_s$ of the voltage source. The output is the voltage $v_o$ measured by the voltmeter. Design the voltage divider; that is, specify values of the resistances $R_1$ and $R_2$ to satisfy both of these specifications.

**Specification 1:** The input and output voltages are related by $v_o = 0.8\ v_s$.

**Specification 2:** The voltage source is required to supply no more than 1 mW of power when the input to the voltage divider is $v_s = 20$ V.

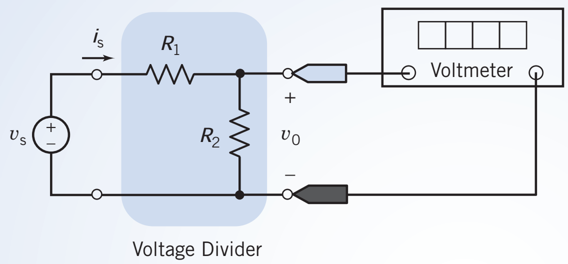


**FIGURE 3.3-5** A voltage divider.

### Solution

We'll examine each specification to see what it tells us about the resistor values.

**Specification 1:** The input and output voltages of the voltage divider are related by

$$v_o = \frac{R_2}{R_1 + R_2} v_s$$

So specification 1 requires

$$\frac{R_2}{R_1 + R_2} = 0.8 \quad \Rightarrow \quad R_2 = 4R_1$$

**Specification 2:** The power supplied by the voltage source is given by

$$p_s = i_s v_s = \left(\frac{v_s}{R_1 + R_2}\right) v_s = \frac{{v_s}^2}{R_1 + R_2}$$

So specification 2 requires

$$0.001 \geq \frac{20^2}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 \geq 400 \times 10^3 = 400\ \text{k}\Omega$$

Combining these results gives

$$5R_1 \geq 400\ \text{k}\Omega$$

The solution is not unique. One solution is

$$R_1 = 100\ \text{k}\Omega \text{ and } R_2 = 400\ \text{k}\Omega$$

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***EXERCISE 3.3-1*** Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-1*a*.

***Hint:*** Figure E 3.3-1*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter $v_m$.

***Answer:*** $v_m = 2$ V

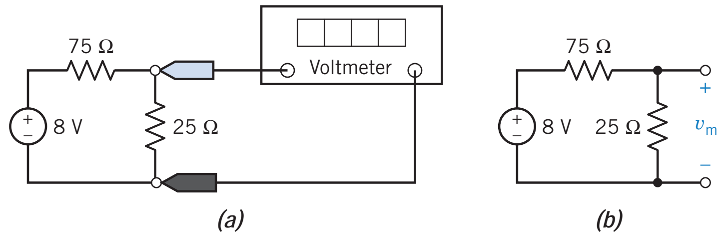


**FIGURE E 3.3-1** (*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter $v_m$.

**EXERCISE 3.3-2** Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-2*a*.

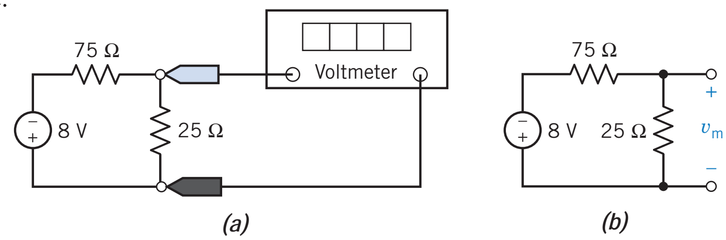


**FIGURE E 3.3-2** (*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter $v_m$.

***Hint:*** Figure E 3.3-2*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter $v_m$.

***Answer:*** $v_m = -2$ V

## 3.4 Parallel Resistors and Current Division

Circuit elements, such as resistors, are connected in *parallel* when the voltage across each element is identical. The resistors in Figure 3.4-1 are connected in *parallel*. Notice, for example, that resistors $R_1$ and $R_2$ are each connected to both node a and node b. Consequently, $v_1 = v_2$, so both resistors have the same voltage. A similar argument shows that resistors $R_2$ and $R_3$ are also connected in parallel. Noticing that $R_2$ is connected in parallel with both $R_1$ and $R_3$, we say that all three resistors are connected in parallel. The order of parallel resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.4-1 will not change if we interchange the positions $R_2$ and $R_3$.

The defining characteristic of parallel elements is that they have the same voltage. To identify a pair of parallel elements, we look for two elements connected between the same pair of nodes.

Consider the circuit with two resistors and a current source shown in Figure 3.4-2. Note that both resistors are connected to terminals a and b and that the voltage $v$ appears across each parallel

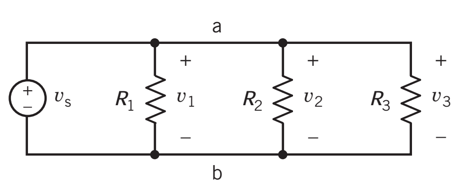


**FIGURE 3.4-1** A circuit with parallel resistors.

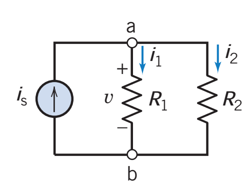


**FIGURE 3.4-2** Parallel circuit with a current source.

element. In anticipation of using Ohm's law, the passive convention is used to assign reference directions to the resistor voltages and currents. We may write KCL at node a (or at node b) to obtain

$$i_s - i_1 - i_2 = 0$$

or

$$i_s = i_1 + i_2$$

Next, from Ohm's law

$$i_1 = \frac{v}{R_1} \quad \text{and} \quad i_2 = \frac{v}{R_2}$$

Then

$$i_s = \frac{v}{R_1} + \frac{v}{R_2} \tag{3.4-1}$$

Recall that we defined conductance $G$ as the inverse of resistance $R$. We may therefore rewrite Eq. 3.4-1 as

$$i_s = G_1 v + G_2 v = (G_1 + G_2)v \tag{3.4-2}$$

Thus, the equivalent circuit for this parallel circuit is a conductance $G_p$, as shown in Figure 3.4-3, where

$$G_p = G_1 + G_2$$

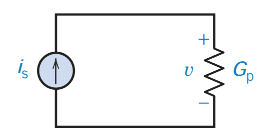


**FIGURE 3.4-3** Equivalent circuit for a parallel circuit.

The equivalent resistance for the two-resistor circuit is found from

$$G_p = \frac{1}{R_1} + \frac{1}{R_2}$$

Because $G_p = 1/R_p$, we have

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \tag{3.4-3}$$

Note that the total conductance, $G_p$, increases as additional parallel elements are added and that the total resistance, $R_p$, declines as each resistor is added.

The circuit shown in Figure 3.4-2 is called a *current divider* circuit because it divides the source current. Note that

$$i_1 = G_1 v \tag{3.4-4}$$

Also, because $i_s = (G_1 + G_2)v$, we solve for $v$, obtaining

$$v = \frac{i_s}{G_1 + G_2} \tag{3.4-5}$$

Substituting $v$ from Eq. 3.4-5 into Eq. 3.4-4, we obtain

$$i_1 = \frac{G_1 i_s}{G_1 + G_2} \tag{3.4-6}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_1 + G_2}$$

Note that we may use $G_2 = 1/R_2$ and $G_1 = 1/R_1$ to obtain the current $i_2$ in terms of two resistances as follows:

$$i_2 = \frac{R_1 i_s}{R_1 + R_2}$$

The current of the source divides between conductances $G_1$ and $G_2$ in proportion to their conductance values.

Let us consider the more general case of current division with a set of $N$ parallel conductors as shown in Figure 3.4-4. The KCL gives

$$i_s = i_1 + i_2 + i_3 + \cdots + i_N \tag{3.4-7}$$

for which

$$i_n = G_n v \tag{3.4-8}$$

for $n = 1, \ \ldots, N$. We may write Eq. 3.4-7 as

$$i_s = (G_1 + G_2 + G_3 + \cdots + G_N)v \tag{3.4-9}$$

Therefore,

$$i_s = v \sum_{n=1}^{N} G_n \tag{3.4-10}$$

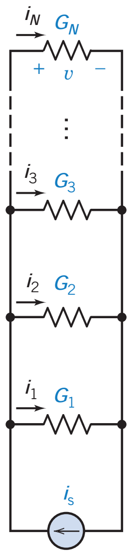


**FIGURE 3.4-4** Set of $N$ parallel conductances with a current source $i_s$.

Because $i_n = G_n v$, we may obtain $v$ from Eq. 3.4-10 and substitute it in Eq. 3.4-8, obtaining

$$i_n = \frac{G_n i_s}{\sum_{n=1}^{N} G_n} \tag{3.4-11}$$

Recall that the equivalent circuit, Figure 3.4-3, has an equivalent conductance $G_p$ such that

$$G_p = \sum_{n=1}^{N} G_n \tag{3.4-12}$$

Therefore,

$$i_n = \frac{G_n i_s}{G_p} \tag{3.4-13}$$

which is the basic equation for the current divider with $N$ conductances. Of course, Eq. 3.4-12 can be rewritten as

$$\frac{1}{R_p} = \sum_{n=1}^{N} \frac{1}{R_n} \tag{3.4-14}$$

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## EXAMPLE 3.4-1 Parallel Resistors

For the circuit in Figure 3.4-5, find (a) the current in each branch, (b) the equivalent circuit, and (c) the voltage $v$. The resistors are

$$R_1 = \frac{1}{2}\ \Omega, \quad R_2 = \frac{1}{4}\ \Omega, \quad R_3 = \frac{1}{8}\ \Omega$$

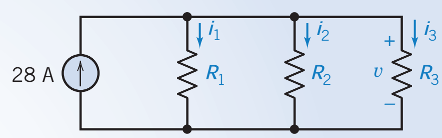


**FIGURE 3.4-5** Parallel circuit for Example 3.3-2.

### Solution

The current divider follows the equation

$$i_n = \frac{G_n i_s}{G_p}$$

so it is wise to find the equivalent circuit, as shown in Figure 3.4-6, with its equivalent conductance $G_p$. We have

$$G_p = \sum_{n=1}^{N} G_n = G_1 + G_2 + G_3 = 2 + 4 + 8 = 14\ \text{S}$$

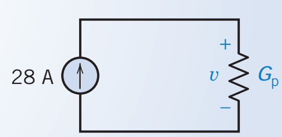


**FIGURE 3.4-6** Equivalent circuit for the parallel circuit of Figure 3.4-5.

Recall that the units for conductance are siemens (S). Then

$$i_1 = \frac{G_1 i_s}{G_p} = \frac{2}{14}(28) = 4 \text{ A}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_p} = \frac{4(28)}{14} = 8 \text{ A}$$

and

$$i_3 = \frac{G_3 i_s}{G_p} = 16 \text{ A}$$

Because $i_n = G_n v$, we have

$$v = \frac{i_1}{G_1} = \frac{4}{2} = 2 \text{ V}$$

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## EXAMPLE 3.4-2 Parallel Resistors

INTERACTIVE EXAMPLE

For the circuit of Figure 3.4-7*a*, find the voltage measured by the voltmeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

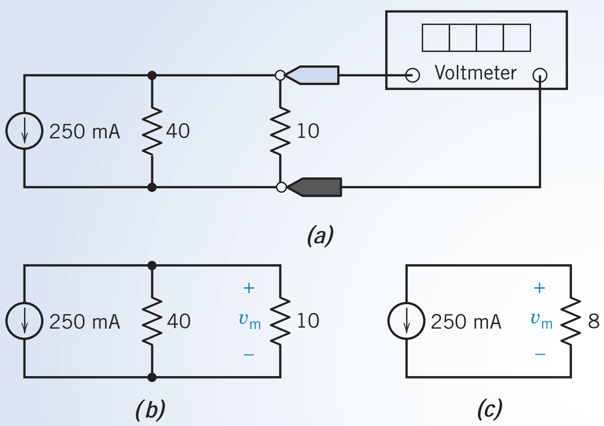


**FIGURE 3.4-7** (*a*) A circuit containing parallel resistors. (*b*) The circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter $v_m$. (*c*) The circuit after the parallel resistors have been replaced by an equivalent resistance.

### Solution

Figure 3.4-7*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit, and a label has been added to indicate the voltage measured by the voltmeter $v_m$. The two resistors are connected in parallel and can be replaced with a single equivalent resistor. The resistance of this equivalent resistor is calculated as

$$\frac{40 \cdot 10}{40 + 10} = 8 \ \Omega$$

Figure 3.4-7*c* shows the circuit after the parallel resistors have been replaced by the equivalent resistor. The current in the equivalent resistor is 250 mA, directed upward. This current and the voltage $v_m$ do not adhere to the passive convention. The current in the equivalent resistance can also be expressed as −250 mA, directed downward. This current and the voltage $v_m$ do adhere to the passive convention. Ohm's law gives

$$v_m = 8(-0.25) = -2 \text{ V}$$

The voltage $v_m$ in Figure 3.4-7*b* is equal to the voltage $v_m$ in Figure 3.4-7*c*. This is a consequence of the equivalence of the 8-Ω resistor to the parallel combination of the 40-Ω and 10-Ω resistors. Looking at Figure 3.4-7*b*, we see that the power absorbed by the resistors is

$$p_R = \frac{{v_m}^2}{40} + \frac{{v_m}^2}{10} = \frac{2^2}{40} + \frac{2^2}{10} = 0.1 + 0.4 = 0.5 \text{ W}$$

The voltage $v_m$ and the current of the current source adhere to the passive convention, so

$$p_s = v_m(0.25) = (-2)(0.25) = -0.5 \text{ W}$$

is the power received by the current source. The current source supplies 0.5 W.

Thus, the power absorbed by the two resistors is equal to that supplied by the source.

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## EXAMPLE 3.4-3 Current Divider Design

The input to the current divider in Figure 3.4-8 is the current $i_s$ of the current source. The output is the current, $i_o$, measured by the ammeter. Specify values of the resistances $R_1$ and $R_2$ to satisfy both of these specifications:

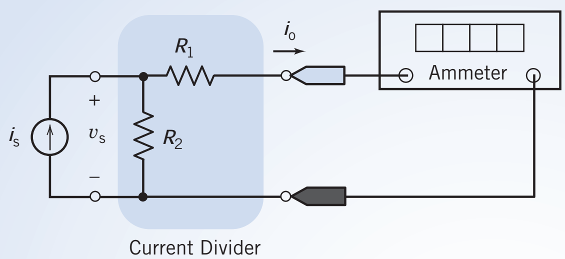



**FIGURE 3.4-8** A current divider circuit.

**Specification 1:** The input and output currents are related by $i_o = 0.8\ i_s$.
**Specification 2:** The current source is required to supply no more than 10 mW of power when the input to the current divider is $i_s = 2$ mA.

### Solution

We'll examine each specification to see what it tells us about the resistor values.
**Specification 1:** The input and output currents of the current divider are related by

$$i_o = \frac{R_2}{R_1 + R_2} i_s$$

So specification 1 requires

$$\frac{R_2}{R_1 + R_2} = 0.8 \quad \Rightarrow \quad R_2 = 4R_1$$

**Specification 2:** The power supplied by the current source is given by

$$p_s = i_s v_s = i_s \left( i_s \left( \frac{R_1 R_2}{R_1 + R_2} \right) \right) = {i_s}^2 \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

So specification 2 requires

$$0.01 \geq (0.002)^2 \left( \frac{R_1 R_2}{R_1 + R_2} \right) \quad \Rightarrow \quad \frac{R_1 R_2}{R_1 + R_2} \leq 2500$$

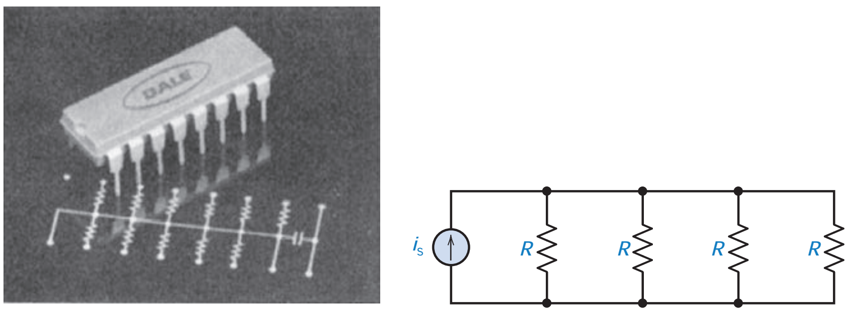
Combining these results gives

$$\frac{R_1(4R_2)}{R_1 + 4R_2} \leq 2500 \quad \Rightarrow \quad \frac{4}{5}R_1 \leq 2500 \quad \Rightarrow \quad R_1 \leq 3125\ \Omega$$

The solution is not unique. One solution is

$$R_1 = 3\ \text{k}\Omega \quad \text{and} \quad R_2 = 12\ \text{k}\Omega$$

**EXERCISE 3.4-1** A resistor network consisting of parallel resistors is shown in a package used for printed circuit board electronics in Figure E 3.4-1*a*. This package is only 2 cm × 0.7 cm, and each resistor is 1 kΩ. The circuit is connected to use four resistors as shown in Figure E 3.4-1*b*. Find the equivalent circuit for this network. Determine the current in each resistor when $i_s = 1$ mA.




Courtesy of Vishay Intertechnology, Inc.

(a) (b)

**FIGURE E 3.4-1** (*a*) A parallel resistor network. (*b*) The connected circuit uses four resistors where $R = 1$ kΩ.

***Answer:*** $R_p = 250\ \Omega$

**EXERCISE 3.4-2** Determine the current measured by the ammeter in the circuit shown in Figure E 3.4-2*a*.

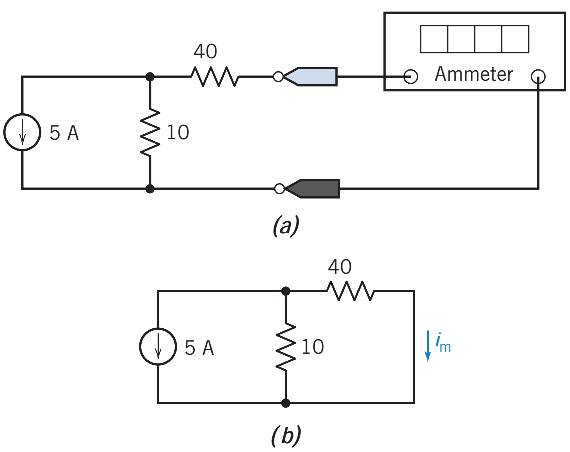



**FIGURE E 3.4-2** (*a*) A current divider. (*b*) The current divider after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter $i_m$.

***Hint:*** Figure E 3.4-2*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit, and a label has been added to indicate the current measured by the ammeter $i_m$.

***Answer:*** $i_m = -1$ A

## 3.5 Series Voltage Sources and Parallel Current Sources

Voltage sources connected in series are equivalent to a single voltage source. The voltage of the equivalent voltage source is equal to the algebraic sum of voltages of the series voltage sources.

Consider the circuit shown in Figure 3.5-1*a*. Notice that the currents of both voltage sources are equal. Accordingly, define the current $i_s$ to be

$$i_s = i_a = i_b \tag{3.5-1}$$

Next, define the voltage $v_s$ to be

$$v_s = v_a + v_b \tag{3.5-2}$$

Using KCL, KVL, and Ohm's law, we can represent the circuit in Figure 3.5-1*a* by the equations

$$i_c = \frac{v_1}{R_1} + i_s \tag{3.5-3}$$

$$i_s = \frac{v_2}{R_2} + i_3 \tag{3.5-4}$$

$$v_c = v_1 \tag{3.5-5}$$

$$v_1 = v_s + v_2 \tag{3.5-6}$$

$$v_2 = i_3 R_3 \tag{3.5-7}$$

where $i_s = i_a = i_b$ and $v_s = v_a + v_b$. These same equations result from applying KCL, KVL, and Ohm's law to the circuit in Figure 3.5-1*b*. If $i_s = i_a = i_b$ and $v_s = v_a + v_b$, then the circuits shown in Figures 3.5-1*a* and 3.5-1*b* are equivalent because they are both represented by the same equations.

For example, suppose that $i_c = 4$ A, $R_1 = 2\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 3\ \Omega$, $v_a = 1$ V, and $v_b = 3$ V. The equations describing the circuit in Figure 3.5-1*a* become

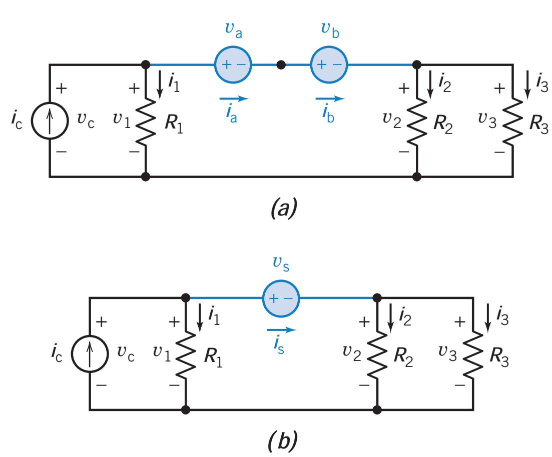
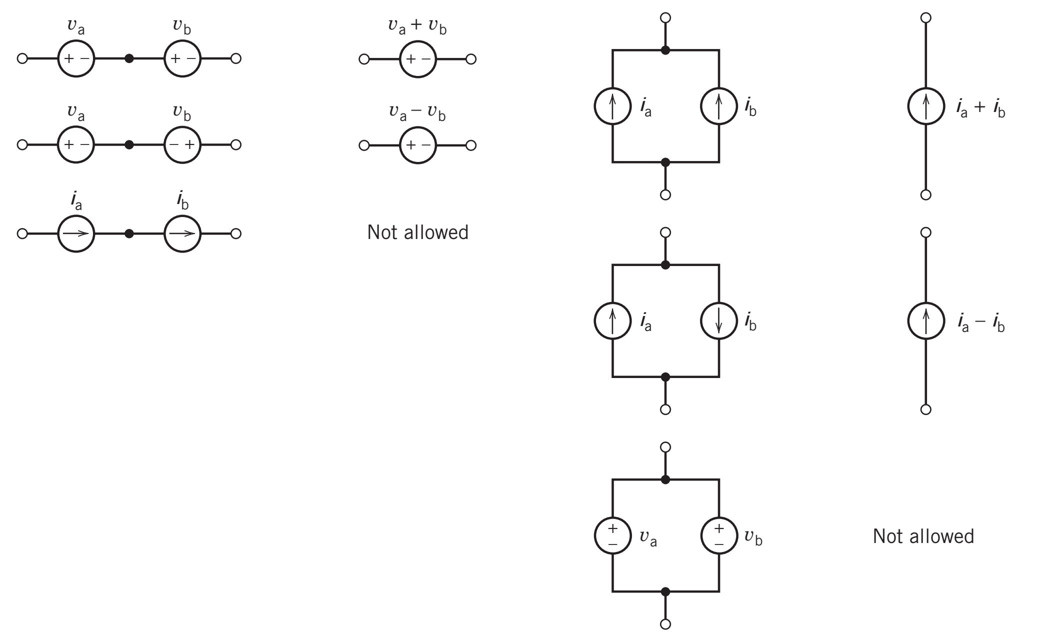



**FIGURE 3.5-1** (*a*) A circuit containing voltage sources connected in series and (*b*) an equivalent circuit.

**Table 3.5-1 Parallel and Series Voltage and Current Sources**

| CIRCUIT | EQUIVALENT CIRCUIT | CIRCUIT | EQUIVALENT CIRCUIT |
|---|---|---|---|




$$4 = \frac{v_1}{2} + i_s \tag{3.5-8}$$

$$i_s = \frac{v_2}{6} + i_3 \tag{3.5-9}$$

$$v_c = v_1 \tag{3.5-10}$$

$$v_1 = 4 + v_2 \tag{3.5-11}$$

$$v_2 = 3i_3 \tag{3.5-12}$$

The solution to this set of equations is $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V. Eqs. 3.5-8 to 3.5-12 also describe the circuit in Figure 3.5-1*b*. Thus, $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V in both circuits. Replacing series voltage sources by a single, equivalent voltage source does not change the voltage or current of other elements of the circuit.

Figure 3.5-2*a* shows a circuit containing parallel current sources. The circuit in Figure 3.5-2*b* is obtained by replacing these parallel current sources by a single, equivalent current source. The current of the equivalent current source is equal to the algebraic sum of the currents of the parallel current sources.

We are not allowed to connect independent current sources in series. Series elements have the same current. This restriction prevents series current sources from being independent. Similarly, we are not allowed to connect independent voltage sources in parallel.

Table 3.5-1 summarizes the parallel and series connections of current and voltage sources.

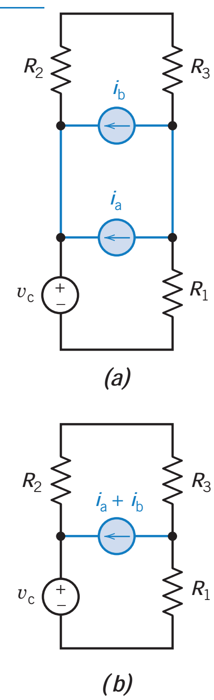



**FIGURE 3.5-2** (*a*) A circuit containing parallel current sources and (*b*) an equivalent circuit.

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## EXAMPLE 3.5-1 Series and Parallel Sources

Figures 3.5-3*a* and *c* show two similar circuits. Both contain series voltage sources and parallel current sources. In each circuit, replace the series voltage sources with an equivalent voltage source and the parallel current sources with an equivalent current source.

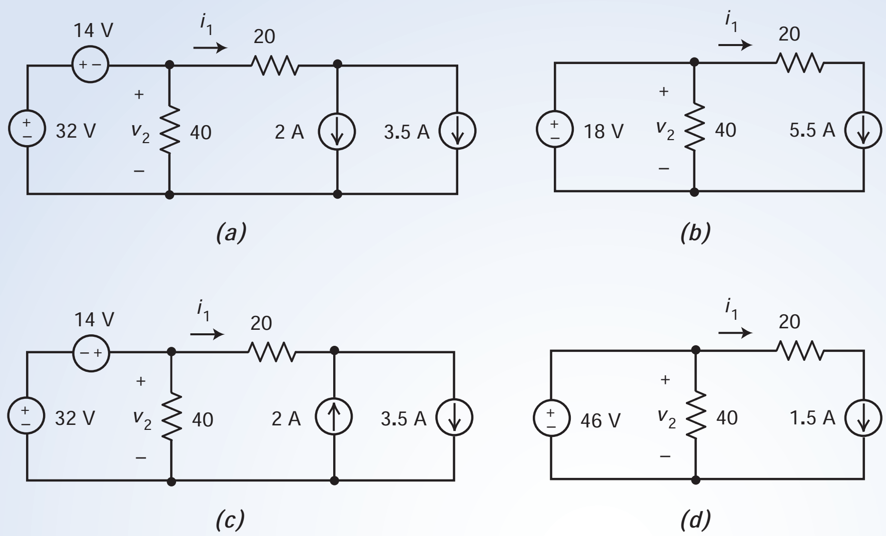


**FIGURE 3.5-3** The circuits considered in Example 3.5-1.

## Solution

Consider first the circuit in Figure 3.5-3*a*. Apply KVL to the left mesh to get

$$14 + v_2 - 32 = 0 \quad \Rightarrow \quad v_2 - 18 = 0$$

Next apply KCL at the right node of the 20 Ω to get

$$i_1 = 2 + 3.5 \quad \Rightarrow \quad i_1 = 5.5$$

These equations suggest that we replace the series voltage sources by a single 18-V source and replace the parallel current sources by a single 5.5-A source. Figure 3.5-3*b* shows the result.

Notice that

$$v_2 - 18 = 0$$

is the KVL equation corresponding to the left mesh of the circuit in Figure 3.5-3*b* and

$$i_1 = 5.5$$

is the KCL equation corresponding to the right node of the 20 Ω to Figure 3.5-3*b*.

Next, consider first the circuit in Figure 3.5-3*c*. Apply KVL to the left mesh to get

$$-14 + v_2 - 32 = 0 \quad \Rightarrow \quad v_2 - 46 = 0$$

Next apply KCL at the right node of the 20 Ω to get

$$i_1 + 2 = 3.5 \quad \Rightarrow \quad i_1 = 1.5$$

These equations suggest that we replace the series voltage sources by a single 46-V source and replace the parallel current sources by a single 1.5-A source. Figure 3.5-3*d* shows the result.

Notice that
$$v_2 - 46 = 0$$
is the KVL equation corresponding to the left mesh of the circuit in Figure 3.5-3*d* and
$$i_1 = 1.5$$
is the KCL equation corresponding to the right node of the 20 Ω to Figure 3.5-3*d*.

## 3.6 *Circuit Analysis*

In this section, we consider the analysis of a circuit by replacing a set of resistors with an equivalent resistance, thus reducing the network to a form easily analyzed.

Consider the circuit shown in Figure 3.6-1. Note that it includes a set of resistors that is connected in series and another set of resistors that is connected in parallel. It is desired to find the output voltage $v_o$, so we wish to reduce the circuit to the equivalent circuit shown in Figure 3.6-2.

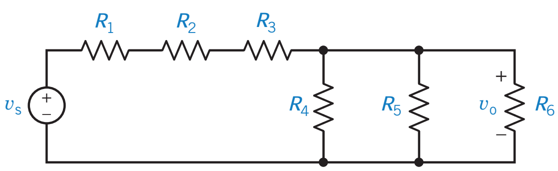


**FIGURE 3.6-1** Circuit with a set of series resistors and a set of parallel resistors.

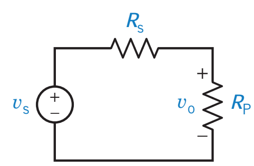


**FIGURE 3.6-2** Equivalent circuit for the circuit of Figure 3.6-2.

We note that the equivalent series resistance is
$$R_s = R_1 + R_2 + R_3$$
and the equivalent parallel resistance is
$$R_p = \frac{1}{G_p}$$
where
$$G_p = G_4 + G_5 + G_6$$
Then, using the voltage divider principle, with Figure 3.6-2, we have
$$v_o = \frac{R_p}{R_s + R_p} v_s$$
Replacing the series resistors by the equivalent resistor $R_s$ did not change the current or voltage of any other circuit element. In particular, the voltage $v_o$ did not change. Also, the voltage $v_o$ across the equivalent resistor $R_p$ is equal to the voltage across each of the parallel resistors. Consequently, the voltage $v_o$ in Figure 3.6-2 is equal to the voltage $v_o$ in Figure 3.6-1. We can analyze the simple circuit in Figure 3.6-2 to find the value of the voltage $v_o$ and know that the voltage $v_o$ in the more complicated circuit shown in Figure 3.6-1 has the same value.

In general, we may find the equivalent resistance for a portion of a circuit consisting only of resistors and then replace that portion of the circuit with the equivalent resistance. For example, consider the circuit shown in Figure 3.6-3. The resistive circuit in (*a*) is equivalent to the single 56 Ω resistor in (*b*). Let's denote the equivalent resistance as $R_{eq}$. We say that $R_{eq}$ is "the equivalent resistance seen looking into the circuit of Figure 3.6-3*a* from terminals *a*-*b*." Figure 3.6-3*c* shows a notation used to indicate the equivalent resistance. Equivalent resistance is an important concept that occurs in a variety of situations and has a variety of names. "Input resistance," "output resistance," "Thévenin resistance," and "Norton resistance" are some names used for equivalent resistance.

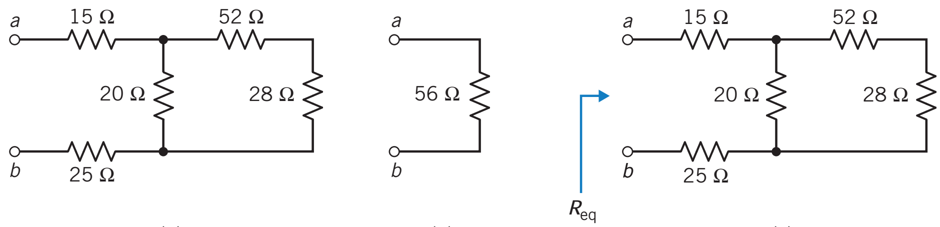


**FIGURE 3.6-3** The resistive circuit in (*a*) is equivalent to the single resistor in (*b*). The notation used to indicate the equivalent resistance is shown in (*c*).

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## EXAMPLE 3.6-1 Series and Parallel Resistors

Determine the value of the current $i$ for the circuit shown in Figure 3.6-4.

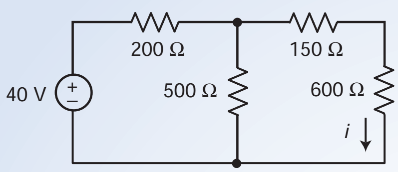


**FIGURE 3.6-4** The circuit considered in Example 3.6-1.

### Solution

The 150- and 600-Ω resistors are connected in series. These series resistors are equivalent to a single resistor. The resistance of the equivalent resistance given by

$$R_s = 150 + 600 = 750\ \Omega$$

Figure 3.6-5*a* shows the circuit after replacing the series resistors by an equivalent resistor. Notice that the current in the equivalent resistor has been labeled as $i$ because it is known to be equal to the currents in the individual series resistors.

The 500- and 750-Ω resistors in Figure 3.6-5*a* are connected parallel. These parallel resistors are equivalent to a single resistor. The resistance of the equivalent resistance given by

$$R_p = \frac{500(750)}{500 + 750} = 300\ \Omega$$

Figure 3.6-5*b* shows the circuit after replacing the parallel resistors by an equivalent resistor. Notice that there is no place in Figure 3.6-5*b* to label the current $i$.

The 200- and 300-Ω resistors in Figure 3.6-5*b* are connected series. The voltage across the 300-Ω resistor can be calculated using voltage division:

$$v_2 = \frac{300}{200 + 300}(40) = 24\ \text{V}$$

The current in the series 200- and 300-Ω resistors in Figure 3.6-5*b* is

$$i_1 = \frac{40}{200 + 300} = 0.08\ \text{A} = 80\ \text{mA}$$

Figure 3.6-5*c* shows the circuit as it was before replacing the parallel 500- and 750-Ω resistors by an equivalent resistor. Replacing these parallel resistors by an equivalent resistance did not change the current in the 200-Ω resistor so the current in the 200-Ω in Figure 3.6-5*d* is labeled as $i_1$. Also, the voltage across the equivalent 300-Ω resistor is equal to the voltage across the individual 500- and 750-Ω parallel resistors. Consequently, the voltage labeled $v_2$ in Figure 3.6-5*c* is equal to the voltage labeled $v_2$ in Figure 3.6-5*b*.

The current $i$ in Figure 3.6-5*c* is related to the current $i_1$ by current division:

$$i = \frac{500}{500 + 750} i_1 = (0.4)(80) = 32 \text{ mA}$$

As a check, we can also calculate the current $i$ using Ohm's law:

$$i = \frac{v_2}{750} = \frac{24}{750} = 32 \text{ mA}$$

(As noted earlier, the current $i$ in Figures 3.6-4*a* and *c* have the same value as the current $i$ in Figure 3.6-5.)

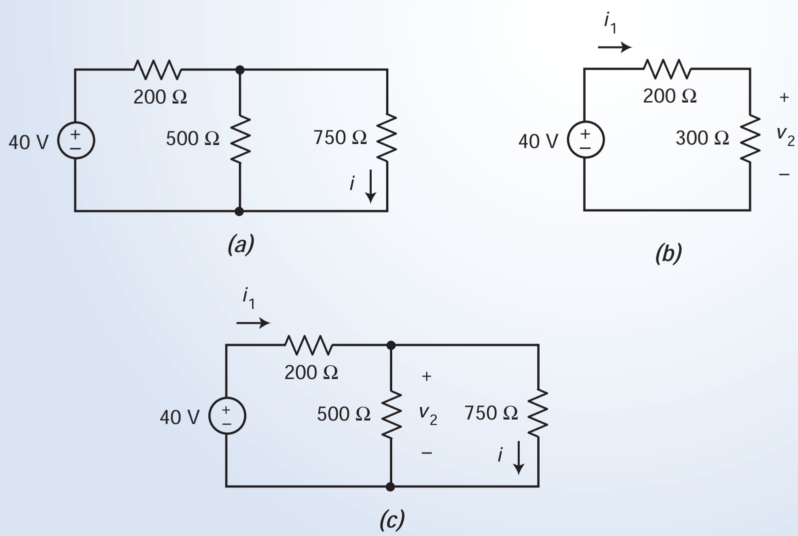


**FIGURE 3.6-5** Analyzing the circuit in Figure 3.6-4 using equivalent resistances.

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## EXAMPLE 3.6-2 Equivalent Resistance

The circuit in Figure 3.6-6*a* contains an ohmmeter. An ohmmeter is an instrument that measures resistance in ohms. The ohmmeter will measure the equivalent resistance of the resistor circuit connected to its terminals. Determine the resistance measured by the ohmmeter in Figure 3.6-6*a*.

### Solution

Working from left to right, the 30-Ω resistor is parallel to the 60-Ω resistor. The equivalent resistance is

$$\frac{60 \cdot 30}{60 + 30} = 20 \, \Omega$$

In Figure 3.6-6*b*, the parallel combination of the 30-Ω and 60-Ω resistors has been replaced with the equivalent 20-Ω resistor. Now the two 20-Ω resistors are in series.

The equivalent resistance is

$$20 + 20 = 40\ \Omega$$

In Figure 3.6-6*c*, the series combination of the two 20-Ω resistors has been replaced with the equivalent 40-Ω resistor. Now the 40-Ω resistor is parallel to the 10-Ω resistor. The equivalent resistance is

$$\frac{40 \cdot 10}{40 + 10} = 8\ \Omega$$

In Figure 3.6-6*d* the parallel combination of the 40-Ω and 10-Ω resistors has been replaced with the equivalent 8-Ω resistor. Thus, the ohmmeter measures a resistance equal to 8 Ω.

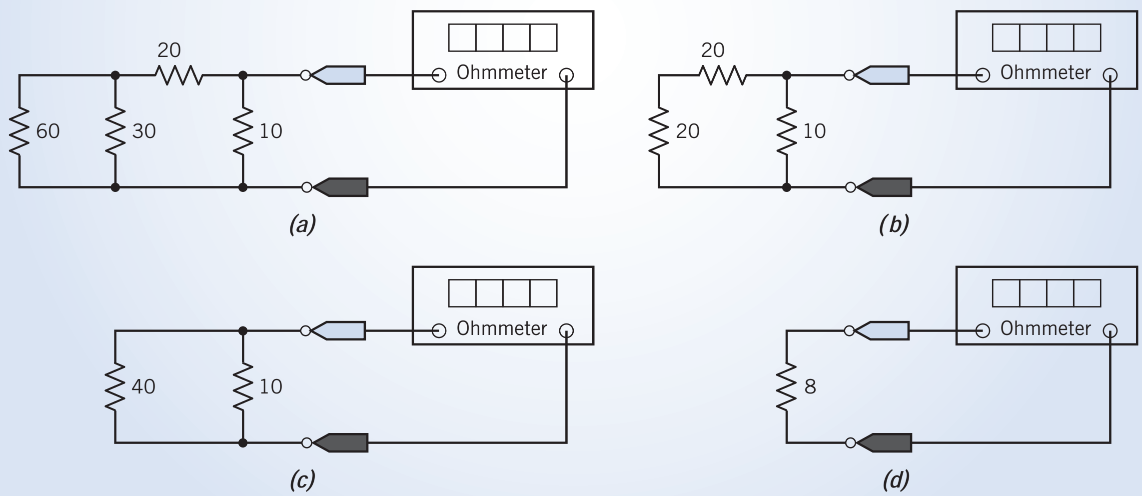



**FIGURE 3.6-6**

## EXAMPLE 3.6-3 Circuit Analysis Using Equivalent Resistances

Determine the values of $i_3$, $v_4$, $i_5$, and $v_6$ in circuit shown in Figure 3.6-7.

## Solution

The circuit shown in Figure 3.6-8 has been obtained from the circuit shown in Figure 3.6-7 by replacing series and parallel combinations of resistances by equivalent resistances. We can use this equivalent circuit to solve this problem in three steps:

1. Determine the values of the resistances $R_1$, $R_2$, and $R_3$ in Figure 3.6-8 that make the circuit in Figure 3.6-8 equivalent to the circuit in Figure 3.6-7.

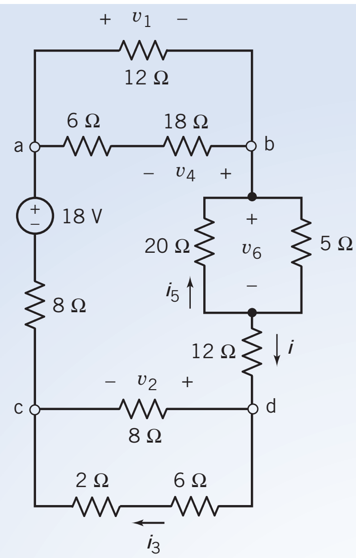



**FIGURE 3.6-7** The circuit considered in Example 3.6-3.

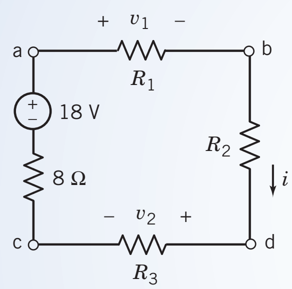



**FIGURE 3.6-8** An equivalent circuit for the circuit in Figure 3.6-7.

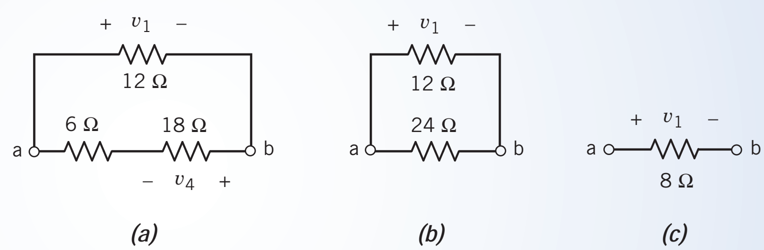



**FIGURE 3.6-9**

2. Determine the values of $v_1$, $v_2$, and $i$ in Figure 3.6-8.
3. Because the circuits are equivalent, the values of $v_1$, $v_2$, and $i$ in Figure 3.6-7 are equal to the values of $v_1$, $v_2$, and $i$ in Figure 3.6-8. Use voltage and current division to determine the values of $i_3$, $v_4$, $i_5$, and $v_6$ in Figure 3.6-7.

**Step 1:** Figure 3.6-9*a* shows the three resistors at the top of the circuit in Figure 3.6-7. We see that the 6-Ω resistor is connected in series with the 18-Ω resistor. In Figure 3.6-9*b*, these series resistors have been replaced by the equivalent 24-Ω resistor. Now the 24-Ω resistor is connected in parallel with the 12-Ω resistor. Replacing series resistors by an equivalent resistance does not change the voltage or current in any other element of the circuit. In particular, $v_1$, the voltage across the 12-Ω resistor, does not change when the series resistors are replaced by the equivalent resistor. In contrast, $v_4$ is not an element voltage of the circuit shown in Figure 3.6-9*b*.

In Figure 3.6-9*c*, the parallel resistors have been replaced by the equivalent 8-Ω resistor. The voltage across the equivalent resistor is equal to the voltage across each of the parallel resistors, $v_1$ in this case. In summary, the resistance $R_1$ in Figure 3.6-8 is given by

$$R_1 = 12 \parallel (6 + 18) = 8\ \Omega$$

Similarly, the resistances $R_2$ and $R_3$ in Figure 3.6-7 are given by

$$R_2 = 12 + (20 \parallel 5) = 16\ \Omega$$

$$R_3 = 8 \parallel (2 + 6) = 4\ \Omega$$

**Step 2:** Apply KVL to the circuit of Figure 3.6-7 to get

$$R_1 i + R_2 i + R_3 i + 8i - 18 = 0 \quad \Rightarrow \quad i = \frac{18}{R_1 + R_2 + R_3 + 8} = \frac{18}{8 + 16 + 4 + 8} = 0.5\ \text{A}$$

Next, Ohm's law gives

$$v_1 = R_1 i = 8(0.5) = 4\ \text{V} \quad \text{and} \quad v_2 = R_3 i = 4(0.5) = 2\ \text{V}$$

**Step 3:** The values of $v_1$, $v_2$, and $i$ in Figure 3.6-7 are equal to the values of $v_1$, $v_2$, and $i$ in Figure 3.6-8. Returning our attention to Figure 3.6-7, and paying attention to reference directions, we can determine the values of $i_3$, $v_4$, $i_5$, and $v_6$ using voltage division, current division, and Ohm's law:

$$\begin{aligned} i_3 &= \frac{8}{8 + (2 + 6)} i = \frac{1}{2}(0.5) = 0.25\ \text{A} \\ v_4 &= -\frac{18}{6 + 18} v_1 = -\frac{3}{4}(4) = -3\ \text{V} \\ i_5 &= -\frac{5}{20 + 5} i = -\left(\frac{1}{5}\right)(0.5) = -0.1\ \text{A} \\ v_6 &= (20 \parallel 5) i = 4(0.5) = 2\ \text{V} \end{aligned}$$

**EXERCISE 3.6-1** Determine the resistance measured by the ohmmeter in Figure E 3.6-1.

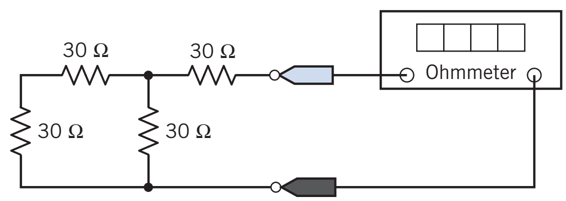



FIGURE E 3.6-1

*Answer:* $\dfrac{(30 + 30) \cdot 30}{(30 + 30) + 30} + 30 = 50\ \Omega$

## 3.7 Analyzing Resistive Circuits Using MATLAB

We can analyze simple circuits by writing and solving a set of equations. We use Kirchhoff's law and the element equations, for instance, Ohm's law, to write these equations. As the following example illustrates, MATLAB provides a convenient way to solve the equations describing an electric circuit.

### EXAMPLE 3.7-1 MATLAB for Simple Circuits

Determine the values of the resistor voltages and currents for the circuit shown in Figure 3.7-1.

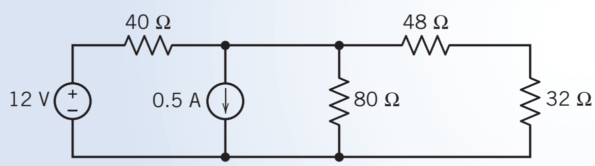



FIGURE 3.7-1 The circuit considered in Example 3.7-1.

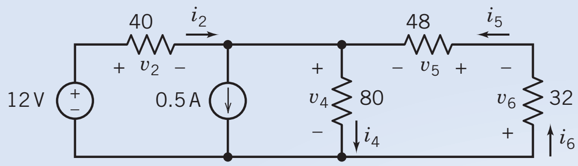



**FIGURE 3.7-2** The circuit from Figure 3.7-1 after labeling the resistor voltages and currents.

## Solution

Let's label the resistor voltages and currents. In anticipation of using Ohm's law, we will label the voltage and current of each resistor to adhere to the passive convention. (Pick one of the variables—the resistor current or the resistor voltage—and label the reference direction however you like. Label the reference direction of the other variable to adhere to the passive convention with the first variable.) Figure 3.7-2 shows the labeled circuit.

Next, we will use Kirchhoff's laws. First, apply KCL to the node at which the current source and the 40-Ω, 48-Ω, and 80-Ω resistors are connected together to write

$$i_2 + i_5 = 0.5 + i_4 \quad (3.7\text{-}1)$$

Next, apply KCL to the node at which the 48-Ω and 32-Ω resistors are connected together to write

$$i_5 = i_6 \quad (3.7\text{-}2)$$

Apply KVL to the loop consisting of the voltage source and the 40-Ω and 80-Ω resistors to write

$$12 = v_2 + v_4 \quad (3.7\text{-}3)$$

Apply KVL to the loop consisting of the 48-Ω, 32-Ω, and 80-Ω resistors to write

$$v_4 + v_5 + v_6 = 0 \quad (3.7\text{-}4)$$

Apply Ohm's law to the resistors.

$$v_2 = 40\, i_2,\ v_4 = 80\, i_4,\ v_5 = 48\, i_5,\ v_6 = 32\, i_6 \quad (3.7\text{-}5)$$

We can use the Ohm's law equations to eliminate the variables representing resistor voltages. Doing so enables us to rewrite Eq. 3.7-3 as:

$$12 = 40\, i_2 + 80\, i_4 \quad (3.7\text{-}6)$$

Similarly, we can rewrite Eq. 3.7-4 as

$$80\, i_4 + 48\, i_5 + 32\, i_6 = 0 \quad (3.7\text{-}7)$$

Next, use Eq. 3.7-2 to eliminate $i_6$ from Eq. 3.7-6 as follows

$$80\, i_4 + 48\, i_5 + 32\, i_5 = 0 \quad \Rightarrow \quad 80\, i_4 + 80\, i_5 = 0 \quad \Rightarrow \quad i_4 = -i_5 \quad (3.7\text{-}8)$$

Use Eq. 3.7-8 to eliminate $i_5$ from Eq. 3.7-1.

$$i_2 - i_4 = 0.5 + i_4 \quad \Rightarrow \quad i_2 = 0.5 + 2\, i_4 \quad (3.7\text{-}9)$$

Use Eq. 3.7-9 to eliminate $i_4$ from Eq. 3.7-6. Solve the resulting equation to determine the value of $i_2$.

$$12 = 40\, i_2 + 80\left(\frac{i_2 - 0.5}{2}\right) = 80\, i_2 - 20 \quad \Rightarrow \quad i_2 = \frac{12 + 20}{80} = 0.4\ \text{A} \quad (3.7\text{-}10)$$

Now we are ready to calculate the values of the rest of the resistor voltages and currents as follows:

$$i_4 = \frac{i_2 - 0.5}{2} = \frac{0.4 - 0.5}{2} = -0.05\ \text{A},$$
$$i_6 = i_5 = -i_4 = 0.05\ \text{A},$$
$$v_2 = 40\, i_2 = 40(0.4) = 16\ \text{V},$$
$$v_4 = 80\, i_4 = 80(-0.05) = -4\ \text{V},$$
$$v_5 = 48\, i_5 = 48(0.05) = 2.4\ \text{V},$$

and

$$v_6 = 32\, i_6 = 32(0.05) = 1.6\ \text{V}.$$

***MATLAB Solution 1***

The preceding algebra shows that this circuit can be represented by these equations:

$$12 = 80\, i_2 - 20, i_4 = \frac{i_2 - 0.5}{2}, i_6 = i_5 = -i_4, v_2 = 40\, i_2,\ v_4 = 80\, i_4,$$
$$v_5 = 48\, i_5, \text{ and } v_6 = 32\, i_6$$

These equations can be solved consecutively, using MATLAB as shown in Figure 3.7-3.

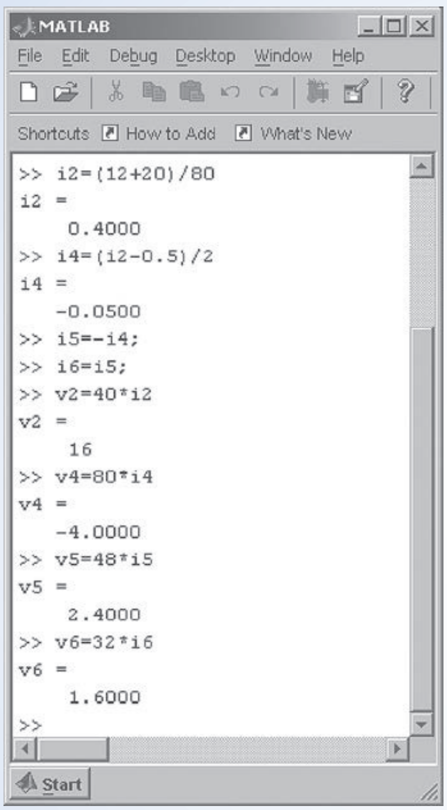



**FIGURE 3.7-3** Consecutive equations.

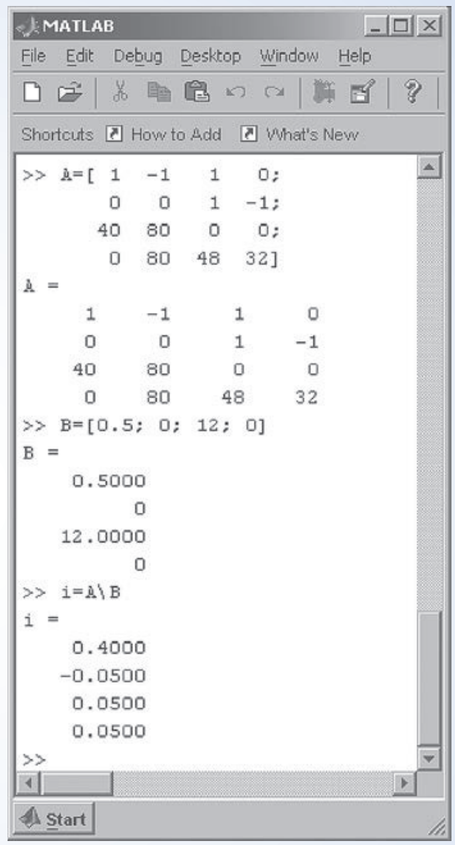



**FIGURE 3.7-4** Simultaneous equations.

***MATLAB Solution 2***

We can avoid some algebra if we are willing to solve simultaneous equations.

After applying Kirchhoff's laws and then using the Ohm's law equations to eliminate the variables representing resistor voltages, we have Eqs. 3.7-1, 2, 6, and 7:

$$i_2 + i_5 = 0.5 + i_4,\ i_5 = i_6, 12 = 40\, i_2 + 80\, i_4,$$

and

$$80\, i_4 + 48\, i_5 + 32\, i_6 = 0$$

This set of four simultaneous equations in $i_2$, $i_4$, $i_5$, and $i_6$ can be written as a single matrix equation.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 40 & 80 & 0 & 0 \\ 0 & 80 & 48 & 32 \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 12 \\ 0 \end{bmatrix} \tag{3.7-11}$$

We can write this equation as

$$Ai = B \tag{3.7-12}$$

where

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 40 & 80 & 0 & 0 \\ 0 & 80 & 48 & 32 \end{bmatrix}, \ i = \begin{bmatrix} i_2 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.5 \\ 0 \\ 12 \\ 0 \end{bmatrix}$$

This matrix equation can be solved using MATLAB as shown in Figure 3.7-4. After entering matrices A and B, the statement

$$\mathrm{i} = \mathrm{A} \backslash \mathrm{B}$$

tells MATLAB to calculate $i$ by solving Eq. 3.7-12.

A circuit consisting of $n$ elements has $n$ currents and $n$ voltages. A set of equations representing that circuit could have as many as $2n$ unknowns. We can reduce the number of unknowns by labeling the currents and voltages carefully. For example, suppose two of the circuit elements are connected in series. We can choose the reference directions for the currents in those elements so that they are equal and use one variable to represent both currents. Table 3.7-1 presents some guidelines that will help us reduce the number of unknowns in the set of equations describing a given circuit.

**Table 3.7-1 Guidelines for Labeling Circuit Variables**

| CIRCUIT FEATURE | GUIDELINE |
|---|---|
| Resistors | Label the voltage and current of each resistor to adhere to the passive convention. Use Ohm's law to eliminate either the current or voltage variable. |
| Series elements | Label the reference directions for series elements so that their currents are equal. Use one variable to represent the currents of series elements. |
| Parallel elements | Label the reference directions for parallel elements so that their voltages are equal. Use one variable to represent the voltages of parallel elements. |
| Ideal Voltmeter | Replace each (ideal) voltmeter by an open circuit. Label the voltage across the open circuit to be equal to the voltmeter voltage. |
| Ideal Ammeter | Replace each (ideal) ammeter by a short circuit. Label the current in the short circuit to be equal to the ammeter current. |

## 3.8 *How Can We Check . . . ?*

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able to quickly identify those solutions that need more work.

The following example illustrates techniques useful for checking the solutions of the sort of problem discussed in this chapter.

### EXAMPLE 3.8-1 How Can We Check Voltage and Current Values?

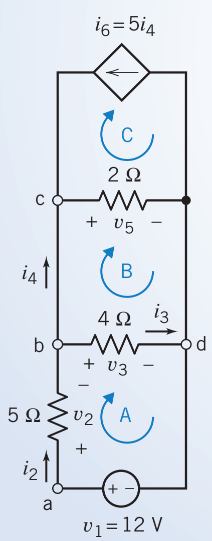
The circuit shown in Figure 3.8-1*a* was analyzed by writing and solving a set of simultaneous equations:

$$12 = v_2 + 4i_3,\ i_4 = \frac{v_2}{5} + i_3,\ v_5 = 4i_3,\ \text{and}\ \frac{v_5}{2} = i_4 + 5i_4$$

The computer program Mathcad (*Mathcad User's Guide*, 1991) was used to solve the equations as shown in Figure 3.8-1*b*. It was determined that

$$v_2 = -60\ \text{V},\ i_3 = 18\ \text{A},\ i_4 = 6\ \text{A},\ \text{and}\ v_5 = 72\ \text{V}.$$

**How can we check** that these currents and voltages are correct?




```
v2 := 0   i3 := 0   i4 := 0   v5 := 0
Given
    12 ≈ v2 + 4 · i3            Apply KVL to loop A.
    i4 ≈ v2/5 + i3              Apply KCL at node b.
    v5 ≈ 4 · i3                 Apply KVL to loop B.
    v5/2 ≈ i4 + 5 · i4          Apply KCL at node c.

  Find (v2,i3,i4,v5) = [-60, 18, 6, 72]
```

(*a*) (*b*)

**FIGURE 3.8-1** (*a*) An example circuit and (*b*) computer analysis using Mathcad.

## Solution

The current $i_2$ can be calculated from $v_2$, $i_3$, $i_4$, and $v_5$ in a couple of different ways. First, Ohm's law gives

$$i_2 = \frac{v_2}{5} = \frac{-60}{5} = -12 \text{ A}$$

Next, applying KCL at node b gives

$$i_2 = i_3 + i_4 = 18 + 6 = 24 \text{ A}$$

Clearly, $i_2$ cannot be both $-12$ and 24 A, so the values calculated for $v_2$, $i_3$, $i_4$, and $v_5$ cannot be correct. Checking the equations used to calculate $v_2$, $i_3$, $i_4$, and $v_5$, we find a sign error in the KCL equation corresponding to node b. This equation should be

$$i_4 = \frac{v_2}{5} - i_3$$

After making this correction, $v_2$, $i_3$, $i_4$, and $v_5$ are calculated to be

$$v_2 = 7.5 \text{ V}, i_3 = 1.125 \text{ A}, i_4 = 0.375 \text{ A}, v_5 = 4.5 \text{ V}$$

Now
$$i_2 = \frac{v_2}{5} = \frac{7.5}{5} = 1.5 \text{ A}$$

and
$$i_2 = i_3 + i_4 = 1.125 + 0.375 = 1.5 \text{ A}$$

This checks as we expected.

As an additional check, consider $v_3$. First, Ohm's law gives

$$v_3 = 4i_3 = 4(1.125) = 4.5 \text{ V}$$

Next, applying KVL to the loop consisting of the voltage source and the 4-Ω and 5-Ω resistors gives

$$v_3 = 12 - v_2 = 12 - 7.5 = 4.5 \text{ V}$$

Finally, applying KVL to the loop consisting of the 2-Ω and 4-Ω resistors gives

$$v_3 = v_5 = 4.5 \text{ V}$$

The results of these calculations agree with each other, indicating that

$$v_2 = 7.5 \text{ V}, \; i_3 = 1.125 \text{ A}, \; i_4 = 0.375 \text{ A}, \; v_5 = 4.5 \text{ V}$$

are the correct values.

## 3.9 Design Example Adjustable Voltage Source

A circuit is required to provide an adjustable voltage. The specifications for this circuit are that:

1. It should be possible to adjust the voltage to any value between $-5$ V and $+5$ V. It should not be possible accidentally to obtain a voltage outside this range.
2. The load current will be negligible.
3. The circuit should use as little power as possible.

The available components are:

1. Potentiometers: resistance values of 10 kΩ, 20 kΩ, and 50 kΩ are in stock.
2. A large assortment of standard 2 percent resistors having values between 10 Ω and 1 MΩ (see Appendix D).
3. Two power supplies (voltage sources): one 12-V supply and one $-12$-V supply, both rated at 100 mA (maximum).

### Describe the Situation and the Assumptions

Figure 3.9-1 shows the situation. The voltage $v$ is the adjustable voltage. The circuit that uses the output of the circuit being designed is frequently called the load. In this case, the load current is negligible, so $i = 0$.

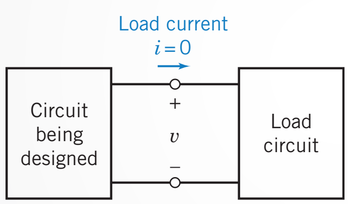



**FIGURE 3.9-1** The circuit being designed provides an adjustable voltage, $v$, to the load circuit.

### State the Goal

A circuit providing the adjustable voltage

$$-5\text{V} \leq v \leq +5\text{V}$$

must be designed using the available components.

### Generate a Plan

Make the following observations.

1. The adjustability of a potentiometer can be used to obtain an adjustable voltage $v$.
2. Both power supplies must be used so that the adjustable voltage can have both positive and negative values.
3. The terminals of the potentiometer cannot be connected directly to the power supplies because the voltage $v$ is not allowed to be as large as 12 V or $-12$ V.

These observations suggest the circuit shown in Figure 3.9-2*a*. The circuit in Figure 3.9-2*b* is obtained by using the simplest model for each component in Figure 3.9-2*a*.

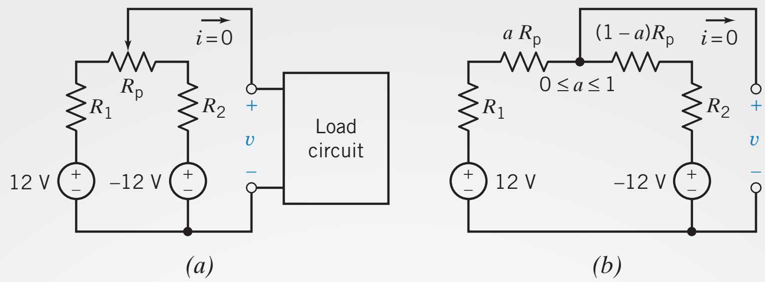



**FIGURE 3.9-2** (*a*) A proposed circuit for producing the variable voltage, *v*, and (*b*) the equivalent circuit after the potentiometer is modeled.

To complete the design, values need to be specified for $R_1$, $R_2$, and $R_p$. Then several results need to be checked and adjustments made, if necessary.

1. Can the voltage $v$ be adjusted to any value in the range $-5$ V to $+5$V?
2. Are the voltage source currents less than 100 mA? This condition must be satisfied if the power supplies are to be modeled as ideal voltage sources.
3. Is it possible to reduce the power absorbed by $R_1$, $R_2$, and $R_p$?

## Act on the Plan

It seems likely that $R_1$ and $R_2$ will have the same value, so let $R_1 = R_2 = R$. Then it is convenient to redraw Figure 3.9-2*b* as shown in Figure 3.9-3.

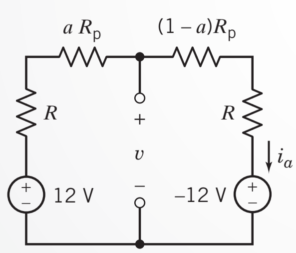



**FIGURE 3.9-3** The circuit after setting $R_1 = R_2 = R$.

Applying KVL to the outside loop yields

$$-12 + Ri_a + aR_p\, i_a + (1 - a)R_p\, i_a + Ri_a - 12 = 0$$

so

$$i_a = \frac{24}{2R + R_p}$$

Next, applying KVL to the left loop gives

$$v = 12 - (R + aR_p)i_a$$

Substituting for $i_a$ gives

$$v = 12 - \frac{24\left(R + aR_p\right)}{2R + R_p}$$

When $a = 0$, $v$ must be 5 V, so

$$5 = 12 - \frac{24R}{2R + R_p}$$

Solving for $R$ gives

$$R = 0.7R_p$$

Suppose the potentiometer resistance is selected to be $R_p = 20$ kΩ, the middle of the three available values. Then,

$$R = 14\ \text{k}\Omega$$

## Verify the Proposed Solution

As a check, notice that when $a = 1$,

$$v = 12 - \left(\frac{14{,}000 + 20{,}000}{28{,}000 + 20{,}000}\right)24 = -5$$

as required. The specification that

$$-5\ \text{V} \le v \le 5\ \text{V}$$

has been satisfied. The power absorbed by the three resistances is

$$p = {i_a}^2(2R + R_p) = \frac{24^2}{2R + R_p}$$

so

$$p = 12\ \text{mW}$$

Notice that this power can be reduced by choosing $R_p$ to be as large as possible, 50 kΩ in this case. Changing $R_p$ to 50 kΩ requires a new value of $R$:

$$R = 0.7 \times R_p = 35\ \text{k}\Omega$$

Because

$$-5\ \text{V} = 12 - \left(\frac{35{,}000 + 50{,}000}{70{,}000 + 50{,}000}\right)24 \le v \le 12 - \left(\frac{35{,}000}{70{,}000 + 50{,}000}\right)24 = 5\ \text{V}$$

the specification that

$$-5\ \text{V} \le v \le 5\ \text{V}$$

has been satisfied. The power absorbed by the three resistances is now

$$p = \frac{24^2}{50{,}000 + 70{,}000} = 5\ \text{mW}$$

Finally, the power supply current is

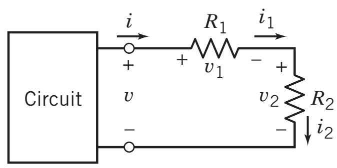
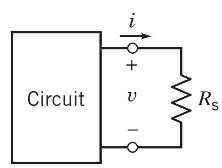
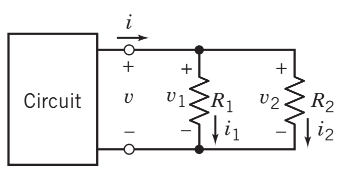
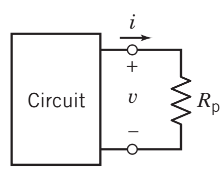
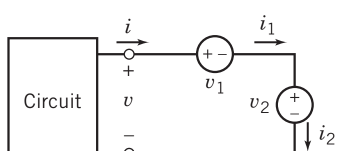
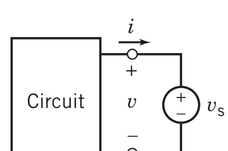
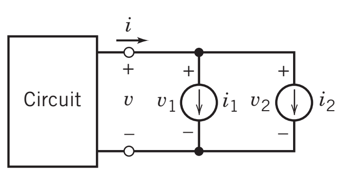
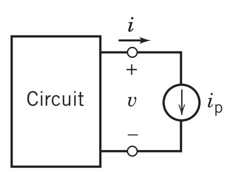
$$i_a = \frac{24}{50{,}000 + 70{,}000} = 0.2\ \text{mA}$$

which is well below the 100 mA that the voltage sources are able to supply. The design is complete.

## 3.10 SUMMARY

- Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering a node is zero. Kirchhoff's voltage law (KVL) states that the algebraic sum of the voltages around a closed path (loop) is zero.
- Simple electric circuits can be analyzed using only Kirchhoff's laws and the constitutive equations of the circuit elements.
- Series resistors act like a "voltage divider," and parallel resistors act like a "current divider." The first two rows of Table 3.10-1 summarize the relevant equations.
- Series resistors are equivalent to a single "equivalent resistor." Similarly, parallel resistors are equivalent to a single "equivalent resistor." The first two rows of Table 3.10-1 summarize the relevant equations.
- Series voltage sources are equivalent to a single "equivalent voltage source." Similarly, parallel current sources are equivalent to a single "equivalent current." The last two rows of Table 3.10-1 summarize the relevant equations.
- Often circuits consisting entirely of resistors can be reduced to a single equivalent resistor by repeatedly replacing series and/or parallel resistors by equivalent resistors.

**Table 3.10-1 Equivalent Circuits for Series and Parallel Elements**

| | | |
|---|---|---|
| Series resistors | <br>$i = i_1 = i_2, v_1 = \frac{R_1}{R_1 + R_2} v$, and $v_2 = \frac{R_2}{R_1 + R_2} v$ | <br>$R_s = R_1 + R_2$ and $v = R_s i$ |
| Parallel resistors | <br>$v = v_1 = v_2, i_1 = \frac{R_2}{R_1 + R_2} i$, and $i_2 = \frac{R_1}{R_1 + R_2} i$ | <br>$R_p = \frac{R_1 R_2}{R_1 + R_2}$ and $v = R_p i$ |
| Series voltage sources | <br>$i = i_1 = i_2$ and $v = v_1 + v_2$ | <br>$v_s = v_1 + v_2$ |
| Parallel current sources | <br>$v = v_1 = v_2$ and $i = i_1 + i_2$ | <br>$i_p = i_1 + i_2$ |

# PROBLEMS

⊕ Problem available in WileyPLUS at instructor's discretion.

## Section 3.2 Kirchhoff's Laws

**P 3.2-1** ⊕ Consider the circuit shown in Figure P 3.2-1. Determine the values of the power supplied by branch *B* and the power supplied by branch *F*.

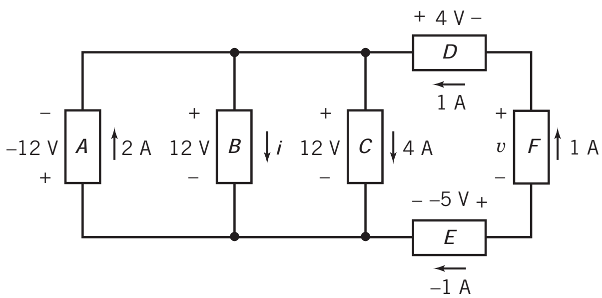



Figure P 3.2-1

**P 3.2-2** Determine the values of $i_2$, $i_4$, $v_2$, $v_3$, and $v_6$ in Figure P 3.2-2.

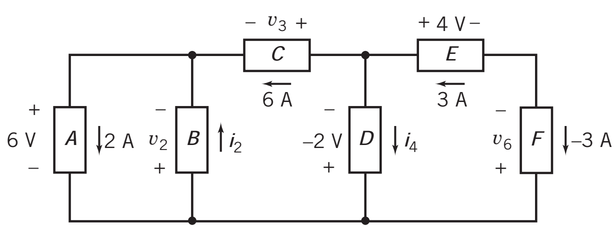



Figure P 3.2-2

**P 3.2-3** Consider the circuit shown in Figure P 3.2-3.

**(a)** Suppose that $R_1 = 8\ \Omega$ and $R_2 = 4\ \Omega$. Find the current *i* and the voltage *v*.

**(b)** Suppose, instead, that $i = 2.25$ A and $v = 42$ V. Determine the resistances $R_1$ and $R_2$.

**(c)** Suppose, instead, that the voltage source supplies 24 W of power and that the current source supplies 9 W of power. Determine the current *i*, the voltage *v*, and the resistances $R_1$ and $R_2$.

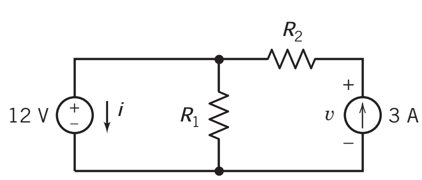



Figure P 3.2-3

**P 3.2-4** Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-4.

*Answer:* The 4-Ω resistor absorbs 100 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 72 W.

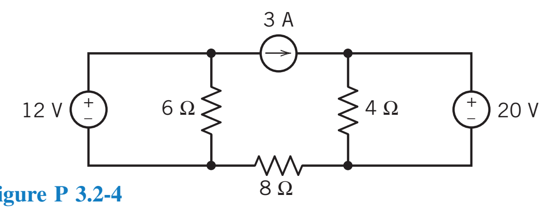



Figure P 3.2-4

**P 3.2-5** ⊕ Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-5.

*Answer:* The 4-Ω resistor absorbs 16 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 8 W.

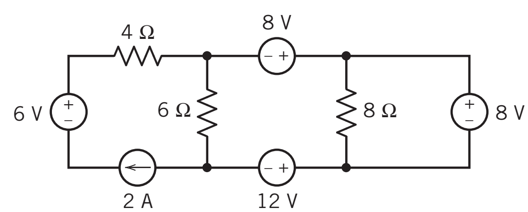



Figure P 3.2-5

**P 3.2-6** Determine the power supplied by each voltage source in the circuit of Figure P 3.2-6.

*Answer:* The 2-V voltage source supplies 2 mW and the 3-V voltage source supplies –6 mW.

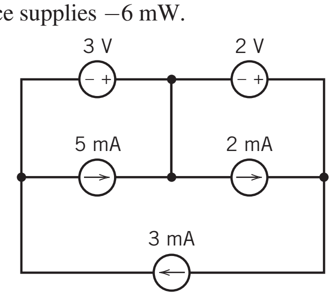



Figure P 3.2-6

**P 3.2-7** ⊕ What is the value of the resistance *R* in Figure P 3.2-7.

*Hint:* Assume an ideal ammeter. An ideal ammeter is equivalent to a short circuit.

*Answer:* $R = 4\ \Omega$

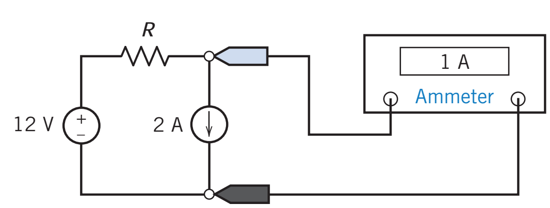



Figure P 3.2-7

**P 3.2-8** The voltmeter in Figure P 3.2-8 measures the value of the voltage across the current source to be 56 V. What is the value of the resistance $R$?

***Hint:*** Assume an ideal voltmeter. An ideal voltmeter is equivalent to an open circuit.

***Answer:*** $R = 10\ \Omega$

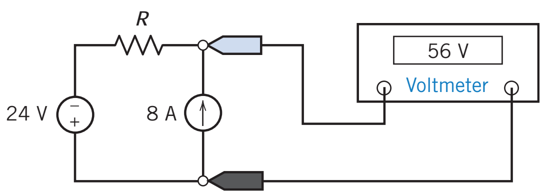



**Figure P 3.2-8**

**P 3.2-9** ⊕ Determine the values of the resistances $R_1$ and $R_2$ in Figure P 3.2-9.

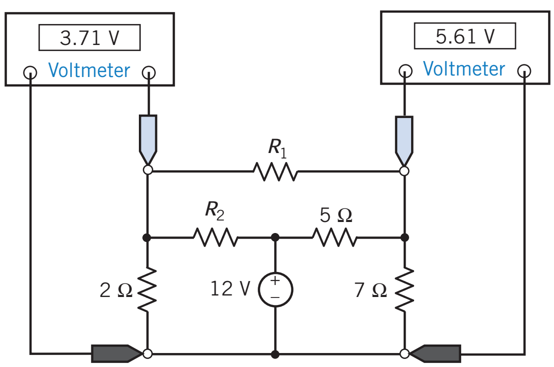



**Figure P 3.2-9**

**P 3.2-10** ⊕ The circuit shown in Figure P 3.2-10 consists of five voltage sources and four current sources. Express the power supplied by each source in terms of the voltage source voltages and the current source currents.

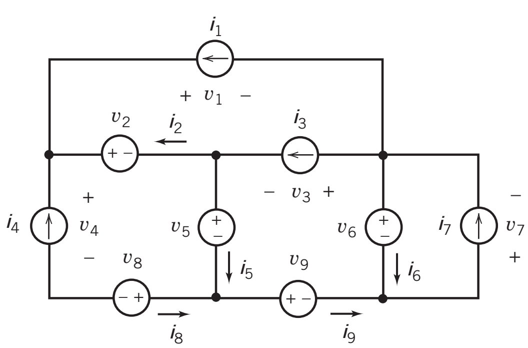



**Figure P 3.2-10**

**P 3.2-11** Determine the power received by each of the resistors in the circuit shown in Figure P 3.2-11.

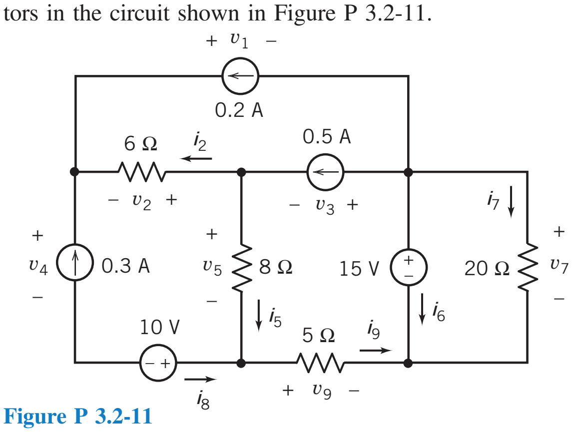



**Figure P 3.2-11**

**P 3.2-12** Determine the voltage and current of each of the circuit elements in the circuit shown in Figure P 3.2-12.

***Hint:*** You'll need to specify reference directions for the element voltages and currents. There is more than one way to do that, and your answers will depend on the reference directions that you choose.

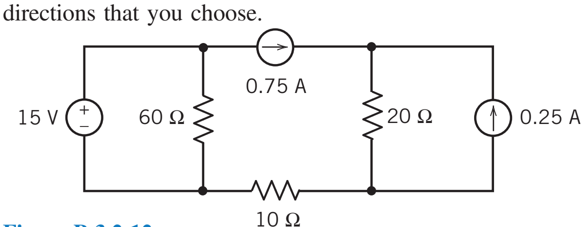



**Figure P 3.2-12**

**P 3.2-13** Determine the value of the current that is measured by the meter in Figure P 3.2-13.

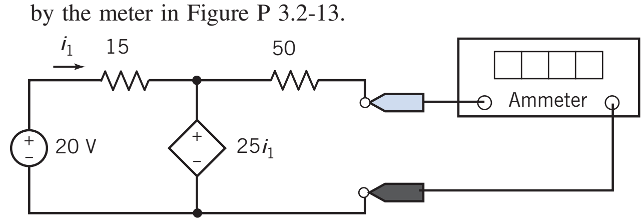



**Figure P 3.2-13**

**P 3.2-14** Determine the value of the voltage that is measured by the meter in Figure P 3.2-14.

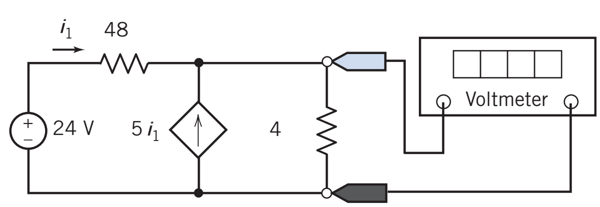



**Figure P 3.2-14**

**P 3.2-15** ⊕ Determine the value of the voltage that is measured by the meter in Figure P 3.2-15.

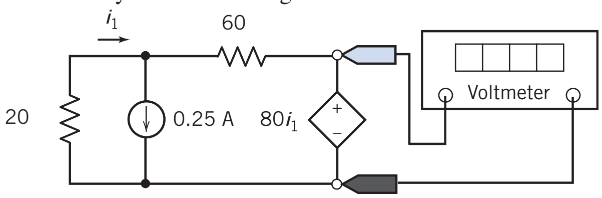



**Figure P 3.2-15**

**P 3.2-16** The voltage source in Figure P 3.2-16 supplies 3.6 W of power. The current source supplies 4.8 W. Determine the values of the resistances $R_1$ and $R_2$.

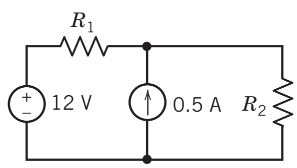



**Figure P 3.2-16**

**P 3.2-17** ⊕ Determine the current $i$ in Figure P 3.2-17.

***Answer:*** $i = 4$ A

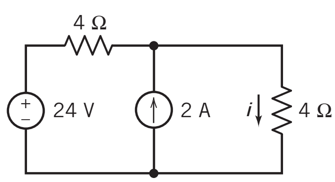



**Figure P 3.2-17**

**P 3.2-18** Determine the value of the current $i_m$ in Figure P 3.2-18*a*.

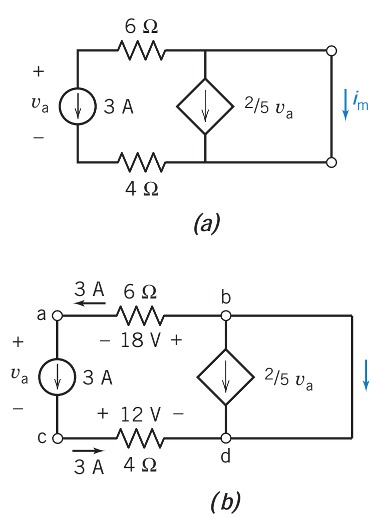



**Figure P 3.2-18** (*a*) A circuit containing a VCCS. (*b*) The circuit after labeling the nodes and some element currents and voltages.

***Hint:*** Apply KVL to the closed path a-b-d-c-a in Figure P 3.2-18*b* to determine $v_a$. Then apply KCL at node b to find $i_m$.

***Answer:*** $i_m = 9$ A

**P 3.2-19** ⊕ Determine the value of the voltage $v_6$ for the circuit shown in Figure P 3.2-19.

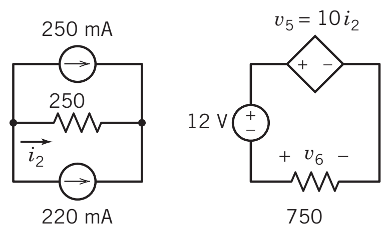



**Figure P 3.2-19**

**P 3.2-20** Determine the value of the voltage $v_6$ for the circuit shown in Figure P 3.2-20.

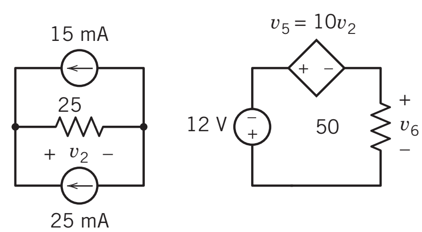



**Figure P 3.2-20**

**P 3.2-21** ⊕ Determine the value of the voltage $v_5$ for the circuit shown in Figure P 3.2-21.

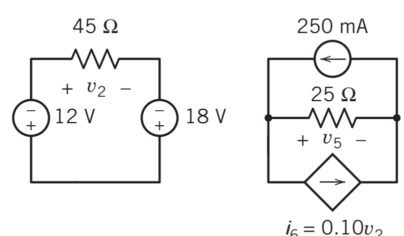



**Figure P 3.2-21**

**P 3.2-22** Determine the value of the voltage $v_5$ for the circuit shown in Figure P 3.2-22.

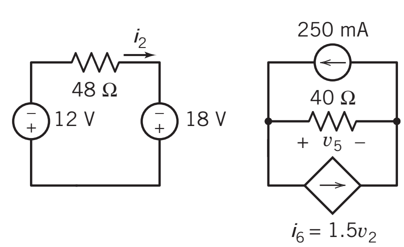



**Figure P 3.2-22**

**P 3.2-23** Determine the value of the voltage $v_6$ for the circuit shown in Figure P 3.2-23.

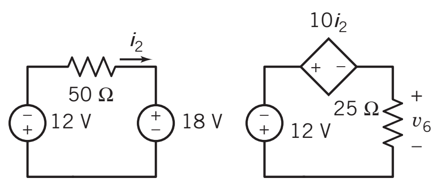



**Figure P 3.2-23**

**P 3.2-24** Determine the value of the voltage $v_5$ for the circuit shown in Figure P 3.2-24.

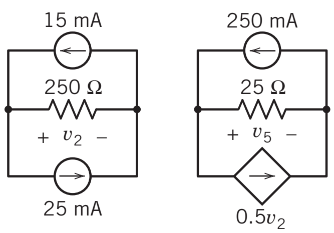



**Figure P 3.2-24**

**P 3.2-25** The voltage source in the circuit shown in Figure P 3.2-25 supplies 2 W of power. The value of the voltage across the 25-Ω resistor is $v_2 = 4$ V. Determine the values of the resistance $R_1$ and of the gain $G$ of the VCCS.

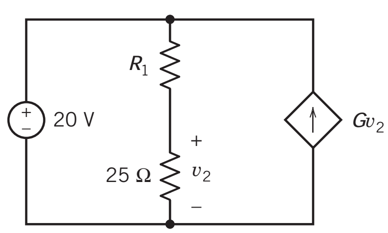



**Figure P 3.2-25**

**P 3.2-26** Consider the circuit shown in Figure P 3.2-26. Determine the values of

**(a)** The current $i_a$ in the 20-Ω resistor.
**(b)** The voltage $v_b$ across the 10-Ω resistor.
**(c)** The current $i_c$ in the independent voltage source.

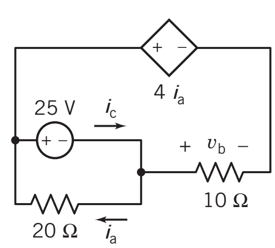



**Figure P 3.2-26**

**P 3.2-27** Consider the circuit shown in Figure P 3.2-27.

**(a)** Determine the values of the resistances.
**(b)** Determine the values of the power supplied by each current source.
**(c)** Determine the values of the power received by each resistor.

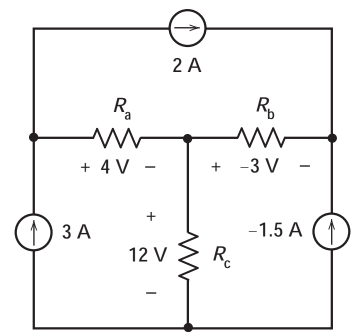



**Figure P 3.2-27**

**P 3.2-28** Consider the circuit shown in Figure P 3.2-28.

**(a)** Determine the value of the power supplied by each independent source.
**(b)** Determine the value of the power received by each resistor.
**(c)** Is power conserved?

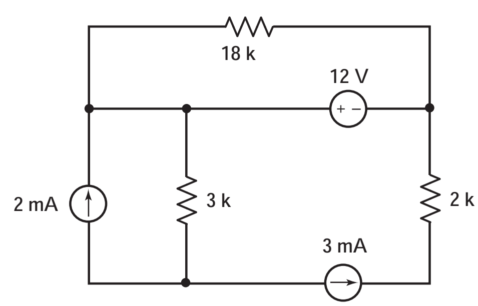



**Figure P 3.2-28**

**P 3.2-29** The voltage across the capacitor in Figure P 3.2-29 is $v(t) = 24 - 10e^{-25t}$ V for $t \geq 0$. Determine the voltage source current $i(t)$ for $t > 0$.

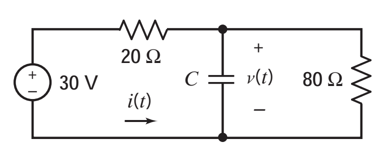



**Figure P 3.2-29**

**P 3.2-30** The current the inductor in Figure P 3.2-30 is given by $i(t) = 8 - 6e^{-25t}$ A for $t \geq 0$. Determine the voltage $v(t)$ across the 80-Ω resistor for $t > 0$.

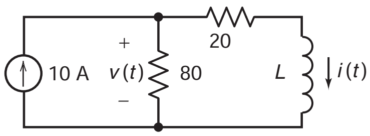



**Figure P 3.2-30**

### Section 3.3 Series Resistors and Voltage Division

**P 3.3-1** Use voltage division to determine the voltages $v_1$, $v_2$, $v_3$, and $v_4$ in the circuit shown in Figure P 3.3-1.

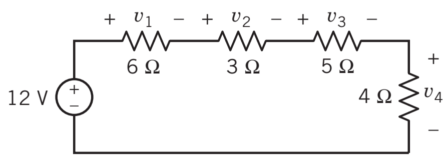



**Figure P 3.3-1**

**P 3.3-2** Consider the circuits shown in Figure P 3.3-2.

**(a)** Determine the value of the resistance $R$ in Figure P 3.3-2*b* that makes the circuit in Figure P 3.3-2*b* equivalent to the circuit in Figure P 3.3-2*a*.
**(b)** Determine the current $i$ in Figure P 3.3-2*b*. Because the circuits are equivalent, the current $i$ in Figure P 3.3-2*a* is equal to the current $i$ in Figure P 3.3-2*b*.
**(c)** Determine the power supplied by the voltage source.

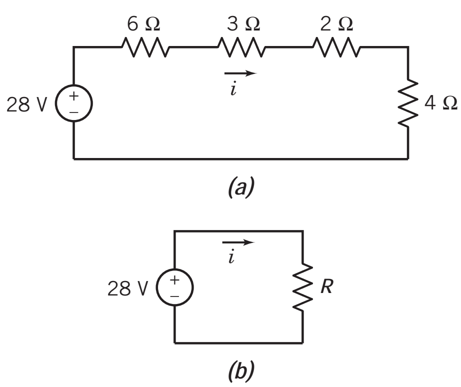



**Figure P 3.3-2**

**P 3.3-3** The ideal voltmeter in the circuit shown in Figure P 3.3-3 measures the voltage $v$.

**(a)** Suppose $R_2 = 50\ \Omega$. Determine the value of $R_1$.
**(b)** Suppose, instead, $R_1 = 50\ \Omega$. Determine the value of $R_2$.
**(c)** Suppose, instead, that the voltage source supplies 1.2 W of power. Determine the values of both $R_1$ and $R_2$.

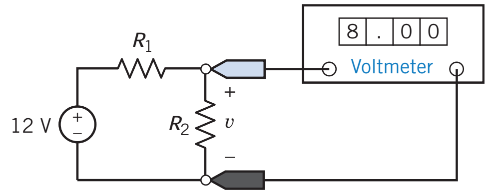



**Figure P 3.3-3**

**P 3.3-4** Determine the voltage $v$ in the circuit shown in Figure P 3.3-4.

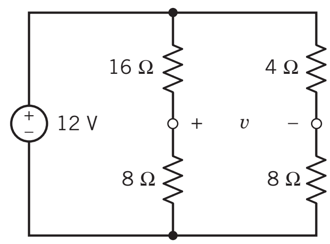



**Figure P 3.3-4**

**P 3.3-5** The model of a cable and load resistor connected to a source is shown in Figure P 3.3-5. Determine the appropriate cable resistance $R$ so that the output voltage $v_o$ remains between 9 V and 13 V when the source voltage $v_s$ varies between 20 V and 28 V. The cable resistance can assume integer values only in the range $20 < R < 100\ \Omega$.

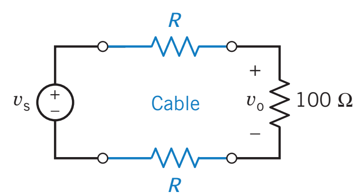



**Figure P 3.3-5** Circuit with a cable.

**P 3.3-6** The input to the circuit shown in Figure P 3.3-6 is the voltage of the voltage source $v_a$. The output of this circuit is the voltage measured by the voltmeter $v_b$. This circuit produces an output that is proportional to the input, that is,

$$v_b = k\, v_a$$

where $k$ is the constant of proportionality.

**(a)** Determine the value of the output, $v_b$, when $R = 180\ \Omega$ and $v_a = 18$ V.
**(b)** Determine the value of the power supplied by the voltage source when $R = 180\ \Omega$ and $v_a = 18$ V.
**(c)** Determine the value of the resistance, $R$, required to cause the output to be $v_b = 2$ V when the input is $v_a = 18$ V.
**(d)** Determine the value of the resistance, $R$, required to cause $v_b = 0.2 v_a$ (that is, the value of the constant of proportionality is $k = 0.2$).

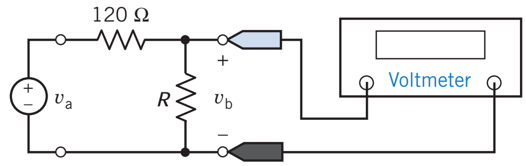



**Figure P 3.3-6**

**P 3.3-7** ⊕ Determine the value of voltage $v$ in the circuit shown in Figure P 3.3-7.

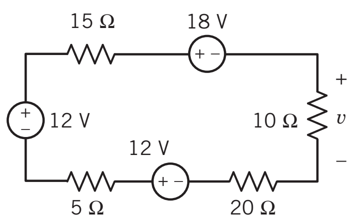



**Figure P 3.3-7**

**P 3.3-8** Determine the power supplied by the dependent source in the circuit shown in Figure P 3.3-8.

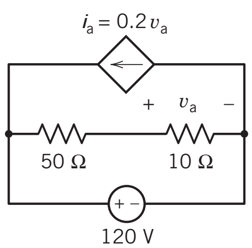



**Figure P 3.3-8**

**P 3.3-9** A potentiometer can be used as a transducer to convert the rotational position of a dial to an electrical quantity. Figure P 3.3-9 illustrates this situation. Figure P 3.3-9*a* shows a potentiometer having resistance $R_p$ connected to a voltage source. The potentiometer has three terminals, one at each end and one connected to a sliding contact called a wiper. A voltmeter measures the voltage between the wiper and one end of the potentiometer.

Figure P 3.3-9*b* shows the circuit after the potentiometer is replaced by a model of the potentiometer that consists of two resistors. The parameter $a$ depends on the angle $\theta$ of the dial. Here $a = \frac{\theta}{360°}$, and $\theta$ is given in degrees. Also, in Figure P 3.3-9*b*, the voltmeter has been replaced by an open circuit, and the voltage measured by the voltmeter $v_m$ has been labeled. The input to the circuit is the angle $\theta$, and the output is the voltage measured by the meter $v_m$.

**(a)** Show that the output is proportional to the input.

**(b)** Let $R_p = 1$ kΩ and $v_s = 24$ V. Express the output as a function of the input. What is the value of the output when $\theta = 45°$? What is the angle when $v_m = 10$ V?

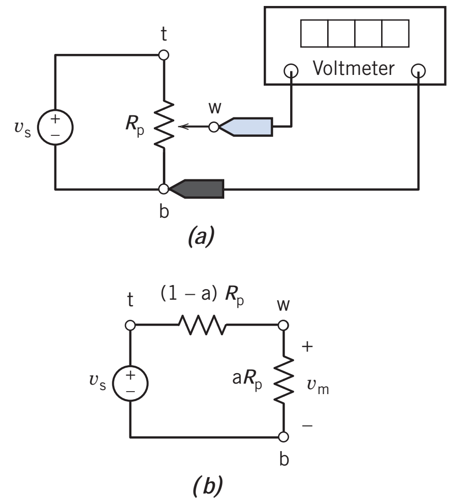



**Figure P 3.3-9**

**P 3.3-10** Determine the value of the voltage measured by the meter in Figure P 3.3-10.

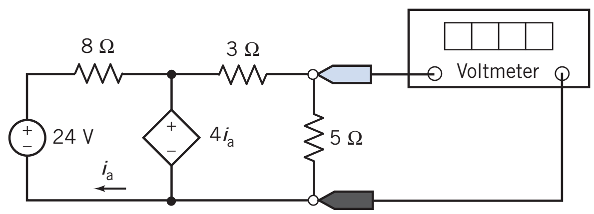



**Figure P 3.3-10**

**P 3.3-11** ⊕ For the circuit of Figure P 3.3-11, find the voltage $v_3$ and the current $i$ and show that the power delivered to the three resistors is equal to that supplied by the source.

***Answer:*** $v_3 = 3$ V, $i = 1$ A

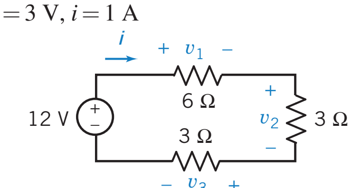



**Figure P 3.3-11**

**P 3.3-12** Consider the voltage divider shown in Figure P 3.3-12 when $R_1 = 8$ Ω. It is desired that the output power absorbed by $R_1$ be 4.5 W. Find the voltage $v_o$ and the required source $v_s$.

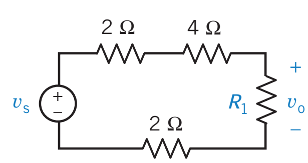



**Figure P 3.3-12**

**P 3.3-13** Consider the voltage divider circuit shown in Figure P 3.3-13. The resistor $R$ represents a temperature sensor. The resistance $R$, in Ω, is related to the temperature $T$, in °C, by the equation

$$R = 50 + \frac{1}{2}T$$

**(a)** Determine the meter voltage, $v_m$, corresponding to temperatures 0°C, 75°C, and 100°C.
**(b)** Determine the temperature $T$ corresponding to the meter voltages 8 V, 10 V, and 15 V.

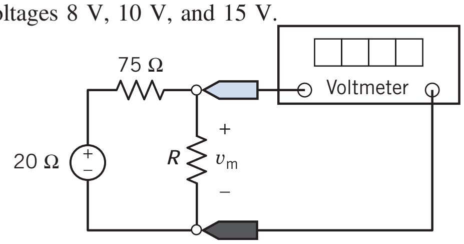



**Figure P 3.3-13**

**P 3.3-14** ⊕ Consider the circuit shown in Figure P 3.3-14.

**(a)** Determine the value of the resistance $R$ required to cause $v_o = 17.07$ V.
**(b)** Determine the value of the voltage $v_o$ when $R$ = 14 Ω.
**(c)** Determine the power supplied by the voltage source when $v_o = 14.22$ V.

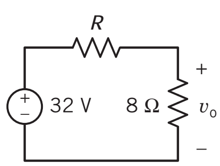



**Figure P 3.3-14**

**P 3.3-15** ⊕ Figure P 3.3-15 shows four similar but slightly different circuits. Determine the values of the voltages $v_1$, $v_2$, $v_3$, and $v_4$.

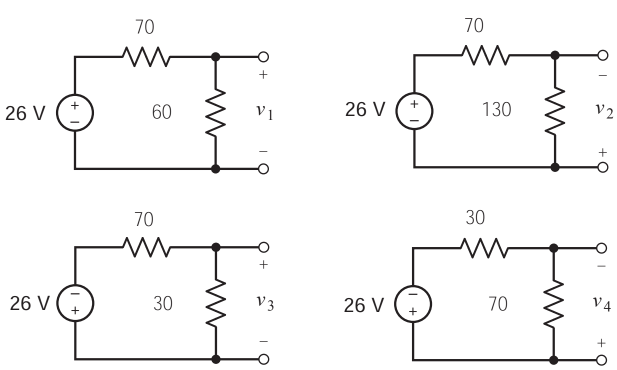



**Figure P 3.3-15**

**P 3.3-16** ⊕ Figure P 3.3-16 shows four similar but slightly different circuits. Determine the values of the voltages $v_1$, $v_2$, $v_3$, and $v_4$.

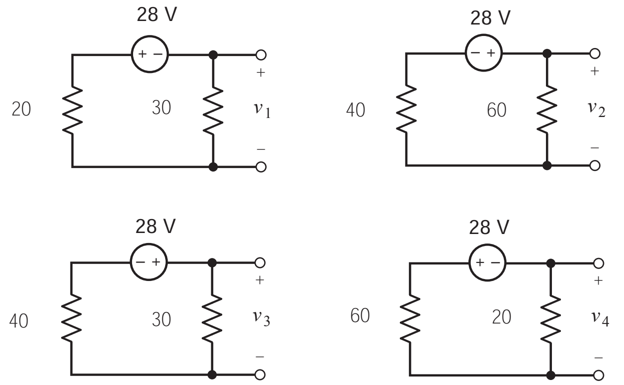



**Figure P 3.3-16**

**P 3.3-17** The input to the circuit shown in Figure P 3.3-17 is the voltage source voltage

$$v_s(t) = 12\cos(377t) \text{ mV}$$

The output is the voltage $v_o$(t). Determine $v_o$(t).

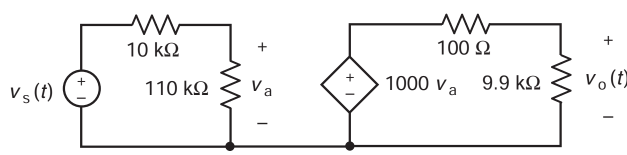



**Figure P 3.3-17**

### Section 3.4 Parallel Resistors and Current Division

**P 3.4-1** ⊕ Use current division to determine the currents $i_1$, $i_2$, $i_3$, and $i_4$ in the circuit shown in Figure P 3.4-1.

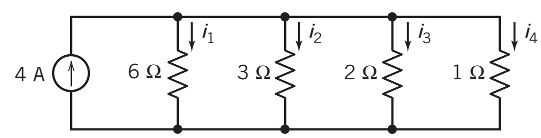



**Figure P 3.4-1**

**P 3.4-2** ⊕ Consider the circuits shown in Figure P 3.4-2.

**(a)** Determine the value of the resistance $R$ in Figure P 3.4-2*b* that makes the circuit in Figure P 3.4-2*b* equivalent to the circuit in Figure P 3.4-2*a*.
**(b)** Determine the voltage $v$ in Figure P 3.4-2*b*. Because the circuits are equivalent, the voltage $v$ in Figure P 3.4-2*a* is equal to the voltage $v$ in Figure P 3.4-2*b*.
**(c)** Determine the power supplied by the current source.

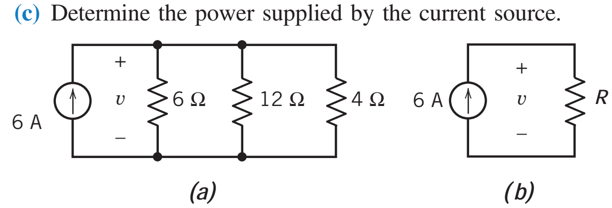



**Figure P 3.4-2**

**P 3.4-3** ⊕ The ideal voltmeter in the circuit shown in Figure P 3.4-3 measures the voltage $v$.

**(a)** Suppose $R_2 = 6\ \Omega$. Determine the value of $R_1$ and of the current $i$.
**(b)** Suppose, instead, $R_1 = 6\ \Omega$. Determine the value of $R_2$ and of the current $i$.
**(c)** Instead, choose $R_1$ and $R_2$ to minimize the power absorbed by any one resistor.

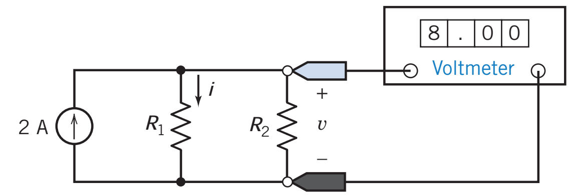



**Figure P 3.4-3**

**P 3.4-4** Determine the current $i$ in the circuit shown in Figure P 3.4-4.

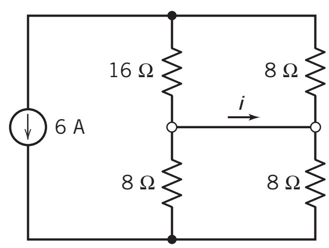



**Figure P 3.4-4**

**P 3.4-5** Consider the circuit shown in Figure P 3.4-5 when $4\ \Omega \le R_1 \le 6\ \Omega$ and $R_2 = 10\ \Omega$. Select the source $i_s$ so that $v_o$ remains between 9 V and 13 V.

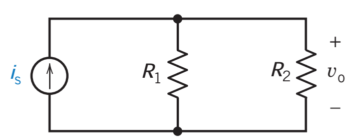



**Figure P 3.4-5**

**P 3.4-6** Figure P 3.4-6 shows a transistor amplifier. The values of $R_1$ and $R_2$ are to be selected. Resistances $R_1$ and $R_2$ are used to bias the transistor, that is, to create useful operating conditions. In this problem, we want to select $R_1$ and $R_2$ so that $v_b = 5$ V. We expect the value of $i_b$ to be approximately 10 μA. When $i_1 \le 10 i_b$, it is customary to treat $i_b$ as negligible, that is, to assume $i_b = 0$. In that case, $R_1$ and $R_2$ comprise a voltage divider.

**(a)** Select values for $R_1$ and $R_2$ so that $v_b = 5$ V, and the total power absorbed by $R_1$ and $R_2$ is no more than 5 mW.
**(b)** An inferior transistor could cause $i_b$ to be larger than expected. Using the values of $R_1$ and $R_2$ from part (a), determine the value of $v_b$ that would result from $i_b = 15$ μA.

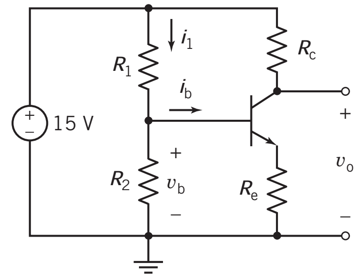



**Figure P 3.4-6**

**P 3.4-7** ⊕ Determine the value of the current $i$ in the circuit shown in Figure P 3.4-7.

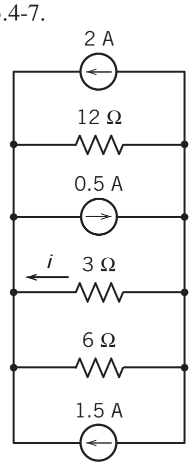



**Figure P 3.4-7**

**P 3.4-8** Determine the value of the voltage $v$ in Figure P 3.4-8.

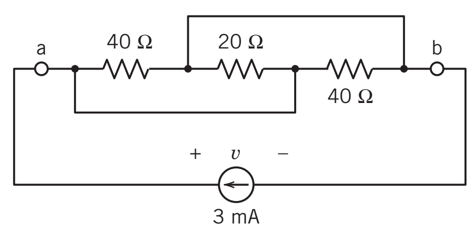



**Figure P 3.4-8**

**P 3.4-9** Determine the power supplied by the dependent source in Figure P 3.4-9.

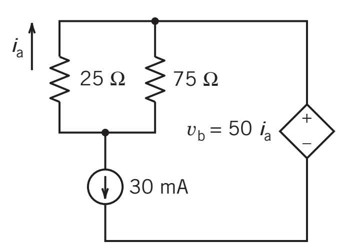



**Figure P 3.4-9**

**P 3.4-10** Determine the values of the resistances $R_1$ and $R_2$ for the circuit shown in Figure P 3.4-10.

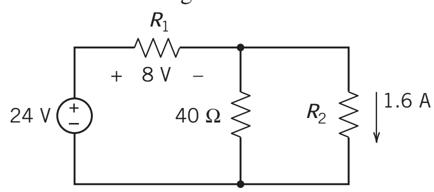



**Figure P 3.4-10**

**P 3.4-11** ⊕ Determine the values of the resistances $R_1$ and $R_2$ for the circuit shown in Figure P 3.4-11.

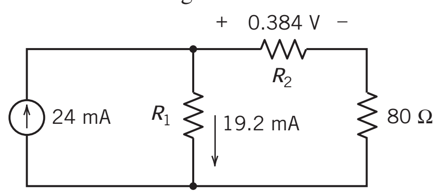



**Figure P 3.4-11**

**P 3.4-12** Determine the value of the current measured by the meter in Figure P 3.4-12.

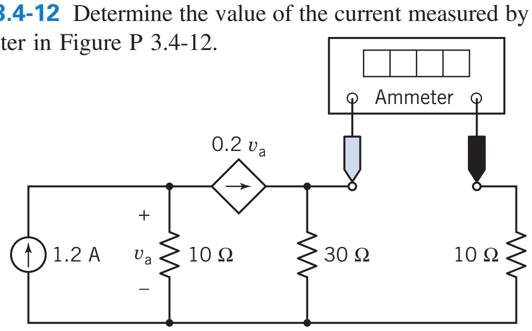



**Figure P 3.4-12**

**P 3.4-13** ⊕ Consider the combination of resistors shown in Figure P 3.4-13. Let $R_p$ denote the equivalent resistance.

**(a)** Suppose $20\ \Omega \le R \le 320\ \Omega$. Determine the corresponding range of values of $R_p$.
**(b)** Suppose, instead, $R = 0$ (a short circuit). Determine the value of $R_p$.
**(c)** Suppose, instead, $R = \infty$ (an open circuit). Determine the value of $R_p$.
**(d)** Suppose, instead, the equivalent resistance is $R_p = 40\ \Omega$. Determine the value of $R$.

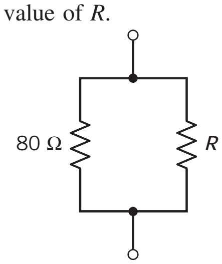



**Figure P 3.4-13**

**P 3.4-14** Consider the combination of resistors shown in Figure P 3.4-l4. Let $R_p$ denote the equivalent resistance.

**(a)** Suppose $40\ \Omega \le R \le 400\ \Omega$. Determine the corresponding range of values of $R_p$.
**(b)** Suppose, instead, $R = 0$ (a short circuit). Determine the value of $R_p$.
**(c)** Suppose, instead, $R = \infty$ (an open circuit). Determine the value of $R_p$.
**(d)** Suppose, instead, the equivalent resistance is $R_p = 80\ \Omega$. Determine the value of $R$.

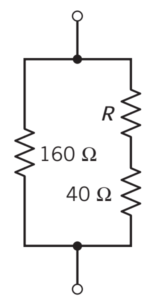



**Figure P 3.4-14**

**P 3.4-15** Consider the combination of resistors shown in Figure P 3.4-15. Let $R_p$ denote the equivalent resistance.

**(a)** Suppose $50\ \Omega \le R \le 800\ \Omega$. Determine the corresponding range of values of $R_p$.
**(b)** Suppose, instead, $R = 0$ (a short circuit). Determine the value of $R_p$.
**(c)** Suppose, instead, $R = \infty$ (an open circuit). Determine the value of $R_p$.
**(d)** Suppose, instead, the equivalent resistance is $R_p = 150\ \Omega$. Determine the value of $R$.

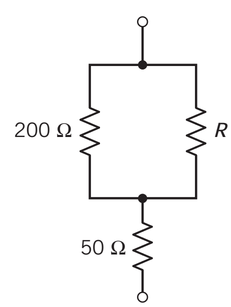



**Figure P 3.4-15**

**P 3.4-16** The input to the circuit shown in Figure P 3.4-16 is the source current $i_s$. The output is the current measured by the meter $i_o$. A current divider connects the source to the meter. Given the following observations:

**(a)** The input $i_s = 5$ A causes the output to be $i_o = 2$ A.
**(b)** When $i_s = 2$ A, the source supplies 48 W.

Determine the values of the resistances $R_1$ and $R_2$.

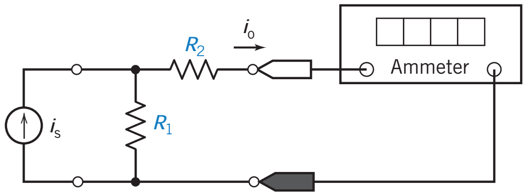



Figure P 3.4-16

**P 3.4-17** Figure P 3.4-17 shows four similar but slightly different circuits. Determine the values of the currents $i_1$, $i_2$, $i_3$, and $i_4$.

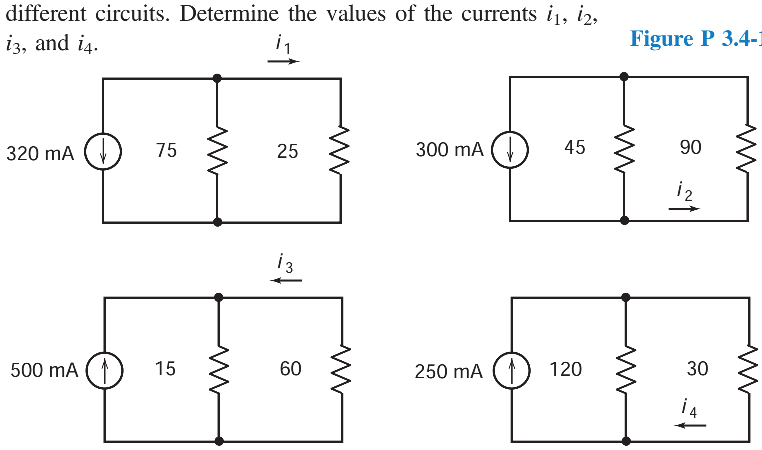



Figure P 3.4-17

**P 3.4-18** Figure P 3.4-18 shows four similar but slightly different circuits. Determine the values of the currents $i_1$, $i_2$, $i_3$, and $i_4$.

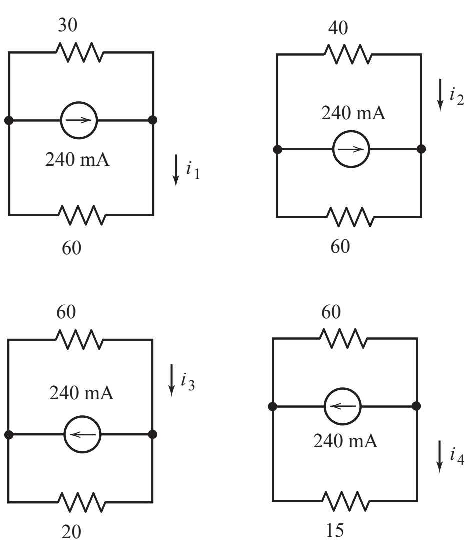



Figure P 3.4-18

**P 3.4-19** The input to the circuit shown in Figure P 3.4-19 is the current source current $I_s$. The output is the current $i_o$. The output of this circuit is proportion to the input, that is

$$i_o = kI_s$$

Determine the value of the constant of proportionality $k$.

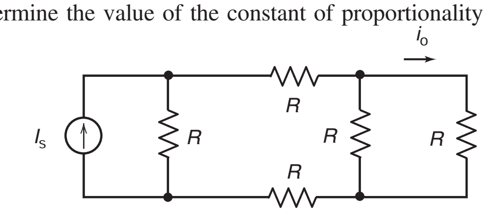



Figure P 3.4-19

**P 3.4-20** The input to the circuit shown in Figure P 3.4-20 is the voltage source voltage $V_s$. The output is the voltage $v_o$. The output of this circuit is proportion to the input, that is

$$v_o = kV_s$$

Determine the value of the constant of proportionality $k$.

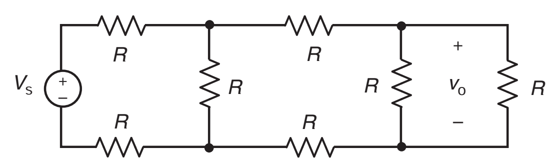



Figure P 3.4-20

## Section 3.5 Series Voltage Sources and Parallel Current Sources

**P 3.5-1** Determine the power supplied by each source in the circuit shown in Figure P 3.5-1.

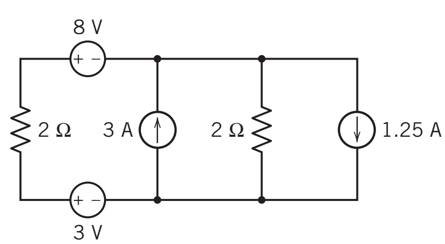



Figure P 3.5-1

**P 3.5-2** Determine the power supplied by each source in the circuit shown in Figure P 3.5-2.

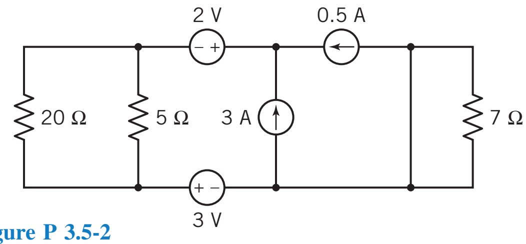



**Figure P 3.5-2**

**P 3.5-3** Determine the power received by each resistor in the circuit shown in Figure P 3.5-3.

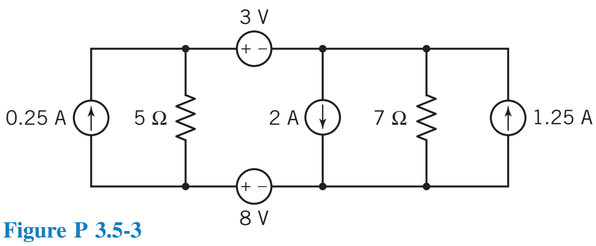



**Figure P 3.5-3**

## Section 3.6 Circuit Analysis

**P 3.6-1** ⊕ The circuit shown in Figure P 3.6-1*a* has been divided into two parts. In Figure P 3.6-1*b*, the right-hand part has been replaced with an equivalent circuit. The left-hand part of the circuit has not been changed.

**(a)** Determine the value of the resistance $R$ in Figure P 3.6-1*b* that makes the circuit in Figure P 3.6-1*b* equivalent to the circuit in Figure P 3.6-1*a*.
**(b)** Find the current $i$ and the voltage $v$ shown in Figure P 3.6-1*b*. Because of the equivalence, the current $i$ and the voltage $v$ shown in Figure P 3.6-1*a* are equal to the current $i$ and the voltage $v$ shown in Figure P 3.6-1*b*.
**(c)** Find the current $i_2$, shown in Figure P 3.6-1*a*, using current division.

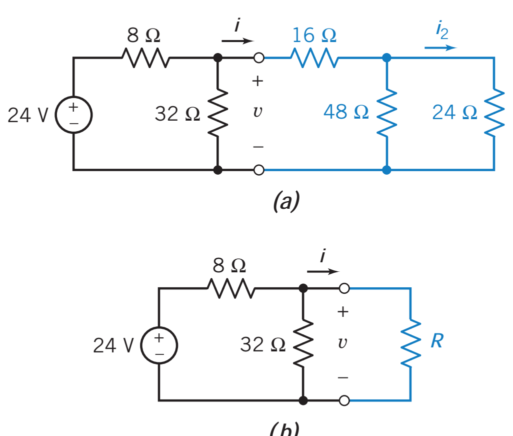



**Figure P 3.6-1**

**P 3.6-2** ⊕ The circuit shown in Figure P 3.6-2*a* has been divided into three parts. In Figure P 3.6-2*b*, the rightmost part has been replaced with an equivalent circuit. The rest of the circuit has not been changed. The circuit is simplified further in Figure 3.6-2*c*. Now the middle and rightmost parts have been replaced by a single equivalent resistance. The leftmost part of the circuit is still unchanged.

**(a)** Determine the value of the resistance $R_1$ in Figure P 3.6-2*b* that makes the circuit in Figure P 3.6-2*b* equivalent to the circuit in Figure P 3.6-2*a*.
**(b)** Determine the value of the resistance $R_2$ in Figure P 3.6-2*c* that makes the circuit in Figure P 3.6-2*c* equivalent to the circuit in Figure P 3.6-2*b*.
**(c)** Find the current $i_1$ and the voltage $v_1$ shown in Figure P 3.6-2*c*. Because of the equivalence, the current $i_1$ and the voltage $v_1$ shown in Figure P 3.6-2*b* are equal to the current $i_1$ and the voltage $v_1$ shown in Figure P 3.6-2*c*.

*Hint:* $24 = 6(i_1 - 2) + i_1 R_2$

**(d)** Find the current $i_2$ and the voltage $v_2$ shown in Figure P 3.6-2*b*. Because of the equivalence, the current $i_2$ and the voltage $v_2$ shown in Figure P 3.6-2*a* are equal to the current $i_2$ and the voltage $v_2$ shown in Figure P 3.6-2*b*.

*Hint:* Use current division to calculate $i_2$ from $i_1$.

**(e)** Determine the power absorbed by the 3-Ω resistance shown at the right of Figure P 3.6-2*a*.

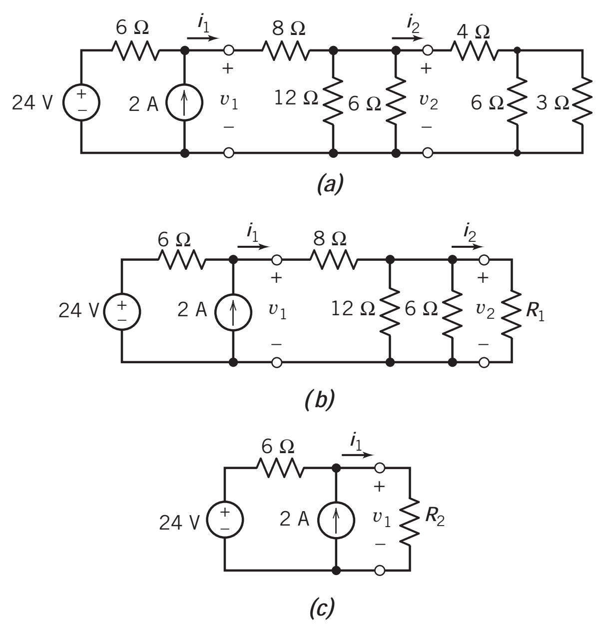



**Figure P 3.6-2**

**P 3.6-3** Find $i$, using appropriate circuit reductions and the current divider principle for the circuit of Figure P 3.6-3.

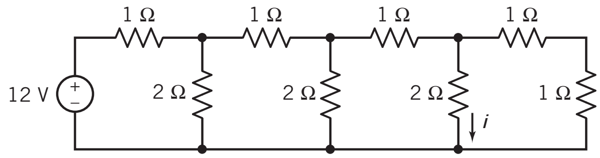



**Figure P 3.6-3**

**P 3.6-4**

**(a)** Determine values of $R_1$ and $R_2$ in Figure P 3.6-4*b* that make the circuit in Figure P 3.6-4*b* equivalent to the circuit in Figure P 3.6-4*a*.

**(b)** Analyze the circuit in Figure P 3.6-4*b* to determine the values of the currents $i_a$ and $i_b$.

**(c)** Because the circuits are equivalent, the currents $i_a$ and $i_b$ shown in Figure P 3.6-4*b* are equal to the currents $i_a$ and $i_b$ shown in Figure P 3.6-4*a*. Use this fact to determine values of the voltage $v_1$ and current $i_2$ shown in Figure P 3.6-4*a*.

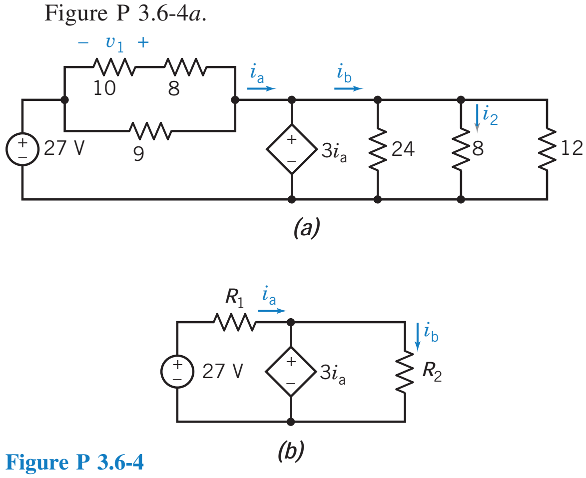



**Figure P 3.6-4**

**P 3.6-5** The voltmeter in the circuit shown in Figure P 3.6-5 shows that the voltage across the 30-Ω resistor is 6 volts. Determine the value of the resistance $R_1$.

***Hint:*** Use the voltage division twice.

***Answer:*** $R_1 = 40\ \Omega$

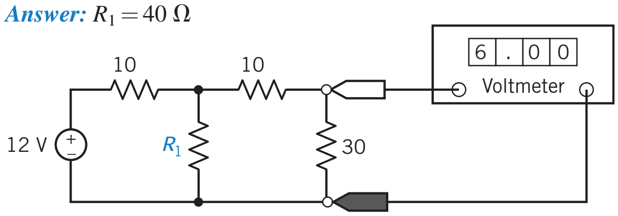



**Figure P 3.6-5**

**P 3.6-6** Determine the voltages $v_a$ and $v_c$ and the currents $i_b$ and $i_d$ for the circuit shown in Figure P 3.6-6.

***Answer:*** $v_a = -2$ V, $v_c = 6$ V, $i_b = -16$ mA, and $i_d = 2$ mA

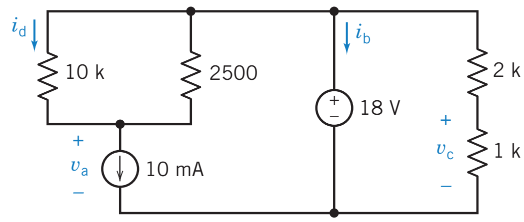



**Figure P 3.6-6**

**P 3.6-7** Determine the value of the resistance $R$ in Figure P 3.6-7.

***Answer:*** $R = 28$ kΩ

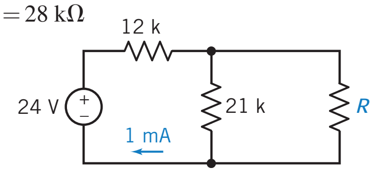



**Figure P 3.6-7**

**P 3.6-8** Most of us are familiar with the effects of a mild electric shock. The effects of a severe shock can be devastating and often fatal. Shock results when current is passed through the body. A person can be modeled as a network of resistances. Consider the model circuit shown in Figure P 3.6-8. Determine the voltage developed across the heart and the current flowing through the heart of the person when he or she firmly grasps one end of a voltage source whose other end is connected to the floor. The heart is represented by $R_h$. The floor has resistance to current flow equal to $R_f$, and the person is standing barefoot on the floor. This type of accident might occur at a swimming pool or boat dock. The upper-body resistance $R_u$ and lower-body resistance $R_L$ vary from person to person.

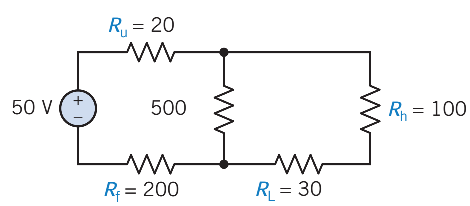



**Figure P 3.6-8**

**P 3.6-9** Determine the value of the current $i$ in Figure P 3.6-9.

***Answer:*** $i = 0.5$ mA

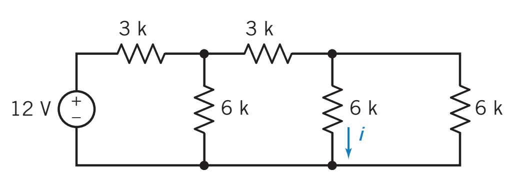



**Figure P 3.6-9**

**P 3.6-10** Determine the values of $i_a$, $i_b$, and $v_c$ in Figure P 3.6-10.

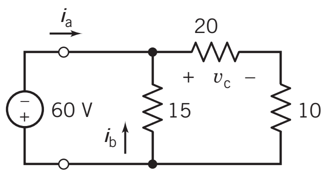



Figure P 3.6-10

**P 3.6-11** Find $i$ and $R_{eq\ a\text{-}b}$ if $v_{ab} = 40$ V in the circuit of Figure P 3.6-11.

*Answer:* $R_{eq\ a-b} = 8\ \Omega$, $i = 5/6$ A

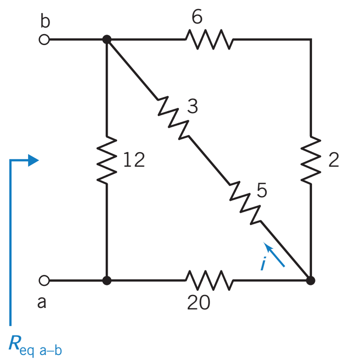



Figure P 3.6-11

**P 3.6-12** The ohmmeter in Figure P 3.6-12 measures the equivalent resistance $R_{eq}$ of the resistor circuit. The value of the equivalent resistance $R_{eq}$ depends on the value of the resistance $R$.

**(a)** Determine the value of the equivalent resistance $R_{eq}$ when $R = 9\ \Omega$.

**(b)** Determine the value of the resistance $R$ required to cause the equivalent resistance to be $R_{eq} = 12\ \Omega$.

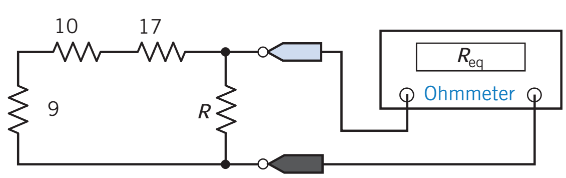



Figure P 3.6-12

**P 3.6-13** Find the $R_{eq}$ at terminals a-b in Figure P 3.6-13. Also determine $i$, $i_1$, and $i_2$.

*Answer:* $R_{eq} = 8\ \Omega$, $i = 5$ A, $i_1 = 5/3$ A, $i_2 = 5/2$ A

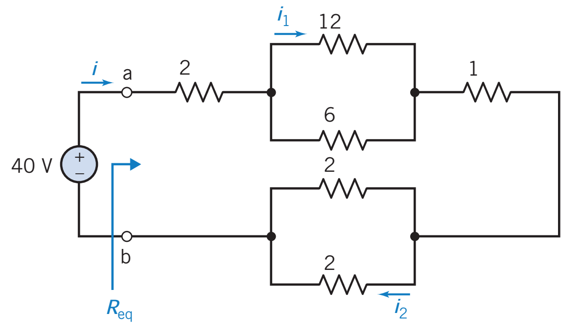



Figure P 3.6-13

**P 3.6-14** All of the resistances in the circuit shown in Figure P 3.6-14 are multiples of $R$. Determine the value of $R$.

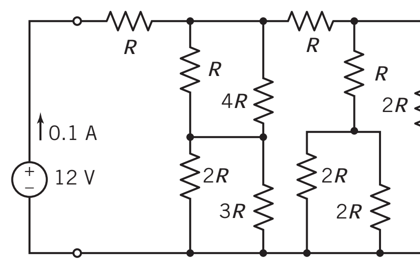



Figure P 3.6-14

**P 3.6-15** The circuit shown in Figure P 3.6-15 contains seven resistors, each having resistance $R$. The input to this circuit is the voltage source voltage $v_s$. The circuit has two outputs, $v_a$ and $v_b$. Express each output as a function of the input.

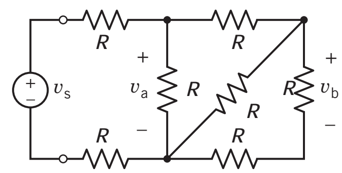



Figure P 3.6-15

**P 3.6-16** The circuit shown in Figure P 3.6-16 contains three 10-Ω, 1/4-W resistors. (Quarter-watt resistors can dissipate 1/4 W safely.) Determine the range of voltage source voltages $v_s$ such that none of the resistors absorbs more than 1/4 W of power.

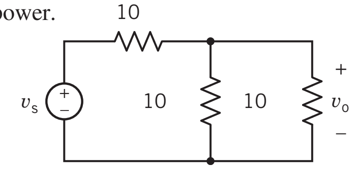



Figure P 3.6-16

**P 3.6-17** The four resistors shown in Figure P 3.6-17 represent strain gauges. Strain gauges are transducers that measure the strain that results when a resistor is stretched or compressed. Strain gauges are used to measure force, displacement, or pressure. The four strain gauges in Figure P 3.6-17 each have a nominal (unstrained) resistance of 200 Ω and can each absorb 0.5 mW safely. Determine the range of voltage source voltages $v_s$ such that no strain gauge absorbs more than 0.5 mW of power.

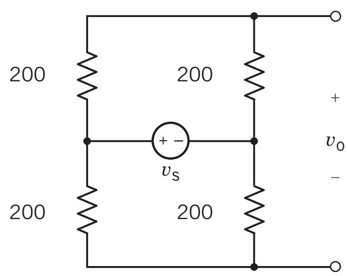



**Figure P 3.6-17**

**P 3.6-18** The circuit shown in Figure P 3.6-18*b* has been obtained from the circuit shown in Figure P 3.6-18*a* by replacing series and parallel combinations of resistances by equivalent resistances.

**(a)** Determine the values of the resistances $R_1$, $R_2$, and $R_3$ in Figure P 3.6-18*b* so that the circuit shown in Figure P 3.6-18*b* is equivalent to the circuit shown in Figure P 3.6-18*a*.

**(b)** Determine the values of $v_1$, $v_2$, and $i$ in Figure P 3.6-18*b*.

**(c)** Because the circuits are equivalent, the values of $v_1$, $v_2$, and $i$ in Figure P 3.6-18*a* are equal to the values of $v_1$, $v_2$, and $i$ in Figure P 3.6-18*b*. Determine the values of $v_4$, $i_5$, $i_6$, and $v_7$ in Figure P 3.6-18*a*.

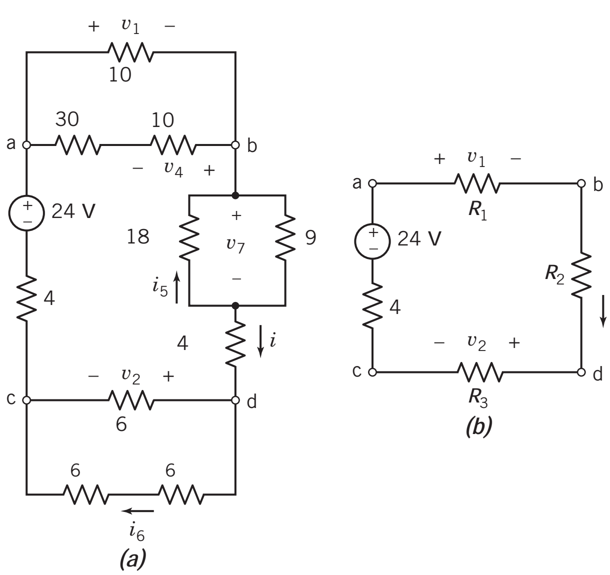



**Figure P 3.6-18**

**P 3.6-19** ⊕ Determine the values of $v_1$, $v_2$, $i_3$, $v_4$, $v_5$, and $i_6$ in Figure P 3.6-19.

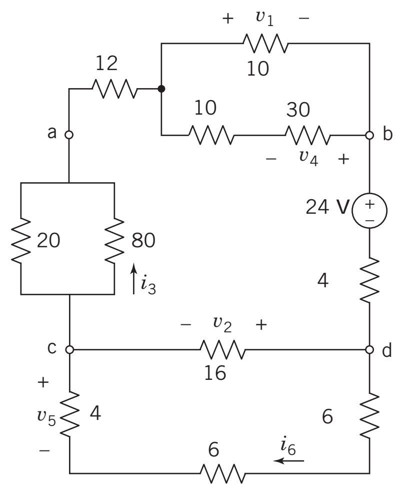



**Figure P 3.6-19**

**P 3.6-20** Determine the values of $i$, $v$, and $R_{eq}$ for the circuit shown in Figure P 3.6-20, given that $v_{ab} = 18$ V.

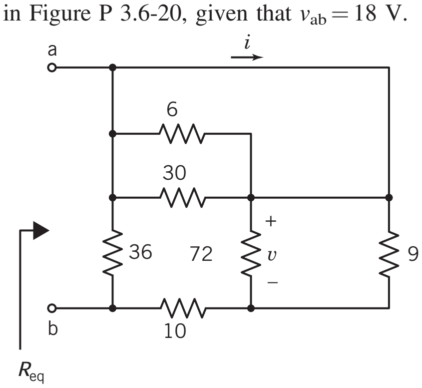



**Figure P 3.6-20**

**P 3.6-21** Determine the value of the resistance $R$ in the circuit shown in Figure P 3.6-21, given that $R_{eq} = 9\ \Omega$ .

***Answer:*** $R = 15\ \Omega$

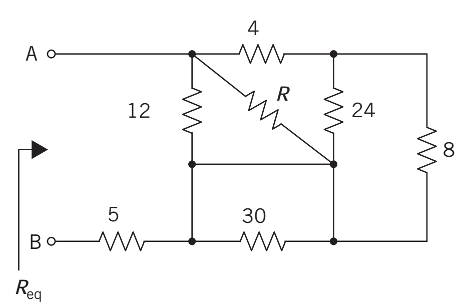



**Figure P 3.6-21**

**P 3.6-22** Determine the value of the resistance $R$ in the circuit shown in Figure P 3.6-22, given that $R_{eq} = 40\ \Omega$.

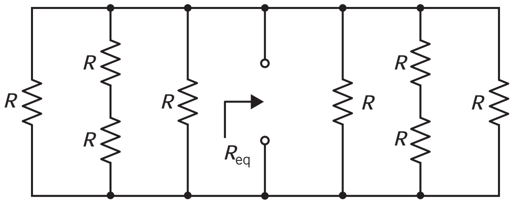



Figure P 3.6-22

**P 3.6-23** Determine the values of $r$, the gain of the CCVS, and $g$, the gain of the VCCS, for the circuit shown in Figure P 3.6-23.

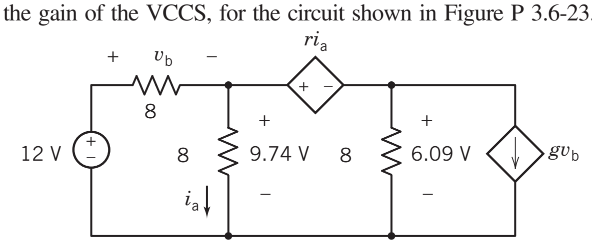



Figure P 3.6-23

**P 3.6-24** The input to the circuit in Figure P 3.6-24 is the voltage of the voltage source $v_s$. The output is the voltage measured by the meter, $v_o$. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

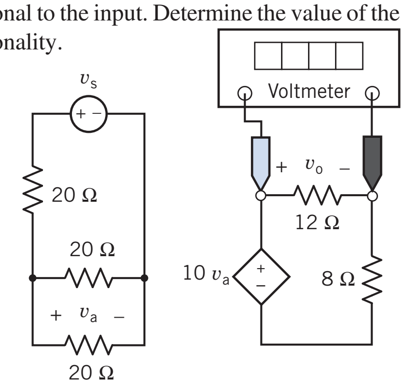



Figure P 3.6-24

**P 3.6-25** ⊕ The input to the circuit in Figure P 3.6-25 is the voltage of the voltage source $v_s$. The output is the current measured by the meter $i_o$. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

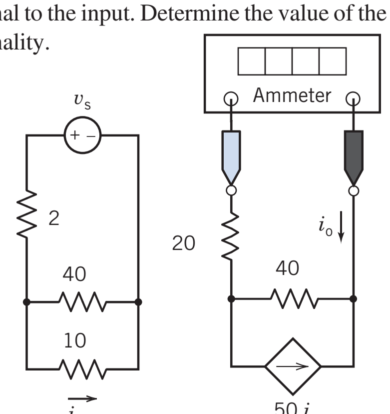



Figure P 3.6-25

**P 3.6-26** Determine the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-26.

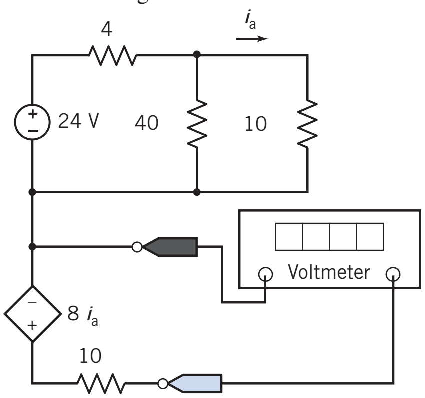



Figure P 3.6-26

**P 3.6-27** ⊕ Determine the current measured by the ammeter in the circuit shown in Figure P 3.6-27.

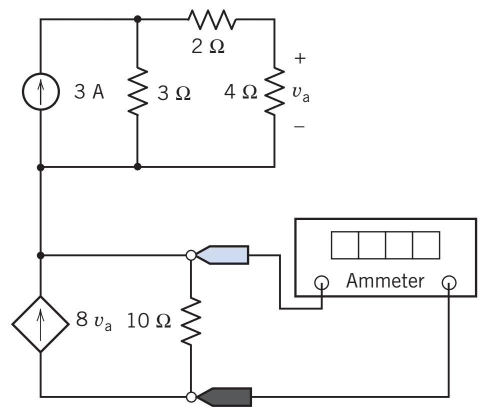



Figure P 3.6-27

**P 3.6-28** Determine the value of the resistance $R$ that causes the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-28 to be 6 V.

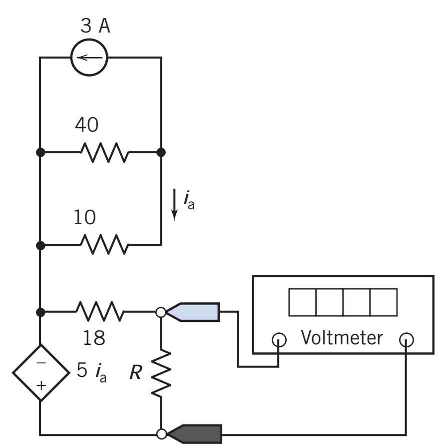



Figure P 3.6-28

**P 3.6-29** ⊕ The input to the circuit shown in Figure P 3.6-29 is the voltage of the voltage source $v_s$. The output is the current measured by the meter $i_m$.

**(a)** Suppose $v_s = 15$ V. Determine the value of the resistance $R$ that causes the value of the current measured by the meter to be $i_m = 12$ A.
**(b)** Suppose $v_s = 15$ V and $R = 80\ \Omega$. Determine the current measured by the ammeter.
**(c)** Suppose $R = 24\ \Omega$. Determine the value of the input voltage $v_s$ that causes the value of the current measured by the meter to be $i_m = 3$ A.

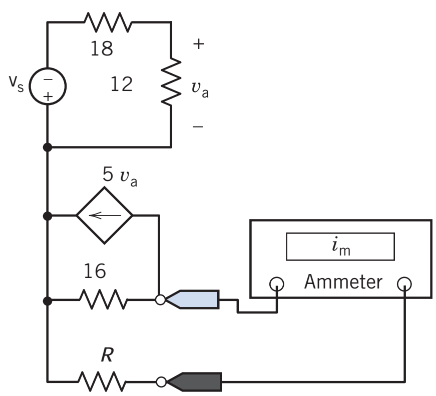



Figure P 3.6-29

**P 3.6-30** ⊕ The ohmmeter in Figure P 3.6-30 measures the equivalent resistance of the resistor circuit connected to the meter probes.

**(a)** Determine the value of the resistance $R$ required to cause the equivalent resistance to be $R_{eq} = 12\ \Omega$.
**(b)** Determine the value of the equivalent resistance when $R = 14\ \Omega$.

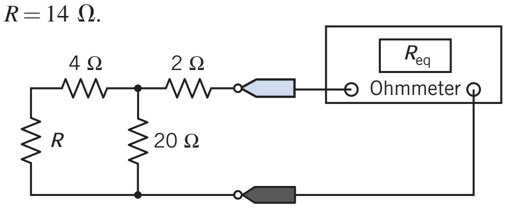



Figure P 3.6-30

**P 3.6-31** The voltmeter in Figure P 3.6-31 measures the voltage across the current source.

**(a)** Determine the value of the voltage measured by the meter.
**(b)** Determine the power supplied by each circuit element.

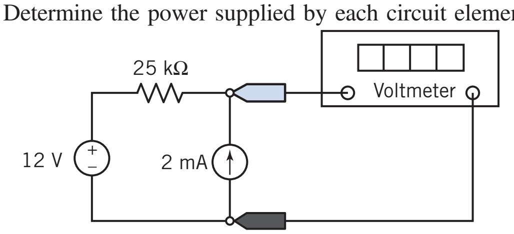



Figure P 3.6-31

**P 3.6-32** Determine the resistance measured by the ohmmeter in Figure P 3.6-32.

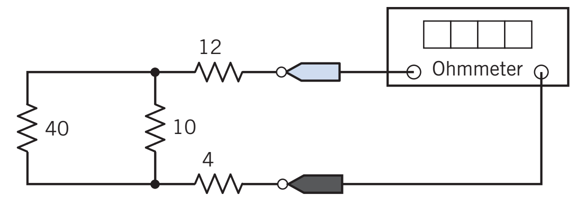



Figure P 3.6-32

**P 3.6-33** Determine the resistance measured by the ohmmeter in Figure P 3.6-33.

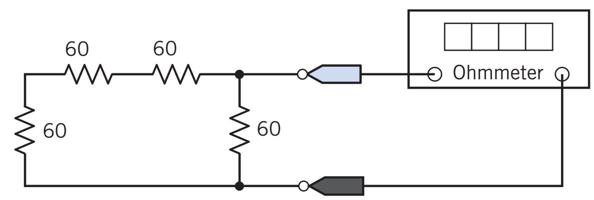



Figure P 3.6-33

**P 3.6-34** ⊕ Consider the circuit shown in Figure P 3.6-34. Given the values of the following currents and voltages:

$$i_1 = 0.625\ \text{A},\ v_2 = -25\ \text{V},\ i_3 = -1.25\ \text{A},\ \text{and}\ v_4 = -18.75\ \text{V},$$

determine the values of $R_1$, $R_2$, $R_3$, and $R_4$.

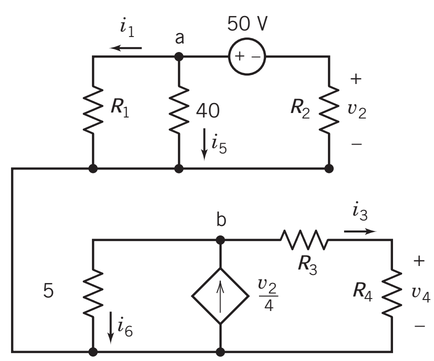



Figure P 3.6-34

**P 3.6-35** Consider the circuits shown in Figure P 3.6-35. The equivalent circuit is obtained from the original circuit by replacing series and parallel combinations of resistors with equivalent resistors. The value of the current in the equivalent circuit is $i_s = 0.8$ A. Determine the values of $R_1$, $R_2$, $R_5$, $v_2$, and $i_3$.

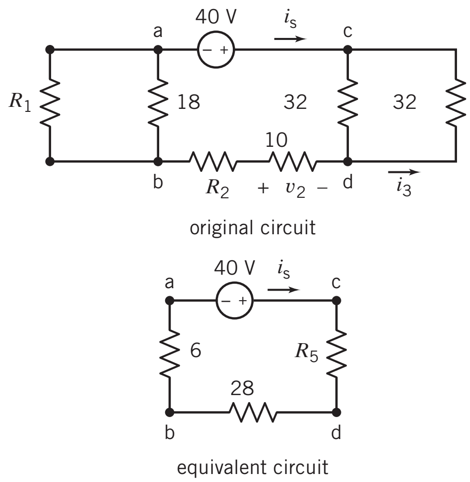



**Figure P 3.6-35**

**P 3.6-36** Consider the circuit shown in Figure P 3.6-36. Given

$$v_2 = \frac{2}{3} v_s,\ i_3 = \frac{1}{5} i_1,\ \text{and } v_4 = \frac{3}{8} v_2,$$

determine the values of $R_1$, $R_2$, and $R_4$.

***Hint:*** Interpret $v_2 = \frac{2}{3} v_s$, $i_3 = \frac{1}{5} i_1$, and $v_4 = \frac{3}{8} v_2$ as current and voltage division.

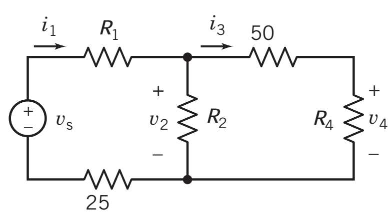



**Figure P 3.6-36**

**P 3.6-37** Consider the circuit shown in Figure P 3.6-37. Given

$$i_2 = \frac{2}{5} i_s,\ v_3 = \frac{2}{3} v_1,\ \text{and } i_4 = \frac{4}{5} i_2,$$

determine the values of $R_1$, $R_2$, and $R_4$.

***Hint:*** Interpret $i_2 = \frac{2}{5} i_s$, $v_3 = \frac{2}{3} v_1$, and $i_4 = \frac{4}{5} i_2$ as current and voltage division.

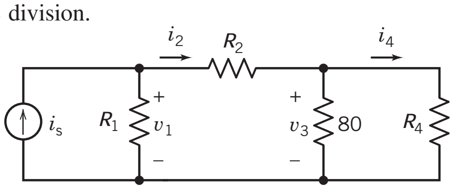



**Figure P 3.6-37**

**P 3.6-38** Consider the circuit shown in Figure P 3.6-38.

**(a)** Suppose $i_3 = \frac{1}{3} i_1$. What is the value of the resistance $R$?

**(b)** Suppose, instead, $v_2 = 4.8$ V. What is the value of the equivalent resistance of the parallel resistors?

**(c)** Suppose, instead, $R = 20\ \Omega$. What is the value of the current in the 40-Ω resistor?

***Hint:*** Interpret $i_3 = \frac{1}{3} i_1$ as current division.

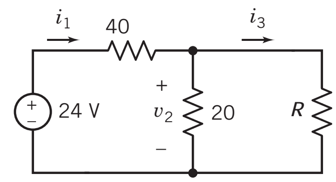



**Figure P 3.6-38**

**P 3.6-39** Consider the circuit shown in Figure P 3.6-39.

**(a)** Suppose $v_3 = \frac{1}{4} v_1$. What is the value of the resistance $R$?

**(b)** Suppose $i_2 = 1.2$ A. What is the value of the resistance $R$?

**(c)** Suppose $R = 70\ \Omega$. What is the voltage across the 20-Ω resistor?

**(d)** Suppose $R = 30\ \Omega$. What is the value of the current in this 30-Ω resistor?

***Hint:*** Interpret $v_3 = \frac{1}{4} v_1$ as voltage division.

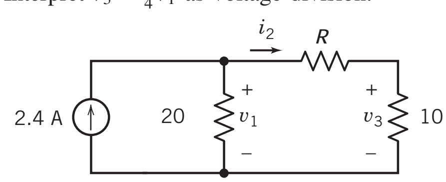



**Figure P 3.6-39**

**P 3.6-40** ⊕ Consider the circuit shown in Figure P 3.6-40. Given that the voltage of the dependent voltage source is $v_a = 8$ V, determine the values of $R_1$ and $v_o$.

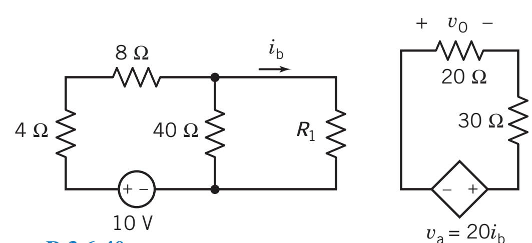



**Figure P 3.6-40**

**P 3.6-41** ⊕ Consider the circuit shown in Figure P 3.6-41. Given that the current of the dependent current source is $i_a = 2$ A, determine the values of $R_1$ and $i_o$.

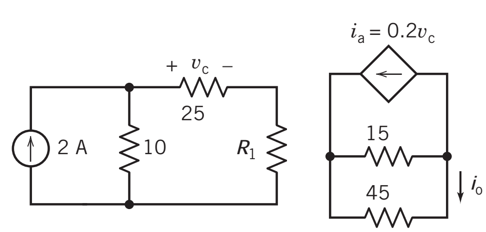



**Figure P 3.6-41**

**P 3.6-42** Determine the values of $i_a$, $i_b$, $i_2$, and $v_1$ in the circuit shown in Figure P 3.6-42.

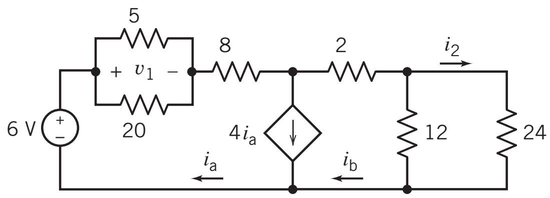



**Figure P 3.6-42**

**P 3.6-43** Determine the values of the resistance $R$ and current $i_a$ in the circuit shown in Figure P 3.6-43.

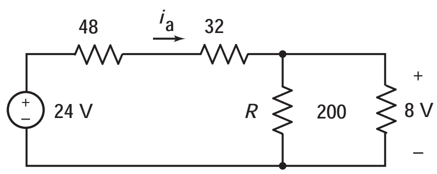



**Figure P 3.6-43**

**P 3.6-44** The input to the circuit shown in Figure P 3.6-44 is the voltage of the voltage source, 32 V. The output is the current in the 10-Ω resistor $i_o$. Determine the values of the resistance $R_1$ and of the gain of the dependent source $G$ that cause both the value of voltage across the 12 Ω to be $v_a$ = 10.38 V and the value of the output current to be $i_o$ = 0.4151 A.

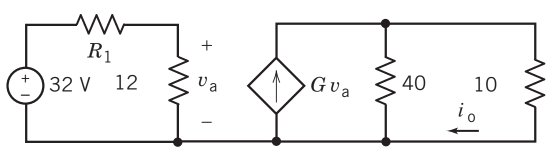



**Figure P 3.6-44**

**P 3.6-45** The equivalent circuit in Figure P 3.6-45 is obtained from the original circuit by replacing series and parallel combinations of resistors by equivalent resistors. The values of the currents in the equivalent circuit are $i_a = 3.5$ A and $i_b = -1.5$ A. Determine the values of the voltages $v_1$ and $v_2$ in the original circuit.

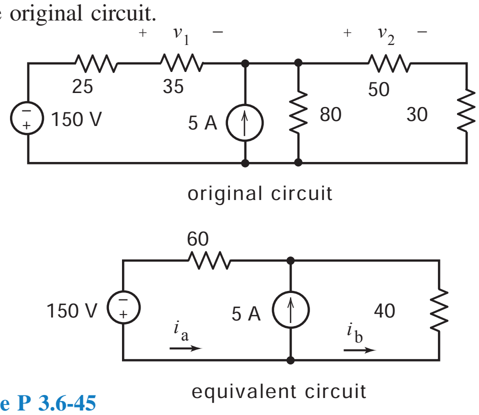



**Figure P 3.6-45**

**P 3.6-46** Figure P 3.6-46 shows three separate, similar circuits. In each a 12-V source is connected to a subcircuit consisting of three resistors. Determine the values of the voltage source currents $i_1$, $i_2$, and $i_3$. Conclude that while the voltage source voltage is 12 V in each circuit, the voltage source current depends on the subcircuit connected to the voltage source.

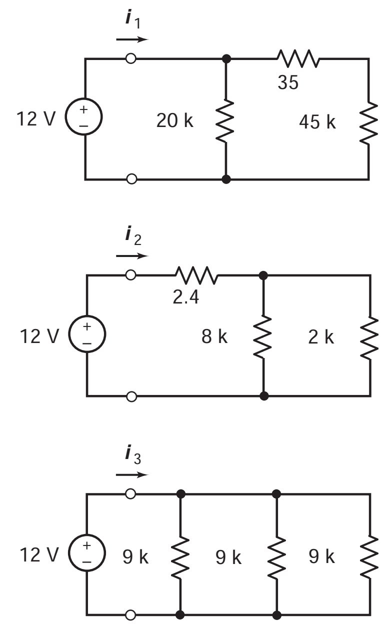



**Figure P 3.6-46**

**P 3.6-47** Determine the values of the voltages $v_1$ and $v_2$ and of the current $i_3$ in the circuit shown in Figure P 3.6-47.

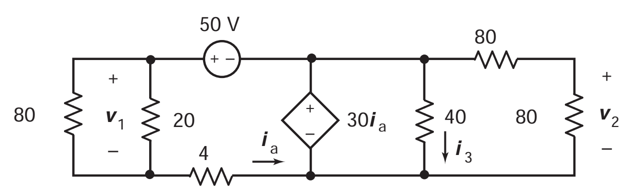



**Figure P 3.6-47**

## Section 3.7 Analyzing Resistive Circuits Using MATLAB

**P 3.7-1** Determine the power supplied by each of the sources, independent and dependent, in the circuit shown in Figure P 3.7-1.

*Hint:* Use the guidelines given in Section 3.7 to label the circuit diagram. Use MATLAB to solve the equations representing the circuit.

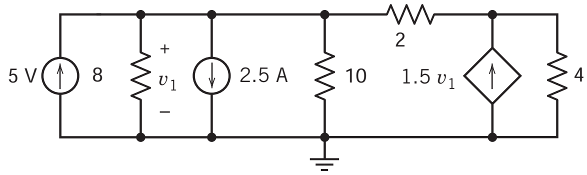



**Figure P 3.7-1**

**P 3.7-2** Determine the power supplied by each of the sources, independent and dependent, in the circuit shown in Figure P 3.7-2.

*Hint:* Use the guidelines given in Section 3.7 to label the circuit diagram. Use MATLAB to solve the equations representing the circuit.

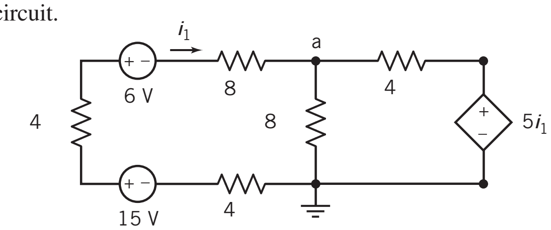



**Figure P 3.7-2**

**P 3.7-3** Determine the power supplied by each of the independent sources in the circuit shown in Figure P 3.7-3.

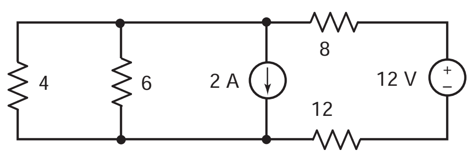



**Figure P 3.7-3**

**P 3.7-4** Determine the power supplied by each of the sources in the circuit shown in Figure P 3.7-4.

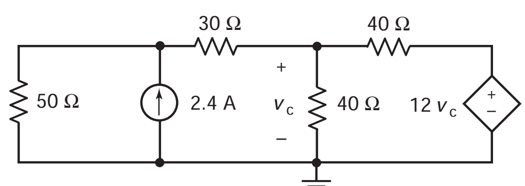



**Figure P 3.7-4**

## Section 3.8 How Can We Check . . . ?

**P 3.8-1** ⊕ A computer analysis program, used for the circuit of Figure P 3.8-1, provides the following branch currents and voltages: $i_1 = -0.833$ A, $i_2 = -0.333$ A, $i_3 = -1.167$ A, and $v = -2.0$ V. Are these answers correct?

*Hint:* Verify that KCL is satisfied at the center node and that KVL is satisfied around the outside loop consisting of the two 6-Ω resistors and the voltage source.

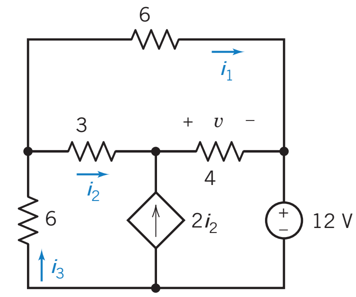



**Figure P 3.8-1**

**P 3.8-2** The circuit of Figure P 3.8-2 was assigned as a homework problem. The answer in the back of the textbook says the current $i$ is 1.25 A. Verify this answer, using current division.

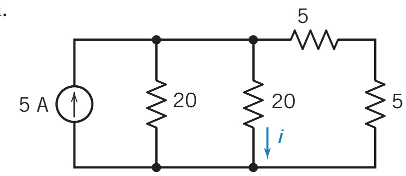



**Figure P 3.8-2**

**P 3.8-3** The circuit of Figure P 3.8-3 was built in the lab, and $v_o$ was measured to be 6.25 V. Verify this measurement, using the voltage divider principle.

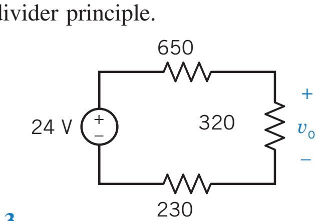



**Figure P 3.8-3**

**P 3.8-4** The circuit of Figure P 3.8-4 represents an auto's electrical system. A report states that $i_H = 9$ A, $i_B = -9$ A, and $i_A = 19.1$ A. Verify that this result is correct.

*Hint:* Verify that KCL is satisfied at each node and that KVL is satisfied around each loop.

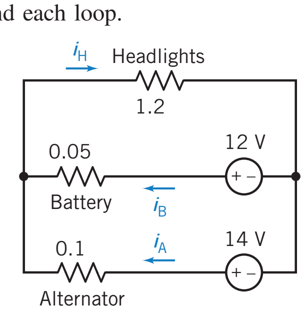



**Figure P 3.8-4** Electric circuit model of an automobile's electrical system.

**P 3.8-5** Computer analysis of the circuit in Figure P 3.8-5 shows that $i_a = -0.5$ mA, and $i_b = -2$ mA. Was the computer analysis done correctly?

***Hint:*** Verify that the KVL equations for all three meshes are satisfied when $i_a = -0.5$ mA, and $i_b = -2$ mA.

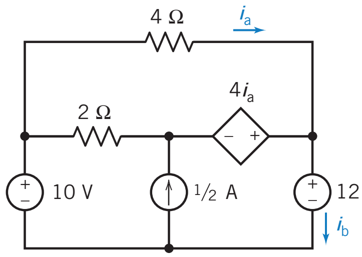



**Figure P 3.8-5**

**P 3.8-6** Computer analysis of the circuit in Figure P 3.8-6 shows that $i_a = 0.5$ mA and $i_b = 4.5$ mA. Was the computer analysis done correctly?

***Hint:*** First, verify that the KCL equations for all five nodes are satisfied when $i_a = 0.5$ mA, and $i_b = 4.5$ mA. Next, verify that the KVL equation for the lower left mesh (a-e-d-a) is satisfied. (The KVL equations for the other meshes aren't useful because each involves an unknown voltage.)

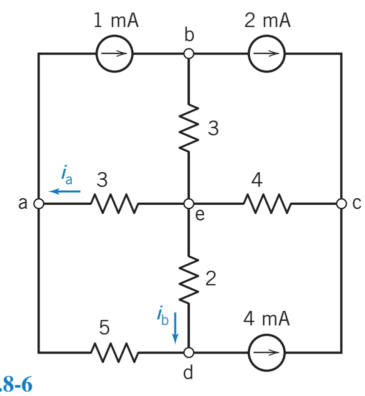



**Figure P 3.8-6**

**P 3.8-7** Verify that the element currents and voltages shown in Figure P 3.8-7 satisfy Kirchhoff's laws:

**(a)** Verify that the given currents satisfy the KCL equations corresponding to nodes a, b, and c.
**(b)** Verify that the given voltages satisfy the KVL equations corresponding to loops a-b-d-c-a and a-b-c-d-a.

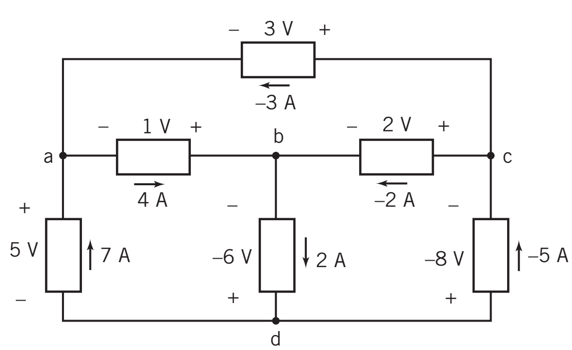
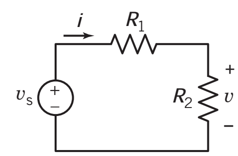



**Figure P 3.8-7**

***P 3.8-8** Figure P 3.8-8 shows a circuit and some corresponding data. The tabulated data provide values of the current $i$ and voltage $v$ corresponding to several values of the resistance $R_2$.

**(a)** Use the data in rows 1 and 2 of the table to find the values of $v_s$ and $R_1$.
**(b)** Use the results of part (a) to verify that the tabulated data are consistent.
**(c)** Fill in the missing entries in the table.




*(a)*

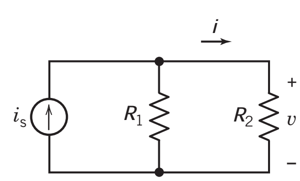
| $R_2$, Ω | $i$, A | $v$, V |
|---|---|---|
| 0 | 2.4 | 0 |
| 10 | 1.2 | 12 |
| 20 | 0.8 | 16 |
| 30 | ? | 18 |
| 40 | 0.48 | ? |

*(b)*

**Figure P 3.8-8**

***P 3.8-9** Figure P 3.8-9 shows a circuit and some corresponding data. The tabulated data provide values of the current $i$ and voltage $v$ corresponding to several values of the resistance $R_2$.

**(a)** Use the data in rows 1 and 2 of the table to find the values of $i_s$ and $R_1$.
**(b)** Use the results of part (a) to verify that the tabulated data are consistent.
**(c)** Fill in the missing entries in the table.




*(a)*

| $R_2$, Ω | $i$, A | $v$, V |
|---|---|---|
| 10 | 4/3 | 40/3 |
| 20 | 6/7 | 120/7 |
| 40 | 1/2 | 20 |
| 80 | ? | ? |

*(b)*

**Figure P 3.8-9**

# Design Problems

**DP 3-1** The circuit shown in Figure DP 3-1 uses a potentiometer to produce a variable voltage. The voltage $v_m$ varies as a knob connected to the wiper of the potentiometer is turned. Specify the resistances $R_1$ and $R_2$ so that the following three requirements are satisfied:

1. The voltage $v_m$ varies from 8 V to 12 V as the wiper moves from one end of the potentiometer to the other end of the potentiometer.
2. The voltage source supplies less than 0.5 W of power.
3. Each of $R_1$, $R_2$, and $R_P$ dissipates less than 0.25 W.

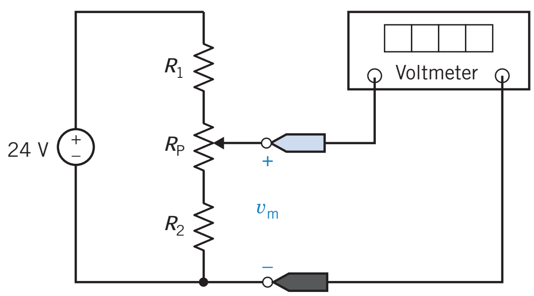



**Figure DP 3-1**

**DP 3-2** The resistance $R_L$ in Figure DP 3-2 is the equivalent resistance of a pressure transducer. This resistance is specified to be 200 Ω ± 5 percent. That is, $190\ \Omega \leq R_L \leq 210\ \Omega$. The voltage source is a 12 V ± 1 percent source capable of supplying 5 W. Design this circuit, using 5 percent, 1/8-watt resistors for $R_1$ and $R_2$, so that the voltage across $R_L$ is

$$v_o = 4\text{ V} \pm 10\%$$

(A 5 percent, 1/8-watt 100-Ω resistor has a resistance between 95 and 105 Ω and can safely dissipate 1/8-W continuously.)

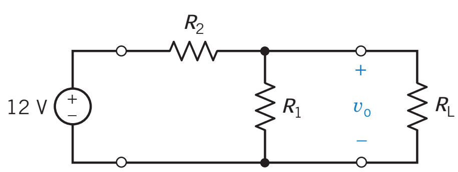
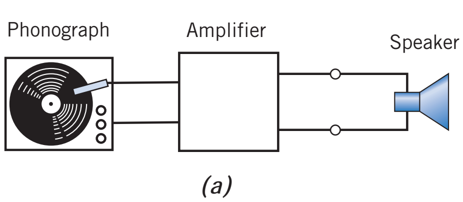



**Figure DP 3-2**

**DP 3-3** A phonograph pickup, stereo amplifier, and speaker are shown in Figure DP 3-3*a* and redrawn as a circuit model as shown in Figure DP 3-3*b*. Determine the resistance $R$ so that the voltage $v$ across the speaker is 16 V. Determine the power delivered to the speaker.



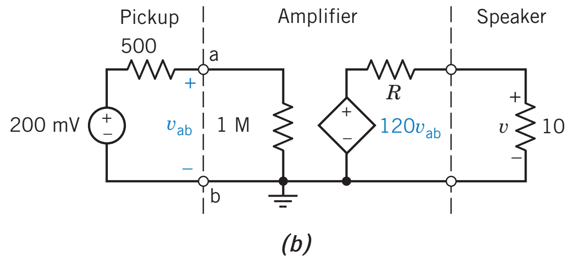




**Figure DP 3-3** A phonograph stereo system.

**DP 3-4** A Christmas tree light set is required that will operate from a 6-V battery on a tree in a city park. The heavy-duty battery can provide 9 A for the four-hour period of operation each night. Design a parallel set of lights (select the maximum number of lights) when the resistance of each bulb is 12 Ω.

**DP 3-5** The input to the circuit shown in Figure DP 3-5 is the voltage source voltage $v_s$. The output is the voltage $v_o$. The output is related to the input by

$$v_o = \frac{R_2}{R_1 + R_2} v_s = g v_s$$

The output of the voltage divider is proportional to the input. The constant of proportionality, $g$, is called the gain of the voltage divider and is given by

$$g = \frac{R_2}{R_1 + R_2}$$

The power supplied by the voltage source is

$$p = v_s i_s = v_s \left( \frac{v_s}{R_1 + R_2} \right) = \frac{{v_s}^2}{R_1 + R_2} = \frac{{v_s}^2}{R_{in}}$$

where

$$R_{in} = R_1 + R_2$$

is called the input resistance of the voltage divider.

**(a)** Design a voltage divider to have a gain, $g = 0.65$.

**(b)** Design a voltage divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 2500\ \Omega$.

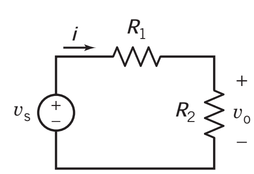



**Figure DP 3-5**

**DP 3-6** The input to the circuit shown in Figure DP 3-6 is the current source current $i_s$. The output is the current $i_o$. The output is related to the input by

$$i_o = \frac{R_1}{R_1 + R_2} i_s = g i_s$$

The output of the current divider is proportional to the input. The constant of proportionality $g$ is called the gain of the current divider and is given by

$$g = \frac{R_1}{R_1 + R_2}$$

The power supplied by the current source is

$$p = v_s i_s = \left[ i_s \left( \frac{R_1 R_2}{R_1 + R_2} \right) \right] i_s = \frac{R_1 R_2}{R_1 + R_2} i_s{}^2 = R_{in} i_s{}^2$$

where

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2}$$

is called the input resistance of the current divider.

**(a)** Design a current divider to have a gain, $g = 0.65$.
**(b)** Design a current divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 10000\ \Omega$.

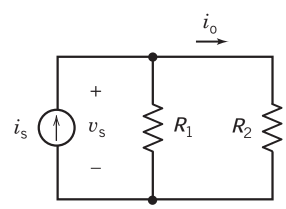



**Figure DP 3-6**

**DP 3-7** Design the circuit shown in Figure DP 3-7 to have an output $v_o = 8.5$ V when the input is $v_s = 12$ V. The circuit should require no more than 1 mW from the voltage source.

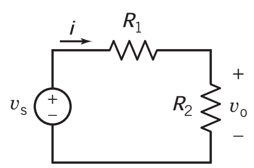



**Figure DP 3-7**

**DP 3-8** Design the circuit shown in Figure DP 3-8 to have an output $i_o = 1.8$ mA when the input is $i_s = 5$ mA. The circuit should require no more than 1 mW from the current source.

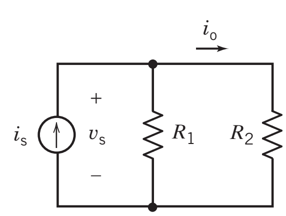



**Figure DP 3-8**

**DP 3-9** A thermistor is a temperature dependent resistor. The thermistor resistance $R_T$ is related to the temperature by the equation

$$R_T = R_T e^{b(1/T - 1/T_o)}$$

where $T$ has units of °K and $R$ is in Ohms. $R_0$ is resistance at temperature $T_0$ and the parameter β is in °K. For example, suppose that a particular thermistor has a resistance $R_0 = 620\ \Omega$ at the temperature $T_0 = 20\,°C = 293\ °K$ and $\beta = 3330\ °K$. At $T = 70\,°C = 343\ °K$ the resistance of this thermistor will be

$$R_T = 620\, e^{3330\,(1/342 - 1/293)} = 121.68\ \ \Omega$$

In Figure DP 3-9 this particular thermistor in used in a voltage divider circuit. Specify the value of the resistor $R$ that will cause the voltage $v_T$ across the thermistor to be 4 V when the temperature is 100 °C.

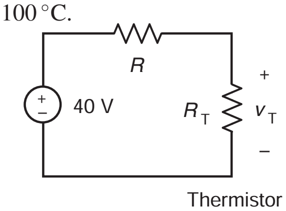



**Figure DP 3-9**

**DP 3-10** The circuit shown in Figure DP 3-10 contains a thermistor that has a resistance $R_0 = 620\ \Omega$ at the temperature $T_0 = 20\,°C = 293\ °K$ and $\beta = 3330\ °K$. (See problem DP 3-9.) Design this circuit (that is, specify the values of $R$ and $V_s$) so that the thermistor voltage is $v_T = 4$ V when $T = 100\,°C$ and $v_T = 20$ V when $T = 0\,°C$.

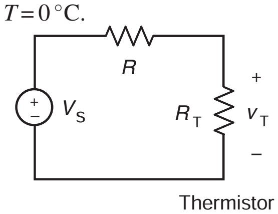



**Figure DP 3-10**

**DP 3-11** The circuit shown in Figure DP 3-11 is designed to help orange growers protect their crops against frost by sounding an alarm when the temperature falls below freezing. It contains a thermistor that has a resistance $R_0 = 620\ \Omega$ at the temperature $T_0 = 20\,°C = 293\ °K$ and $\beta = 3330\ °K$. (See problem DP 3-9.)

The alarm will sound when the voltage at the – input of the comparator is less than the voltage at the + input. Using voltage division twice, we see that the alarm sounds whenever

$$\frac{R_2}{R_T + R_2} < \frac{R_4}{R_3 + R_4}$$

Determine values of $R_2$, $R_3$, and $R_4$ that cause the alarm to sound whenever the temperature is below freezing.

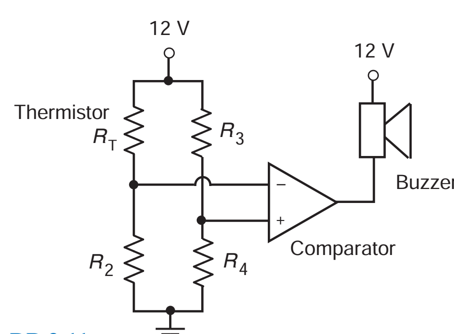



**Figure DP 3-11**

# CHAPTER 4 Methods of Analysis of Resistive Circuits

## IN THIS CHAPTER



## 4.1 *Introduction*

To analyze an electric circuit, we write and solve a set of equations. We apply Kirchhoff's current and voltage laws to get some of the equations. The constitutive equations of the circuit elements, such as Ohm's law, provide the remaining equations. The unknown variables are element currents and voltages. Solving the equations provides the values of the element current and voltages.

This method works well for small circuits, but the set of equations can get quite large for even moderate-sized circuits. A circuit with only 6 elements has 6 element currents and 6 element voltages. We could have 12 equations in 12 unknowns. In this chapter, we consider two methods for writing a smaller set of simultaneous equations:

- The node voltage method.
- The mesh current method.

The node voltage method uses a new type of variable called the node voltage. The "node voltage equations" or, more simply, the "node equations," are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the node voltage equations are the node voltages. After solving the node voltage equations, we determine the values of the element currents and voltages from the values of the node voltages.

It's easier to write node voltage equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write node voltage equations for circuits that consist of:

- Resistors and independent current sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

The mesh current method uses a new type of variable called the mesh current. The "mesh current equations" or, more simply, the "mesh equations," are a set of simultaneous equations that represent a given electric circuit. The unknown variables of the mesh current equations are the mesh currents. After solving the mesh current equations, we determine the values of the element currents and voltages from the values of the mesh currents.

It's easier to write mesh current equations for some types of circuit than for others. Starting with the easiest case, we will learn how to write mesh current equations for circuits that consist of:

- Resistors and independent voltage sources.
- Resistors and independent current and voltage sources.
- Resistors and independent and dependent voltage and current sources.

## 4.2 *Node Voltage Analysis of Circuits with Current Sources*

Consider the circuit shown in Figure 4.2-1*a*. This circuit contains four elements: three resistors and a current source. The *nodes* of a circuit are the places where the elements are connected together. The circuit shown in Figure 4.2-1*a* has three nodes. It is customary to draw the elements horizontally or vertically and to connect these elements by horizontal and vertical lines that represent wires. In other words, nodes are drawn as points or are drawn using horizontal or vertical lines. Figure 4.2-1*b* shows the same circuit, redrawn so that all three nodes are drawn as points rather than lines. In Figure 4.2-1*b*, the nodes are labeled as node a, node b, and node c.

Analyzing a connected circuit containing $n$. nodes will require $n - 1$ KCL equations. One way to obtain these equations is to apply KCL at each node of the circuit except for one. The node at which KCL is not applied is called the reference node. Any node of the circuit can be selected to be the reference node. We will often choose the node at the bottom of the circuit to be the reference node. (When the circuit contains a grounded power supply, the ground node of the power supply is usually selected as the reference node.) In Figure 4.2-1*b*, node c is selected as the reference node and marked with the symbol used to identify the reference node.

The voltage at any node of the circuit, relative to the reference node, is called a **node voltage**. In Figure 4.2-1*b*, there are two node voltages: the voltage at node a with respect to the reference node, node c, and the voltage at node b, again with respect to the reference node, node c. In Figure 4.2-1*c*, voltmeters are added to measure the node voltages. To measure node voltage at node a, connect the red

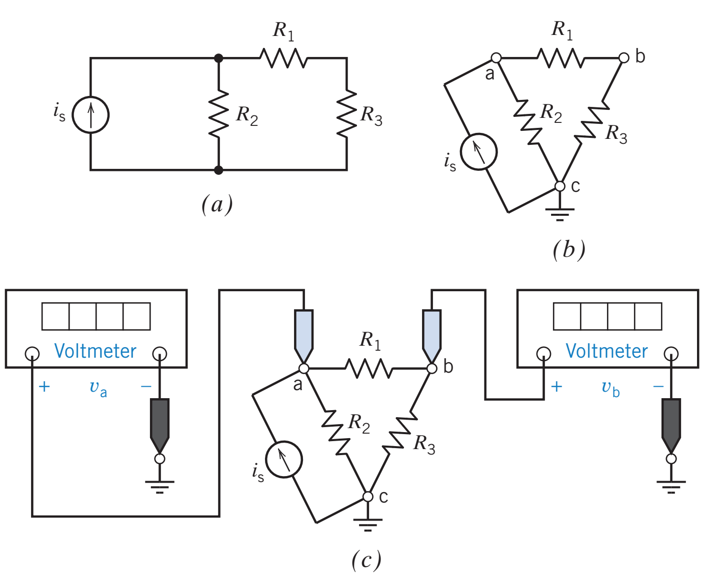



**FIGURE 4.2-1** (*a*) A circuit with three nodes. (*b*) The circuit after the nodes have been labeled and a reference node has been selected and marked. (*c*) Using voltmeters to measure the node voltages.

probe of the voltmeter at node a and connect the black probe at the reference node, node c. To measure node voltage at node b, connect the red probe of the voltmeter at node b and connect the black probe at the reference node, node c.

The node voltages in Figure 4.2-1*c* can be represented as $v_{ac}$ and $v_{bc}$, but it is conventional to drop the subscript c and refer to these as $v_a$ and $v_b$. Notice that the node voltage at the reference node is $v_{cc} = v_c = 0$ V because a voltmeter measuring the node voltage at the reference node would have both probes connected to the same point.

One of the standard methods for analyzing an electric circuit is to write and solve a set of simultaneous equations called the node equations. The unknown variables in the node equations are the node voltages of the circuit. We determine the values of the node voltages by solving the node equations.

To write a set of node equations, we do two things:

1. Express element currents as functions of the node voltages.
2. Apply Kirchhoff's current law (KCL) at each of the nodes of the circuit except for the reference node.

Consider the problem of expressing element currents as functions of the node voltages. Although our goal is to express element *currents* as functions of the node voltages, we begin by expressing element *voltages* as functions of the node voltages. Figure 4.2-2 shows how this is done. The voltmeters in Figure 4.2-2 measure the node voltages $v_1$ and $v_2$ at the nodes of the circuit element. The element voltage has been labeled as $v_a$. Applying Kirchhoff's voltage law to the loop shown in Figure 4.2-2 gives

$$v_a = v_1 - v_2$$

This equation expresses the element voltage $v_a$ as a function of the node voltages $v_1$ and $v_2$. (There is an easy way to remember this equation. Notice the reference polarity of the element voltage $v_a$. The element voltage is equal to the node voltage at the node near the + of the reference polarity minus the node voltage at the node near the – of the reference polarity.)

Now consider Figure 4.2-3. In Figure 4.2-3*a*, we use what we have learned to express the voltage of a circuit element as a function of node voltages. The circuit element in Figure 4.2-3*a* could be anything: a resistor, a current source, a dependent voltage source, and so on. In Figures 4.2-3*b* and *c*, we consider specific types of circuit element. In Figure 4.2-3*b*, the circuit element is a voltage source. The element voltage has been represented twice, once as the voltage source voltage $V_s$ and once as a function of the node voltages $v_1 - v_2$. Noticing that the reference polarities for $V_s$ and $v_1 - v_2$ are the same (both + on the left), we write

$$V_s = v_1 - v_2$$

This is an important result. Whenever we have a voltage source connected between two nodes of a circuit, we can express the voltage source voltage $V_s$ as a function of the node voltages $v_1$ and $v_2$.

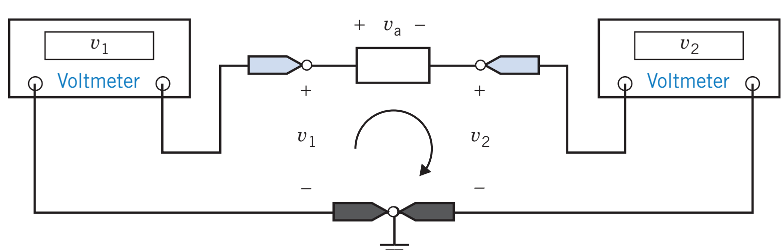



**FIGURE 4.2-2** Node voltages $v_1$ and $v_2$ and element voltage $v_a$ of a circuit element.

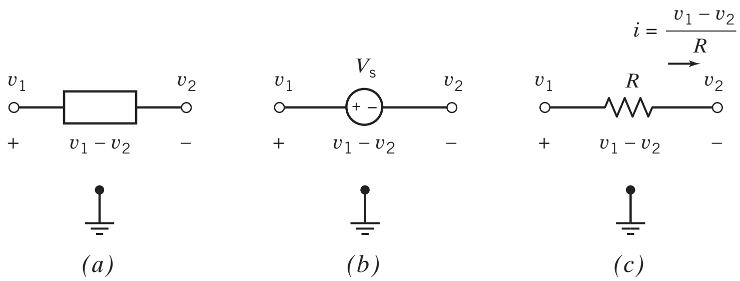



**FIGURE 4.2-3** Node voltages $v_1$ and $v_2$ and element voltage $v_1 - v_2$ of a (*a*) generic circuit element, (*b*) voltage source, and (*c*) resistor.

Frequently, we know the value of the voltage source voltage. For example, suppose that $V_s = 12$ V. Then

$$12 = v_1 - v_2$$

This equation relates the values of two of the node voltages.

Next, consider Figure 4.2-3*c*. In Figure 4.2-3*c*, the circuit element is a resistor. We will use Ohm's law to express the resistor current $i$ as a function of the node voltages. First, we express the resistor voltage as a function of the node voltages $v_1 - v_2$. Noticing that the resistor voltage $v_1 - v_2$ and the current $i$ adhere to the passive convention, we use Ohm's law to write

$$i = \frac{v_1 - v_2}{R}$$

Frequently, we know the value of the resistance. For example, when $R = 8\ \Omega$, this equation becomes

$$i = \frac{v_1 - v_2}{8}$$

This equation expresses the resistor current $i$ as a function of the node voltages $v_1$ and $v_2$.

Next, let's write node equations to represent the circuit shown in Figure 4.2-4*a*. The input to this circuit is the current source current $i_s$. To write node equations, we will first express the resistor currents as functions of the node voltages and then apply Kirchhoff's current law at nodes a and b. The resistor voltages are expressed as functions of the node voltages in Figure 4.2-4*b*, and then the resistor currents are expressed as functions of the node voltages in Figure 4.2-4*c*.

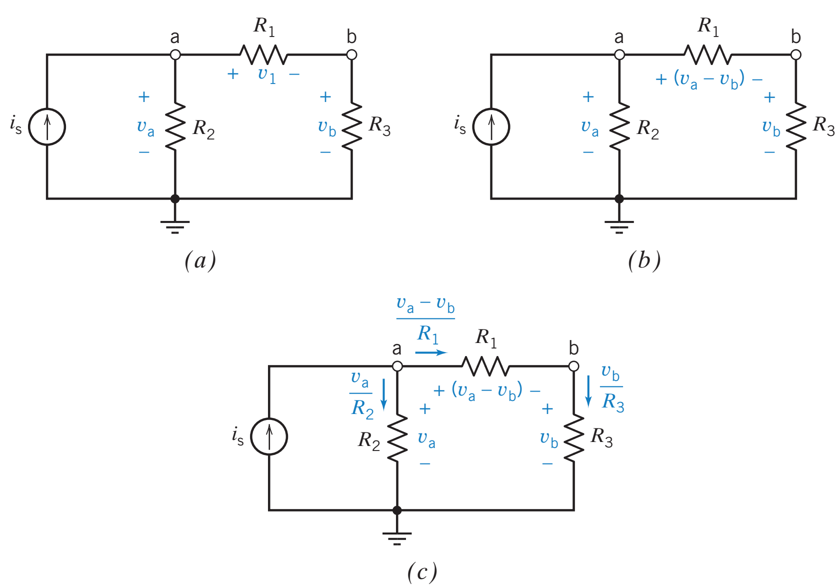



**FIGURE 4.2-4** (*a*) A circuit with three resistors. (*b*) The resistor voltages expressed as functions of the node voltages. (*c*) The resistor currents expressed as functions of the node voltages.

The node equations representing the circuit in Figure 4.2-4 are obtained by applying Kirchhoff's current law at nodes a and b. Using KCL at node a gives

$$i_s = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_1} \tag{4.2-1}$$

Similarly, the KCL equation at node b is

$$\frac{v_a - v_b}{R_1} = \frac{v_b}{R_3} \tag{4.2-2}$$

If $R_1 = 1\ \Omega$, $R_2 = R_3 = 0.5\ \Omega$, and $i_s = 4$ A, and Eqs. 4.2-1 and 4.2-2 may be rewritten as

$$4 = \frac{v_a - v_b}{1} + \frac{v_a}{0.5} \tag{4.2-3}$$

$$\frac{v_a - v_b}{1} = \frac{v_b}{0.5} \tag{4.2-4}$$

Solving Eq. 4.2-4 for $v_b$ gives

$$v_b = \frac{v_a}{3} \tag{4.2-5}$$

Substituting Eq. 4.2-5 into Eq. 4.2-3 gives

$$4 = v_a - \frac{v_a}{3} + 2v_a = \frac{8}{3} v_a \tag{4.2-6}$$

Solving Eq. 4.2-6 for $v_a$ gives

$$v_a = \frac{3}{2}\ \text{V}$$

Finally, Eq. 4.2-5 gives

$$v_b = \frac{1}{2}\ \text{V}$$

Thus, the node voltages of this circuit are

$$v_a = \frac{3}{2}\ \text{V} \quad \text{and} \quad v_b = \frac{1}{2}\ \text{V}$$

Try it yourself in WileyPLUS

## EXAMPLE 4.2-1 Node Equations

Determine the value of the resistance $R$ in the circuit shown in Figure 4.2-5*a*.

### Solution

Let $v_a$ denote the node voltage at node a and $v_b$ denote the node voltage at node b. The voltmeter in Figure 4.2-5 measures the value of the node voltage at node b, $v_b$. In Figure 4.2-5*b*, the resistor currents are expressed as functions of the node voltages. Apply KCL at node a to obtain

$$4 + \frac{v_a}{10} + \frac{v_a - v_b}{5} = 0$$

Using $v_b = 5$ V gives

$$4 + \frac{v_a}{10} + \frac{v_a - 5}{5} = 0$$

Solving for $v_a$, we get

$$v_a = -10\ \text{V}$$

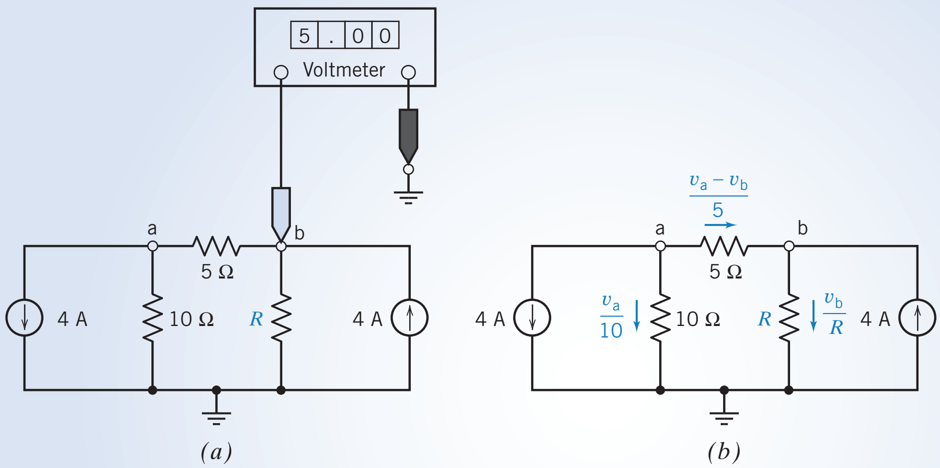



**FIGURE 4.2-5** (*a*) The circuit for Example 4.2-1. (*b*) The circuit after the resistor currents are expressed as functions of the node voltages.

Next, apply KCL at node b to obtain

$$-\left(\frac{v_a - v_b}{5}\right) + \frac{v_b}{R} - 4 = 0$$

Using $v_a = -10$ V and $v_b = 5$ V gives

$$-\left(\frac{-10 - 5}{5}\right) + \frac{5}{R} - 4 = 0$$

Finally, solving for $R$ gives

$$R = 5\ \Omega$$

## Example 4.2-2 Node Equations

Obtain the node equations for the circuit in Figure 4.2-6.

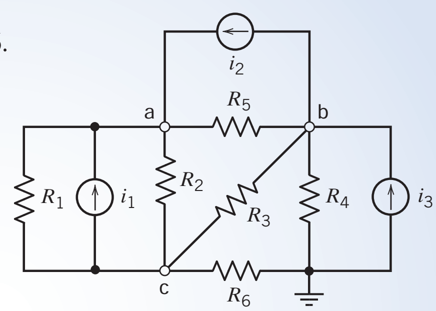



**FIGURE 4.2-6** The circuit for Example 4.2-2.

### Solution

Let $v_a$ denote the node voltage at node a, $v_b$ denote the node voltage at node b, and $v_c$ denote the node voltage at node c. Apply KCL at node a to obtain

$$-\left(\frac{v_a - v_c}{R_1}\right) + i_1 - \left(\frac{v_a - v_c}{R_2}\right) + i_2 - \left(\frac{v_a - v_b}{R_5}\right) = 0$$

Separate the terms of this equation that involve $v_a$ from the terms that involve $v_b$ and the terms that involve $v_c$ to obtain.

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right)v_a - \left(\frac{1}{R_5}\right)v_b - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_c = i_1 + i_2$$

There is a pattern in the node equations of circuits that contain only resistors and current sources. In the node equation at node a, the coefficient of $v_a$ is the sum of the reciprocals of the resistances of all resistors connected to node a. The coefficient of $v_b$ is minus the sum of the reciprocals of the resistances of all resistors connected between node b and node a. The coefficient $v_c$ is minus the sum of the reciprocals of the resistances of all resistors connected between node c and node a. The right-hand side of this equation is the algebraic sum of current source currents directed into node a.

Apply KCL at node b to obtain

$$-i_2 + \left(\frac{v_a - v_b}{R_5}\right) - \left(\frac{v_b - v_c}{R_3}\right) - \left(\frac{v_b}{R_4}\right) + i_3 = 0$$

Separate the terms of this equation that involve $v_a$ from the terms that involve $v_b$ and the terms that involve $v_c$ to obtain

$$-\left(\frac{1}{R_5}\right)v_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)v_b - \left(\frac{1}{R_3}\right)v_c = i_3 - i_2$$

As expected, this node equation adheres to the pattern for node equations of circuits that contain only resistors and current sources. In the node equation at node b, the coefficient of $v_b$ is the sum of the reciprocals of the resistances of all resistors connected to node b. The coefficient of $v_a$ is minus the sum of the reciprocals of the resistances of all resistors connected between node a and node b. The coefficient of $v_c$ is minus the sum of the reciprocals of the resistances of all resistors connected between node c and node b. The right-hand side of this equation is the algebraic sum of current source currents directed into node b.

Finally, use the pattern for the node equations of circuits that contain only resistors and current sources to obtain the node equation at node c:

$$-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_a - \left(\frac{1}{R_3}\right)v_b + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6}\right)v_c = -i_1$$

## EXAMPLE 4.2-3 Node Equations

Determine the node voltages for the circuit in Figure 4.2-6 when $i_1 = 1$ A, $i_2 = 2$ A, $i_3 = 3$ A, $R_1 = 5\,\Omega$, $R_2 = 2V$, $R_3 = 10\,\Omega$, $R_4 = 4\,\Omega$, $R_5 = 5\,\Omega$, and $R_6 = 2\,\Omega$.

### Solution

The node equations are

$$\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{5}\right)v_a - \left(\frac{1}{5}\right)v_b - \left(\frac{1}{5} + \frac{1}{2}\right)v_c = 1 + 2$$

$$-\left(\frac{1}{5}\right)v_a + \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{4}\right)v_b - \left(\frac{1}{10}\right)v_c = -2 + 3$$

$$-\left(\frac{1}{5} + \frac{1}{2}\right)v_a - \left(\frac{1}{10}\right)v_b + \left(\frac{1}{5} + \frac{1}{2} + \frac{1}{10} + \frac{1}{2}\right)v_c = -1$$

$$0.9v_a - 0.2v_b - 0.7v_c = 3$$
$$-0.2v_a + 0.55v_b - 0.1v_c = 1$$
$$-0.7v_a - 0.1v_b + 1.3v_c = -1$$

The node equations can be written using matrices as

$$A\,v = b$$

where

$$A = \begin{bmatrix} 0.9 & -0.2 & -0.7 \\ -0.2 & 0.55 & -0.1 \\ -0.7 & 0.1 & 1.3 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \text{ and, } v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

This matrix equation is solved using MATLAB in Figure 4.2-7.

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 7.1579 \\ 5.0526 \\ 3.4737 \end{bmatrix}$$

Consequently, $v_a = 7.1579$ V, $v_b = 5.0526$ V, and $v_c = 3.4737$ V

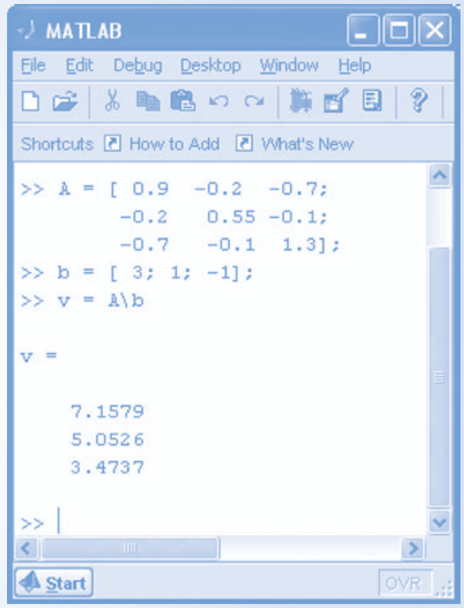



**FIGURE 4.2-7** Using MATLAB to solve the node equation in Example 4.2-3.

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***EXERCISE 4.2-1*** Determine the node voltages $v_a$ and $v_b$ for the circuit of Figure E 4.2-1.

***Answer:*** $v_a = 3$ V and $v_b = 11$ V

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***EXERCISE 4.2-2*** Determine the node voltages $v_a$ and $v_b$ for the circuit of Figure E 4.2-2.

***Answer:*** $v_a = -4/3$ V and $v_b = 4$ V

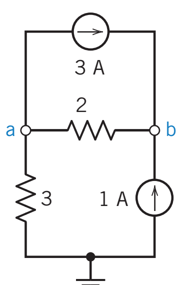


**FIGURE E 4.2-1**

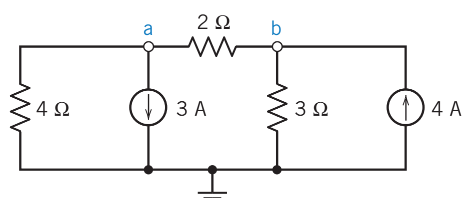


**FIGURE E 4.2-2**

## 4.3 *Node Voltage Analysis of Circuits with Current and Voltage Sources*

In the preceding section, we determined the node voltages of circuits with independent current sources only. In this section, we consider circuits with both independent current and voltage sources.

First we consider the circuit with a voltage source between ground and one of the other nodes. Because we are free to select the reference node, this particular arrangement is easily achieved.

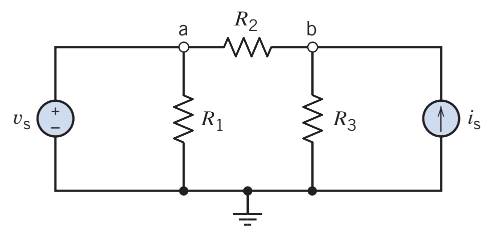


**FIGURE 4.3-1** Circuit with an independent voltage source and an independent current source.

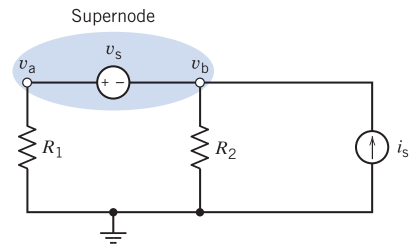


**FIGURE 4.3-2** Circuit with a supernode that incorporates $v_a$ and $v_b$.

Such a circuit is shown in Figure 4.3-1. We immediately note that the source is connected between terminal a and ground and, therefore,

$$v_a = v_s$$

Thus, $v_a$ is known and only $v_b$ is unknown. We write the KCL equation at node b to obtain

$$i_s = \frac{v_b}{R_3} + \frac{v_b - v_a}{R_2}$$

However, $v_a = v_s$. Therefore,

$$i_s = \frac{v_b}{R_3} + \frac{v_b - v_s}{R_2}$$

Then, solving for the unknown node voltage $v_b$, we get

$$v_b = \frac{R_2 R_3 i_s + R_3 v_s}{R_2 + R_3}$$

Next, let us consider the circuit of Figure 4.3-2, which includes a voltage source between two nodes. Because the source voltage is known, use KVL to obtain

$$v_a - v_b = v_s$$

or

$$v_a - v_s = v_b$$

To account for the fact that the source voltage is known, we consider both node a and node b as part of one larger node represented by the shaded ellipse shown in Figure 4.3-2. We require a larger node because $v_a$ and $v_b$ are dependent. This larger node is often called a *supernode* or a *generalized node*. KCL says that the algebraic sum of the currents entering a supernode is zero. That means that we apply KCL to a supernode in the same way that we apply KCL to a node.

A **supernode** consists of two nodes connected by an independent or a dependent voltage source.

We then can write the KCL equation at the supernode as

$$\frac{v_a}{R_1} + \frac{v_b}{R_2} = i_s$$

However, because $v_a = v_s + v_b$, we have

$$\frac{v_s + v_b}{R_1} + \frac{v_b}{R_2} = i_s$$

Then, solving for the unknown node voltage $v_b$, we get

$$v_b = \frac{R_1 R_2 i_s - R_2 v_s}{R_1 + R_2}$$

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## EXAMPLE 4.3-1 Node Equations

Determine the values node voltages, $v_1$ and $v_2$, in the circuit shown in Figure 4.3-3*a*.

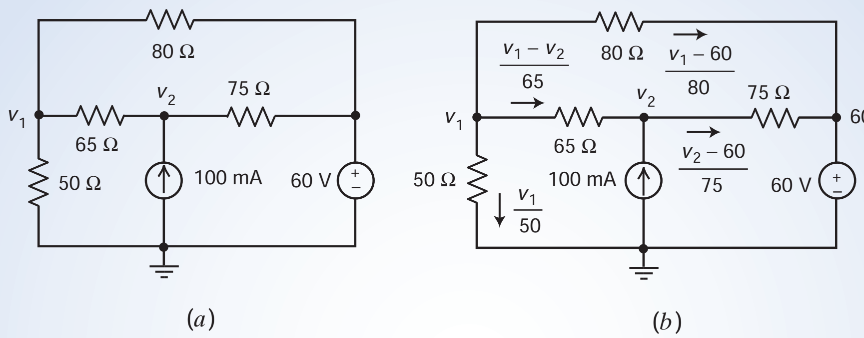


**FIGURE 4.3-3** The circuit considered in Example 4.3-1.

## Solution

First, represent the resistor currents in terms of the node voltages as shown in Figure 4.3-3*b*. Apply at KCL at node 1 to get

$$\frac{v_1}{50} + \frac{v_1 - v_2}{65} + \frac{v_1 - 60}{80} = 0 \Rightarrow \left(\frac{1}{50} + \frac{1}{65} + \frac{1}{80}\right) v_1 - \left(\frac{1}{65}\right) v_2 = \frac{60}{80}$$

Apply KCL at node 2 to get

$$0.1 = \frac{v_2 - v_1}{65} + \frac{v_2 - 60}{75} = \quad \Rightarrow -\left(\frac{1}{65}\right) v_1 + \left(\frac{1}{65} + \frac{1}{75}\right) v_2 = 0.1$$

Organize these equations in matrix form to write

$$\begin{bmatrix} \frac{1}{50} + \frac{1}{65} + \frac{1}{80} & -\frac{1}{65} \\ -\frac{1}{65} & \frac{1}{65} + \frac{1}{75} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{60}{80} \\ 0.1 \end{bmatrix}$$

Solving, we get $v_1 = 30.081\,\text{V}$ and $v_2 = 47.990\,\text{V}$

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## EXAMPLE 4.3-2 Supernodes

Determine the values of the node voltages $v_a$ and $v_b$ for the circuit shown in Figure 4.3-4.

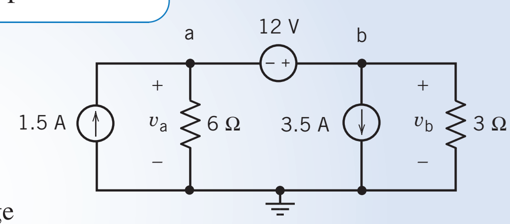


**FIGURE 4.3-4** The circuit for Example 4.3-2.

## Solution

We can write the first node equation by considering the voltage source. The voltage source voltage is related to the node voltages by

$$v_b - v_a = 12 \Rightarrow v_b = v_a + 12$$

To write the second node equation, we must decide what to do about the voltage source current. (Notice that there is no easy way to express the voltage source current in terms of the node voltages.) In this example, we illustrate two methods of writing the second node equation.

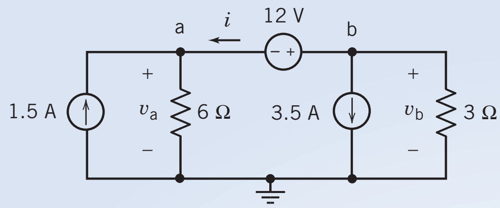


**FIGURE 4.3-5** Method 1 For Example 4.3-2.

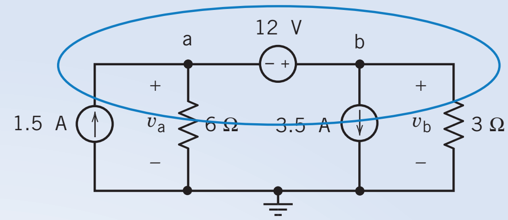


**FIGURE 4.3-6** Method 2 for Example 4.3-2.

**Method 1:** Assign a name to the voltage source current. Apply KCL at both of the voltage source nodes. Eliminate the voltage source current from the KCL equations.

Figure 4.3-5 shows the circuit after labeling the voltage source current. The KCL equation at node a is

$$1.5 + i = \frac{v_a}{6}$$

The KCL equation at node b is

$$i + 3.5 + \frac{v_b}{3} = 0$$

Combining these two equations gives

$$1.5 - \left(3.5 + \frac{v_b}{3}\right) = \frac{v_a}{6} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

**Method 2:** Apply KCL to the supernode corresponding to the voltage source. Shown in Figure 4.3-6, this supernode separates the voltage source and its nodes from the rest of the circuit. (In this small circuit, the rest of the circuit is just the reference node.)

Apply KCL to the supernode to get

$$1.5 = \frac{v_a}{6} + 3.5 + \frac{v_b}{3} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

This is the same equation that was obtained using method 1. Applying KCL to the supernode is a shortcut for doings three things:

1. Labeling the voltage source current as $i$.
2. Applying KCL at both nodes of the voltage source.
3. Eliminating $i$ from the KCL equations.

In summary, the node equations are

$$v_b - v_a = 12$$

and

$$\frac{v_a}{6} + \frac{v_b}{3} = -2.0$$

Solving the node equations gives

$$v_a = -12\text{ V}, \text{ and } v_b = 0\text{ V}$$

(We might be surprised that $v_b$ is 0 V, but it is easy to check that these values are correct by substituting them into the node equations.)

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## EXAMPLE 4.3-3 Node Equations for a Circuit Containing Voltage Sources

Determine the node voltages for the circuit shown in Figure 4.3-7.

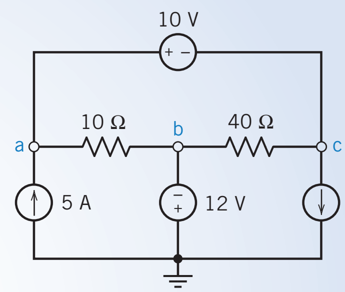


**FIGURE 4.3-7** The circuit for Example 4.3-3.

### Solution

We will calculate the node voltages of this circuit by writing a KCL equation for the supernode corresponding to the 10-V voltage source. First notice that

$$v_b = -12 \text{ V}$$

and that

$$v_a = v_c + 10$$

Writing a KCL equation for the supernode, we have

$$\frac{v_a - v_b}{10} + 2 + \frac{v_c - v_b}{40} = 5$$

or

$$4\, v_a + v_c - 5\, v_b = 120$$

Using $v_a = v_c + 10$ and $v_b = -12$ to eliminate $v_a$ and $v_b$, we have

$$4(v_c + 10) + v_c - 5(-12) = 120$$

Solving this equation for $v_c$, we get

$$v_c = 4 \text{ V}$$

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**EXERCISE 4.3-1** Find the node voltages for the circuit of Figure E 4.3-1.

*Hint:* Write a KCL equation for the supernode corresponding to the 10-V voltage source.

*Answer:* $2 + \frac{v_b + 10}{20} + \frac{v_b}{30} = 5 \quad \Rightarrow \quad v_b = 30 \text{ V} \quad \text{and} \quad v_a = 40 \text{ V}$

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**EXERCISE 4.3-2** Find the voltages $v_a$ and $v_b$ for the circuit of Figure E 4.3-2.

*Answer:* $\frac{(v_b + 8) - (-12)}{10} + \frac{v_b}{40} = 3 \quad \Rightarrow \quad v_b = 8 \text{ V} \quad \text{and} \quad v_a = 16 \text{ V}$

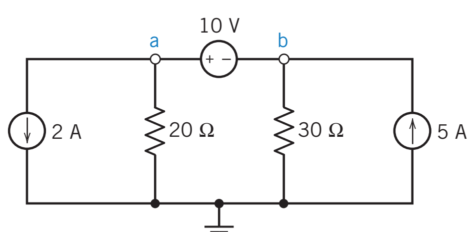


**FIGURE E 4.3-1**

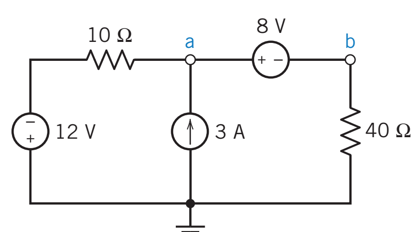


**FIGURE E 4.3-2**

## 4.4 Node Voltage Analysis with Dependent Sources

When a circuit contains a dependent source the controlling current or voltage of that dependent source must be expressed as a function of the node voltages.

It is then a simple matter to express the controlled current or voltage as a function of the node voltages. The node equations are then obtained using the techniques described in the previous two sections.

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### EXAMPLE 4.4-1 Node Equations for a Circuit Containing a Dependent Source

Determine the node voltages for the circuit shown in Figure 4.4-1.

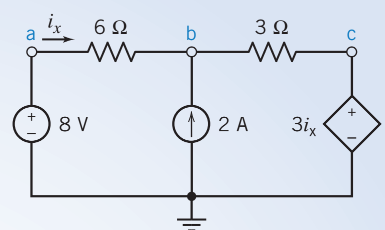


FIGURE 4.4-1 A circuit with a CCVS.

#### Solution

The controlling current of the dependent source is $i_x$. Our first task is to express this current as a function of the node voltages:

$$i_x = \frac{v_a - v_b}{6}$$

The value of the node voltage at node a is set by the 8-V voltage source to be

$$v_a = 8 \text{ V}$$

So

$$i_x = \frac{8 - v_b}{6}$$

The node voltage at node c is equal to the voltage of the dependent source, so

$$v_c = 3i_x = 3\left(\frac{8 - v_b}{6}\right) = 4 - \frac{v_b}{2} \tag{4.4-1}$$

Next, apply KCL at node b to get

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - v_c}{3} \tag{4.4-2}$$

Using Eq. 4.4-1 to eliminate $v_c$ from Eq. 4.4-2 gives

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - \left(4 - \frac{v_b}{2}\right)}{3} = \frac{v_b}{2} - \frac{4}{3}$$

Solving for $v_b$ gives

$$v_b = 7 \text{ V}$$

Then,

$$v_c = 4 - \frac{v_b}{2} = \frac{1}{2} \text{ V}$$

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## EXAMPLE 4.4-2 Node Equations for a Circuit Containing a Dependent Source

Determine the node voltages for the circuit shown in Figure 4.4-2.

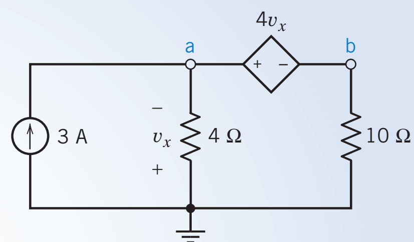


**FIGURE 4.4-2** A circuit with a VCVS.

### Solution

The controlling voltage of the dependent source is $v_x$. Our first task is to express this voltage as a function of the node voltages:

$$v_x = -v_a$$

The difference between the node voltages at nodes a and b is set by voltage of the dependent source:

$$v_a - v_b = 4\,v_x = 4(-v_a) = -4\,v_a$$

Simplifying this equation gives

$$v_b = 5\,v_a \tag{4.4-3}$$

Applying KCL to the supernode corresponding to the dependent voltage source gives

$$3 = \frac{v_a}{4} + \frac{v_b}{10} \tag{4.4-4}$$

Using Eq. 4.4-3 to eliminate $v_b$ from Eq. 4.4-4 gives

$$3 = \frac{v_a}{4} + \frac{5v_a}{10} = \frac{3}{4}v_a$$

Solving for $v_a$, we get

$$v_a = 4\text{ V}$$

Finally,

$$v_b = 5\,v_a = 20\text{ V}$$

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## EXAMPLE 4.4-3 Node Equations for a Circuit Containing a Dependent Source

Determine the node voltages corresponding to nodes a and b for the circuit shown in Figure 4.4-3.

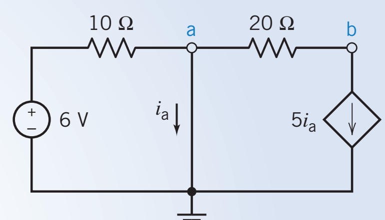


**FIGURE 4.4-3** A circuit with a CCCS.

### Solution

The controlling current of the dependent source is $i_a$. Our first task is to express this current as a function of the node voltages. Apply KCL at node a to get

$$\frac{6 - v_a}{10} = i_a + \frac{v_a - v_b}{20}$$

Node a is connected to the reference node by a short circuit, so $v_a = 0$ V. Substituting this value of $v_a$ into the preceding equation and simplifying gives

$$i_a = \frac{12 + v_b}{20} \tag{4.4-5}$$

Next, apply KCL at node b to get

$$\frac{0 - v_b}{20} = 5\,i_a \tag{4.4-6}$$

Using Eq. 4.4-5 to eliminate $i_a$ from Eq. 4.4-6 gives

$$\frac{0 - v_b}{20} = 5\left(\frac{12 + v_b}{20}\right)$$

Solving for $v_b$ gives

$$v_b = -10\text{ V}$$

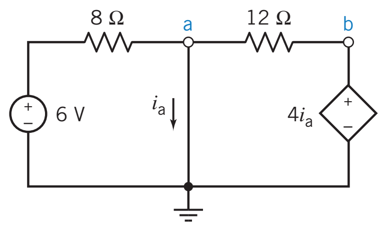


**FIGURE E 4.4-1** A circuit with a CCVS.

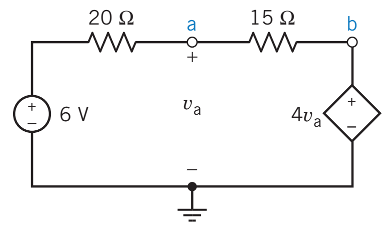


**FIGURE E 4.4-2** A circuit with a VCVS.

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**EXERCISE 4.4-1** Find the node voltage $v_b$ for the circuit shown in Figure E 4.4-1.

*Hint:* Apply KCL at node a to express $i_a$ as a function of the node voltages. Substitute the result into $v_b = 4i_a$ and solve for $v_b$.

*Answer:* $-\frac{6}{8} + \frac{v_b}{4} - \frac{v_b}{12} = 0 \quad \Rightarrow \quad v_b = 4.5 \text{ V}$

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**EXERCISE 4.4-2** Find the node voltages for the circuit shown in Figure E 4.4-2.

*Hint:* The controlling voltage of the dependent source is a node voltage, so it is already expressed as a function of the node voltages. Apply KCL at node a.

*Answer:* $\frac{v_a - 6}{20} + \frac{v_a - 4v_a}{15} = 0 \quad \Rightarrow \quad v_a = -2 \text{ V}$

## 4.5 Mesh Current Analysis with Independent Voltage Sources

In this and succeeding sections, we consider the analysis of circuits using Kirchhoff's voltage law (KVL) around a closed path. A *closed path* or a *loop* is drawn by starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once.

A mesh is a special case of a loop.

A **mesh** is a loop that does not contain any other loops within it.

Mesh current analysis is applicable only to planar networks. A planar circuit is one that can be drawn on a plane, without crossovers. An example of a nonplanar circuit is shown in Figure 4.5-1, in which the crossover is identified and cannot be removed by redrawing the circuit. For planar networks, the meshes in the network look like windows. There are four meshes in the circuit shown in Figure 4.5-2. They are identified as $M_i$. Mesh 2 contains the elements $R_3$, $R_4$, and $R_5$. Note that the resistor $R_3$ is common to both mesh 1 and mesh 2.

We define a mesh current as the current through the elements constituting the mesh. Figure 4.5-3*a* shows a circuit having two meshes with the mesh currents labeled as $i_1$ and $i_2$. We will use the convention of a mesh current in the clockwise direction as shown in Figure 4.5-3*a*. In Figure 4.5-3*b*, ammeters have been inserted into the meshes to measure the mesh currents.

One of the standard methods for analyzing an electric circuit is to write and solve a set of simultaneous equations called the mesh equations. The unknown variables in the mesh equations are the mesh currents of the circuit. We determine the values of the mesh currents by solving the mesh equations.

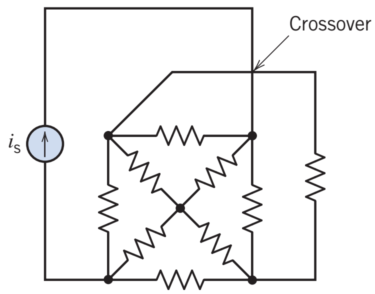


FIGURE 4.5-1 Nonplanar circuit with a crossover.

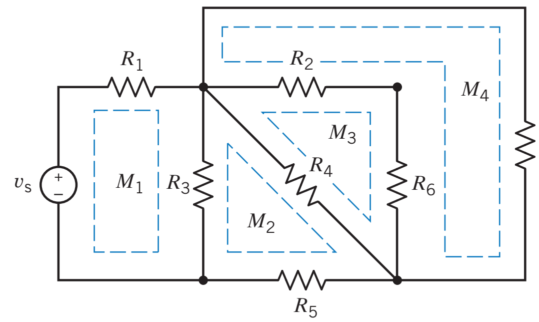


FIGURE 4.5-2 Circuit with four meshes. Each mesh is identified by dashed lines.

To write a set of mesh equations, we do two things:

1. Express element voltages as functions of the mesh currents.
2. Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.

Consider the problem of expressing element voltages as functions of the mesh currents. Although our goal is to express element *voltages* as functions of the mesh currents, we begin by expressing element *currents* as functions of the mesh currents. Figure 4.5-3*b* shows how this is done. The ammeters in Figure 4.5-3*b* measure the mesh currents, $i_1$ and $i_2$. Elements C and E are in the right mesh but not in the left mesh. Apply Kirchhoff's current law at node c and then at node f to see that the currents in elements C and E are equal to the mesh current of the right mesh, $i_2$, as shown in Figure 4.5-3*b*. Similarly, elements A and D are only in the left mesh. The currents in elements A and D are equal to the mesh current of the left mesh, $i_1$, as shown in Figure 4.5-3*b*.

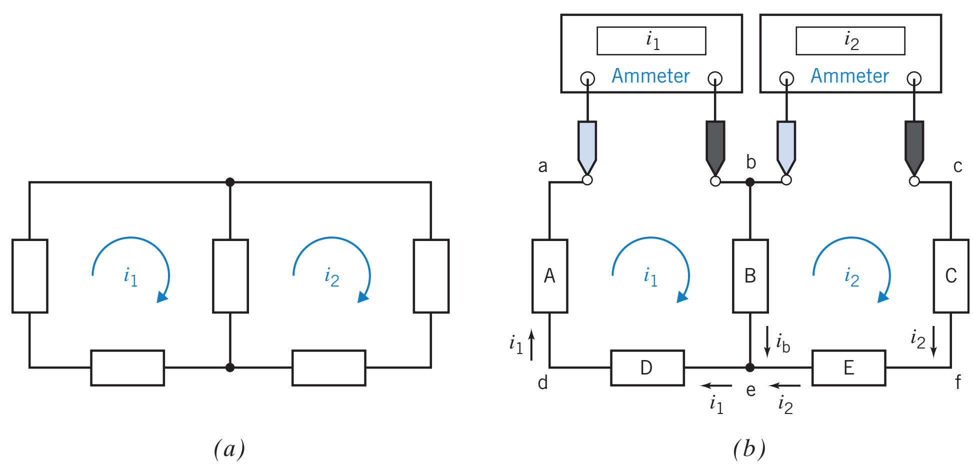


FIGURE 4.5-3 (*a*) A circuit with two meshes. (*b*) Inserting ammeters to measure the mesh currents.

Element B is in both meshes. The current of element B has been labeled as $i_b$. Applying Kirchhoff's current law at node b in Figure 4.5-3*b* gives

$$i_b = i_1 - i_2$$

This equation expresses the element current $i_b$ as a function of the mesh currents $i_1$ and $i_2$.

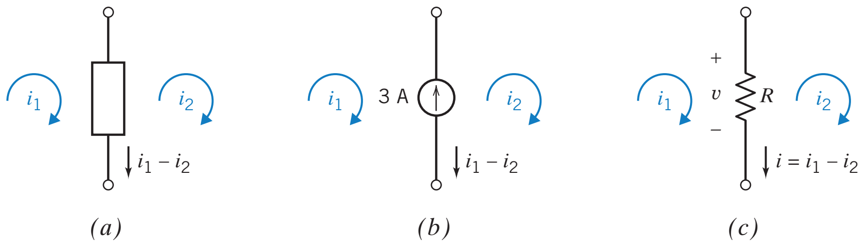


**FIGURE 4.5-4** Mesh currents $i_1$ and $i_2$ and element current $i_1 - i_2$ of a (*a*) generic circuit element, (*b*) current source, and (*c*) resistor.

Figure 4.5-4*a* shows a circuit element that is in two meshes. The current of the circuit element is expressed as a function of the mesh currents of the two meshes. The circuit element in Figure 4.5-4*a* could be anything: a resistor, a current source, a dependent voltage source, and so on. In Figures 4.5-4*b* and *c*, we consider specific types of circuit element. In Figure 4.5-4*b*, the circuit element is a current source. The element current has been represented twice, once as the current source current 3 A and once as a function of the mesh currents $i_1 - i_2$. Noticing that the reference directions for 3 A and $i_1 - i_2$ are different (one points up, the other points down), we write

$$-3 = i_1 - i_2$$

This equation relates the values of two of the mesh currents.

Next consider Figure 4.5-4*c*. In Figure 4.5-4*c*, the circuit element is a resistor. We will use Ohm's law to express the resistor voltage $v$ as functions of the mesh currents. First, we express the resistor current as a function of the mesh currents $i_1 - i_2$. Noticing that the resistor current $i_1 - i_2$ and the voltage $v$ adhere to the passive convention, we use Ohm's law to write

$$v = R(i_1 - i_2)$$

Frequently, we know the value of the resistance. For example, when $R = 8\ \Omega$, this equation becomes

$$v = 8(i_1 - i_2)$$

This equation expresses the resistor voltage $v$ as a function of the mesh currents $i_1$ and $i_2$.

Next, let's write mesh equations to represent the circuit shown in Figure 4.5-5*a*. The input to this circuit is the voltage source voltage $v_s$. To write mesh equations, we will first express the resistor voltages as functions of the mesh currents and then apply Kirchhoff's voltage law to the meshes. The resistor currents are expressed as functions of the mesh currents in Figure 4.5-5*b*, and then the resistor voltages are expressed as functions of the mesh currents in Figure 4.5-5*c*.

We may use Kirchhoff's voltage law around each mesh. We will use the following convention for obtaining the algebraic sum of voltages around a mesh. We will move around the mesh in the clockwise direction. If we encounter the + sign of the voltage reference polarity of an element voltage before the – sign, we add that voltage. Conversely, if we encounter the – of the voltage reference polarity of an element voltage before the + sign, we subtract that voltage. Thus, for the circuit of Figure 4.5-5*c*, we have

$$\text{mesh 1:}\quad -v_s + R_1 i_1 + R_3(i_1 - i_2) = 0 \tag{4.5-1}$$

$$\text{mesh 2:}\quad -R_3(i_1 - i_2) + R_2 i_2 = 0 \tag{4.5-2}$$

Note that the voltage across $R_3$ in mesh 1 is determined from Ohm's law, where

$$v = R_3 i_a = R_3(i_1 - i_2)$$

where $i_a$ is the actual element current flowing downward through $R_3$.

Equations 4.5-1 and 4.5-2 will enable us to determine the two mesh currents $i_1$ and $i_2$. Rewriting the two equations, we have

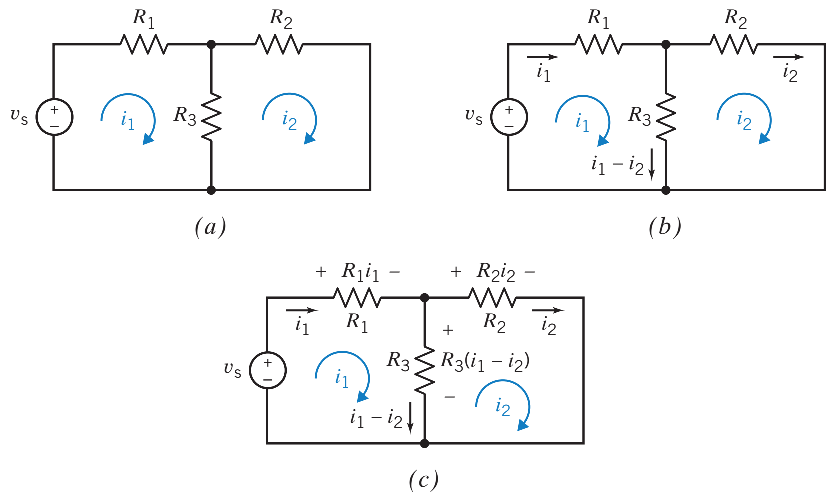


**FIGURE 4.5-5** (*a*) A circuit. (*b*) The resistor currents expressed as functions of the mesh currents. (*c*) The resistor voltages expressed as functions of the mesh currents.

$$i_1(R_1 + R_3) - i_2 R_3 = v_s$$

and

$$-i_1 R_3 + i_2(R_3 + R_2) = 0$$

If $R_1 = R_2 = R_3 = 1\ \Omega$, we have

$$2i_1 - i_2 = v_s$$

and

$$-i_1 + 2i_2 = 0$$

Add twice the first equation to the second equation, obtaining $3i_1 = 2v_s$. Then we have

$$i_1 = \frac{2v_s}{3} \quad \text{and} \quad i_2 = \frac{v_s}{3}$$

Thus, we have obtained two independent mesh current equations that are readily solved for the two unknowns. If we have $N$ meshes and write $N$ mesh equations in terms of $N$ mesh currents, we can obtain $N$ independent mesh equations. This set of $N$ equations is independent and thus guarantees a solution for the $N$ mesh currents.

A circuit that contains only independent voltage sources and resistors results in a specific format of equations that can readily be obtained. Consider a circuit with three meshes, as shown in Figure 4.5-6. Assign the clockwise direction to all of the mesh currents. Using KVL, we obtain the three mesh equations

$$\text{mesh 1:} \quad -v_s + R_1 i_1 + R_4(i_1 - i_2) = 0$$
$$\text{mesh 2:} \quad R_2 i_2 + R_5(i_2 - i_3) + R_4(i_2 - i_1) = 0$$
$$\text{mesh 3:} \quad R_5(i_3 - i_2) + R_3 i_3 + v_g = 0$$

These three mesh equations can be rewritten by collecting coefficients for each mesh current as

$$\text{mesh 1:} \quad (R_1 + R_4)i_1 - R_4 i_2 = v_s$$
$$\text{mesh 2:} \quad -R_4 i_1 + R_5 + (R_4 + R_2 + R_5)i_2 - R_5 i_3 = 0$$
$$\text{mesh 3:} \quad -R_5 i_2 + (R_3 + R_5)i_3 = -v_g$$

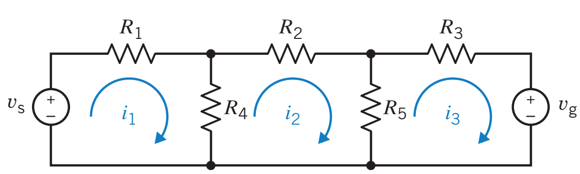


**FIGURE 4.5-6** Circuit with three mesh currents and two voltage sources.

Hence, we note that the coefficient of the mesh current $i_1$ for the first mesh is the sum of resistances in mesh 1, and the coefficient of the second mesh current is the negative of the resistance common to meshes 1 and 2. In general, we state that for mesh current $i_n$, the equation for the $n$th mesh with independent voltage sources only is obtained as follows:

$$-\sum_{q=1}^{Q} R_k i_q + \sum_{j=1}^{P} R_j i_n = -\sum_{n=1}^{N} v_{sn} \tag{4.5-3}$$

That is, for mesh $n$ we multiply $i_n$ by the sum of all resistances $R_j$ around the mesh. Then we add the terms due to the resistances in common with another mesh as the negative of the connecting resistance $R_k$, multiplied by the mesh current in the adjacent mesh $i_q$ for all $Q$ adjacent meshes. Finally, the independent voltage sources around the loop appear on the right side of the equation as the negative of the voltage sources encountered as we traverse the loop in the direction of the mesh current. Remember that the preceding result is obtained assuming all mesh currents flow clockwise.

The general matrix equation for the mesh current analysis for independent voltage sources present in a circuit is

$$\mathbf{R}\,\mathbf{i} = \mathbf{v}_s \tag{4.5-4}$$

where $\mathbf{R}$ is a symmetric matrix with a diagonal consisting of the sum of resistances in each mesh and the off-diagonal elements are the negative of the sum of the resistances common to two meshes. The matrix $\mathbf{i}$ consists of the mesh current as

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ \cdot \\ \cdot \\ \cdot \\ i_N \end{bmatrix}$$

For $N$ mesh currents, the source matrix $\mathbf{v_s}$ is

$$\mathbf{v_s} = \begin{bmatrix} v_{s1} \\ v_{s2} \\ \cdot \\ \cdot \\ \cdot \\ v_{sN} \end{bmatrix}$$

where $v_{sj}$ is the algebraic sum of the voltages of the voltage sources in the $j$th mesh with the appropriate sign assigned to each voltage.

For the circuit of Figure 4.5-6 and the matrix Eq. 4.5-4, we have

$$\mathbf{R} = \begin{bmatrix} (R_1 + R_4) & -R_4 & 0 \\ -R_4 & (R_2 + R_4 + R_5) & -R_5 \\ 0 & -R_5 & (R_3 + R_5) \end{bmatrix}$$

Note that $\mathbf{R}$ is a symmetric matrix, as we expected.

**EXERCISE 4.5-1** Determine the value of the voltage measured by the voltmeter in Figure E 4.5-1.

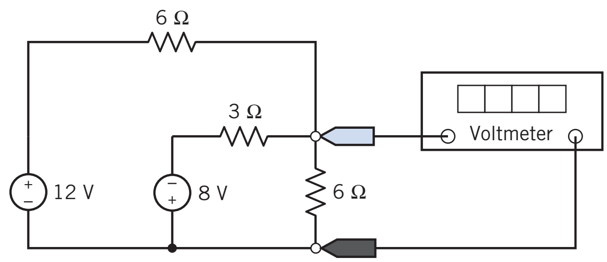


FIGURE E 4.5-1

*Answer:* −1 V

## 4.6 Mesh Current Analysis with Current and Voltage Sources

Heretofore, we have considered only circuits with independent voltage sources for analysis by the mesh current method. If the circuit has an independent current source, as shown in Figure 4.6-1, we recognize that the second mesh current is equal to the negative of the current source current. We can then write

$$i_2 = -i_s$$

and we need only determine the first mesh current $i_1$. Writing KVL for the first mesh, we obtain

$$(R_1 + R_2)i_1 - R_2 i_2 = v_s$$

Because $i_2 = -i_s$, we have

$$i_1 = \frac{v_s - R_2 i_s}{R_1 + R_2} \tag{4.6-1}$$

where $i_s$ and $v_s$ are sources of known magnitude.

If we encounter a circuit as shown in Figure 4.6-2, we have a current source $i_s$ that has an unknown voltage $v_{ab}$ across its terminals. We can readily note that

$$i_2 - i_1 = i_s \tag{4.6-2}$$

by writing KCL at node a. The two mesh equations are

$$\text{mesh 1:}\ \ R_1 i_1 + v_{ab} = v_s \tag{4.6-3}$$

$$\text{mesh 2:}\ \ (R_2 + R_3)i_2 - v_{ab} = 0 \tag{4.6-4}$$

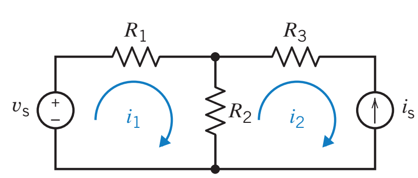


**FIGURE 4.6-1** Circuit with an independent voltage source and an independent current source.

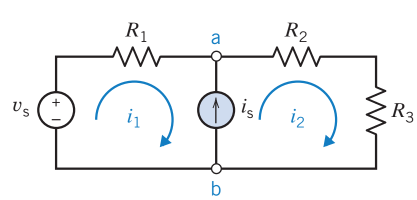


**FIGURE 4.6-2** Circuit with an independent current source common to both meshes.

We note that if we add Eqs. 4.6-3 and 4.6-4, we eliminate $v_{ab}$, obtaining

$$R_1 i_1 + (R_2 + R_3) i_2 = v_s$$

However, because $i_2 = i_s + i_1$, we obtain

$$R_1 i_1 + (R_2 + R_3)(i_s + i_1) = v_s$$

or

$$i_1 = \frac{v_s - (R_2 + R_3) i_s}{R_1 + R_2 + R_3} \tag{4.6-5}$$

Thus, we account for independent current sources by recording the relationship between the mesh currents and the current source current. If the current source influences *only one* mesh current, we write the equation that relates that mesh current to the current source current and write the KVL equations for the remaining meshes. If the current source influences two mesh currents, we write the KVL equation for both meshes, assuming a voltage $v_{ab}$ across the terminals of the current source. Then, adding these two mesh equations, we obtain an equation independent of $v_{ab}$.

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## EXAMPLE 4.6-1 Mesh Equations

Consider the circuit of Figure 4.6-3 where $R_1 = R_2 = 1\ \Omega$ and $R_3 = 2\ \Omega$. Find the three mesh currents.

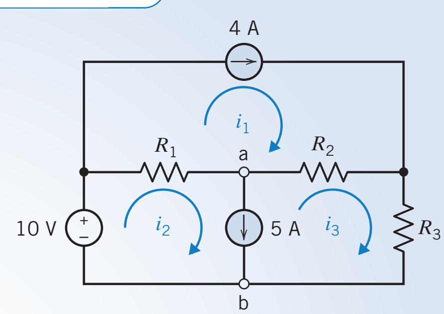


**FIGURE 4.6-3** Circuit with two independent current sources.

### Solution

Because the 4-A source is in mesh 1 only, we note that

$$i_1 = 4$$

For the 5-A source, we have

$$i_2 - i_3 = 5 \tag{4.6-6}$$

Writing KVL for mesh 2 and mesh 3, we obtain

$$\text{mesh 2: } R_1(i_2 - i_1) + v_{ab} = 10 \tag{4.6-7}$$

$$\text{mesh 3: } R_2(i_3 - i_1) + R_3 i_3 - v_{ab} = 0 \tag{4.6-8}$$

We substitute $i_1 = 4$ and add Eqs. 4.6-7 and 4.6-8 to obtain

$$R_1(i_2 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10 \tag{4.6-9}$$

From Eq. 4.6-6, $i_2 = 5 + i_3$, substituting into Eq. 4.6-9, we have

$$R_1(5 + i_3 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10$$

Using the values for the resistors, we obtain

$$i_3 = \frac{13}{4}\text{A} \quad \text{and} \quad i_2 = 5 + i_3 = \frac{33}{4}\text{A}$$

Another technique for the mesh analysis method when a current source is common to two meshes involves the concept of a supermesh. A *supermesh* is one mesh created from two meshes that have a current source in common, as shown in Figure 4.6-4.

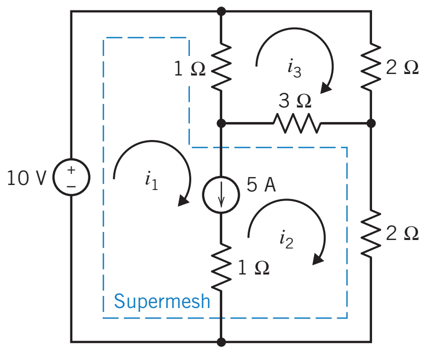


**FIGURE 4.6-4** Circuit with a supermesh that incorporates mesh 1 and mesh 2. The supermesh is indicated by the dashed line.

A **supermesh** is one larger mesh created from two meshes that have an independent or dependent current source in common.

For example, consider the circuit of Figure 4.6-4. The 5-A current source is common to mesh 1 and mesh 2. The supermesh consists of the interior of mesh 1 and mesh 2. Writing KVL around the periphery of the supermesh shown by the dashed lines, we obtain

$$-10 + 1(i_1 - i_3) + 3(i_2 - i_3) + 2i_2 = 0$$

For mesh 3, we have

$$1(i_3 - i_1) + 2i_3 + 3(i_3 - i_2) = 0$$

Finally, the equation that relates the current source current to the mesh currents is

$$i_1 - i_2 = 5$$

Then the three equations may be reduced to

$$\begin{aligned} &\text{supermesh:} && 1i_1 + 5i_2 - 4i_3 &&= 10 \\ &\text{mesh 3:} && -1i_1 - 3i_2 + 6i_3 &&= 0 \\ &\text{current source:} && 1i_1 - 1i_2 &&= 5 \end{aligned}$$

Therefore, solving the three equations simultaneously, we find that $i_2 = 2.5\text{A}, i_1 = 7.5$ A, and $i_3 = 2.5\text{A}$.

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## EXAMPLE 4.6-2 Supermeshes

Determine the values of the mesh currents $i_1$ and $i_2$ for the circuit shown in Figure 4.6-5.

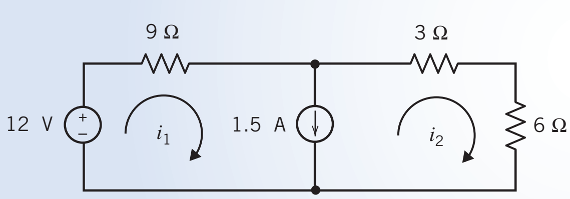


**FIGURE 4.6-5** The circuit for Example 4.6-2.

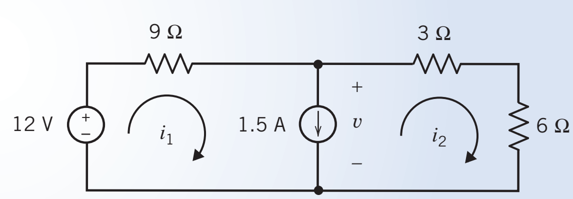


**FIGURE 4.6-6** Method 1 of Example 4.6-2.

## Solution

We can write the first mesh equation by considering the current source. The current source current is related to the mesh currents by

$$i_1 - i_2 = 1.5 \quad \Rightarrow \quad i_1 = i_2 + 1.5$$

To write the second mesh equation, we must decide what to do about the current source voltage. (Notice that there is no easy way to express the current source voltage in terms of the mesh currents.) In this example, we illustrate two methods of writing the second mesh equation.

**Method 1:** Assign a name to the current source voltage. Apply KVL to both of the meshes. Eliminate the current source voltage from the KVL equations.

Figure 4.6-6 shows the circuit after labeling the current source voltage. The KVL equation for mesh 1 is

$$9i_1 + v - 12 = 0$$

The KVL equation for mesh 2 is

$$3i_2 + 6i_2 - v = 0$$

Combining these two equations gives

$$9i_1 + (3i_2 + 6i_2) - 12 = 0 \quad \Rightarrow \quad 9i_1 + 9i_2 = 12$$

**Method 2:** Apply KVL to the supermesh corresponding to the current source. Shown in Figure 4.6-7, this supermesh is the perimeter of the two meshes that each contain the current source. Apply KVL to the supermesh to get

$$9i_1 + 3i_2 + 6i_2 - 12 = 0 \quad \Rightarrow \quad 9i_1 + 9i_2 = 12$$

This is the same equation that was obtained using method 1. Applying KVL to the supermesh is a shortcut for doing three things:

1. Labeling the current source voltage as $v$.
2. Applying KVL to both meshes that contain the current source.
3. Eliminating $v$ from the KVL equations.

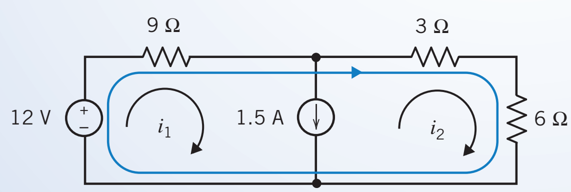


**FIGURE 4.6-7** Method 2 of Example 4.6-2.

In summary, the mesh equations are

$$i_1 = i_2 + 1.5$$

and

$$9i_1 + 9i_2 = 12$$

Solving the node equations gives

$$i_1 = 1.4167\text{A} \quad \text{and} \quad i_2 = -83.3\ \text{mA}$$

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**EXERCISE 4.6-1** Determine the value of the voltage measured by the voltmeter in Figure E 4.6-1.

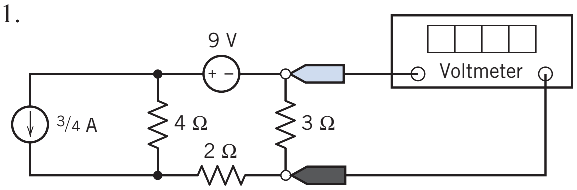


**FIGURE E 4.6-1**

***Hint:*** Write and solve a single mesh equation to determine the current in the 3-Ω resistor.

***Answer:*** −4 V

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**EXERCISE 4.6-2** Determine the value of the current measured by the ammeter in Figure E 4.6-2.

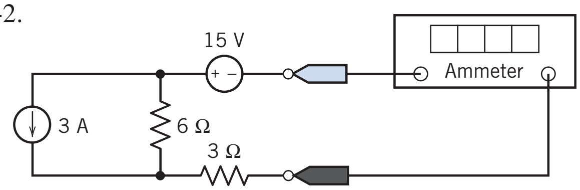


**FIGURE E 4.6-2**

***Hint:*** Write and solve a single mesh equation.

***Answer:*** −3.67 A

## 4.7 Mesh Current Analysis with Dependent Sources

When a circuit contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the mesh currents.

It is then a simple matter to express the controlled current or voltage as a function of the mesh currents. The mesh equations can then be obtained by applying Kirchhoff's voltage law to the meshes of the circuit.

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**EXAMPLE 4.7-1** Mesh Equations and Dependent Sources

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.7-1*a*. Find the value of the voltage measured by the voltmeter.

### Solution

Figure 4.7-1*b* shows the circuit after replacing the voltmeter by an equivalent open circuit and labeling the voltage, $v_m$, measured by the voltmeter. Figure 4.7-1*c* shows the circuit after numbering the meshes. Let $i_1$ and $i_2$ denote the mesh currents in meshes 1 and 2, respectively.

The controlling current of the dependent source, $i_a$, is the current in a short circuit. This short circuit is common to meshes 1 and 2. The short-circuit current can be expressed in terms of the mesh currents as

$$i_a = i_1 - i_2$$

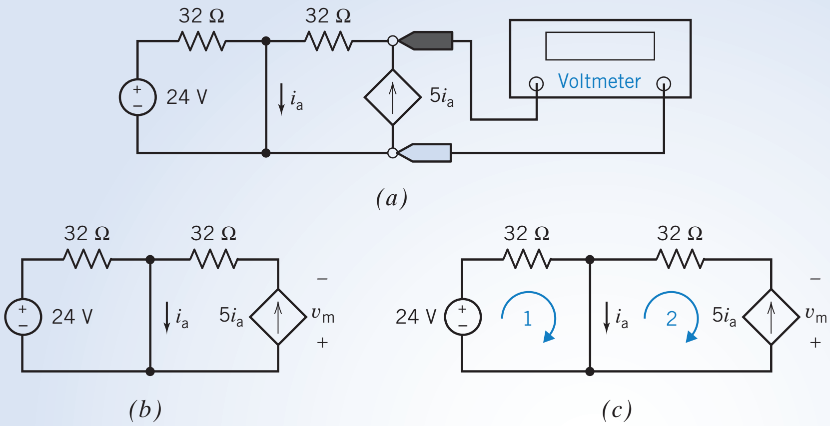


**FIGURE 4.7-1** (*a*) The circuit considered in Example 4.7-1. (*b*) The circuit after replacing the voltmeter by an open circuit. (*c*) The circuit after labeling the meshes.

The dependent source is in only one mesh, mesh 2. The reference direction of the dependent source current does not agree with the reference direction of $i_2$. Consequently,

$$5i_a = -i_2$$

Solving for $i_2$ gives

$$i_2 = -5i_a = -5(i_1 - i_2)$$

Therefore,

$$-4i_2 = -5i_1 \quad \Rightarrow \quad i_2 = \frac{5}{4}i_1$$

Apply KVL to mesh 1 to get

$$32i_1 - 24 = 0 \quad \Rightarrow \quad i_1 = \frac{3}{4}\,\text{A}$$

Consequently, the value of $i_2$ is

$$i_2 = \frac{5}{4}\left(\frac{3}{4}\right) = \frac{15}{16}\,\text{A}$$

Apply KVL to mesh 2 to get

$$32i_2 - v_m = 0 \quad \Rightarrow \quad v_m = 32i_2$$

Finally,

$$v_m = 32\left(\frac{15}{16}\right) = 30\ \text{V}$$

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## EXAMPLE 4.7-2 Mesh Equations and Dependent Sources

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.7-2*a*. Find the value of the gain $A$ of the CCVS.

### Solution

Figure 4 7-2*b* shows the circuit after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter. Figure 4.7-2*c* shows the circuit after numbering the meshes. Let $i_1$ and $i_2$ denote the mesh currents in meshes 1 and 2, respectively.

The voltage across the dependent source is represented in two ways. It is $Ai_a$ with the + of reference direction at the bottom and −7.2 V with the + at the top. Consequently,

$$Ai_a = -(-7.2) = 7.2\ \text{V}$$

The controlling current of the dependent source, $i_a$, is the current in a short circuit. This short circuit is common to meshes 1 and 2. The short-circuit current can be expressed in terms of the mesh currents as

$$i_a = i_1 - i_2$$

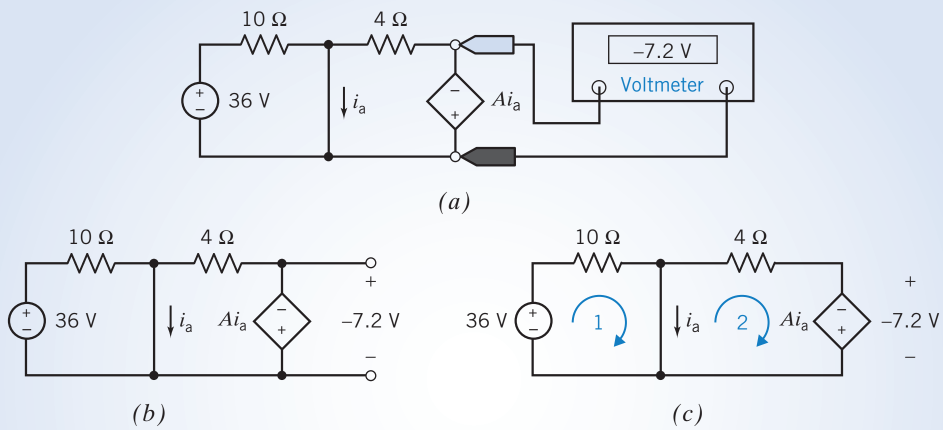


**FIGURE 4.7-2** (*a*) The circuit considered in Example 4.7-2. (*b*) The circuit after replacing the voltmeter by an open circuit. (*c*) The circuit after labeling the meshes.

Apply KVL to mesh 1 to get $$10i_1 - 36 = 0 \Rightarrow i_1 = 3.6 \text{ A}$$

Apply KVL to mesh 2 to get $$4i_2 + (-7.2) = 0 \Rightarrow i_2 = 1.8 \text{ A}$$

Finally, $$A = \frac{Ai_a}{i_a} = \frac{Ai_a}{i_1 - i_2} = \frac{7.2}{3.6 - 1.8} = 4 \text{ V/A}$$

## 4.8 The Node Voltage Method and Mesh Current Method Compared

The analysis of a complex circuit can usually be accomplished by either the node voltage or the mesh current method. The advantage of using these methods is the systematic procedures provided for obtaining the simultaneous equations.

In some cases, one method is clearly preferred over another. For example, when the circuit contains only voltage sources, it is probably easier to use the mesh current method. When the circuit contains only current sources, it will usually be easier to use the node voltage method.

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**EXAMPLE 4.8-1 Mesh Equations**

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.8-1. Find the value of the resistance, *R*.

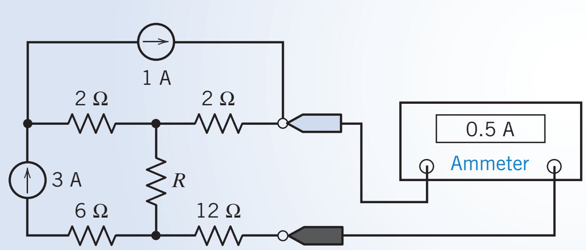


**FIGURE 4.8-1** The circuit considered in Example 4.8-1.

## Solution

Figure 4.8-2*a* shows the circuit from Figure 4.8-1 after replacing the ammeter by an equivalent short circuit and labeling the current measured by the ammeter. This circuit can be analyzed using mesh equations or using node equations. To decide which will be easier, we first count the nodes and meshes. This circuit has five nodes. Selecting a reference node and then applying KCL at the other four nodes will produce a set of four node equations. The circuit has three meshes. Applying KVL to these three meshes will produce a set of three mesh equations. Hence, analyzing this circuit using mesh equations instead of node equations will produce a smaller set of equations. Further, notice that two of the three mesh currents can be determined directly from the current source currents. This makes the mesh equations easier to solve. We will analyze this circuit by writing and solving mesh equations.

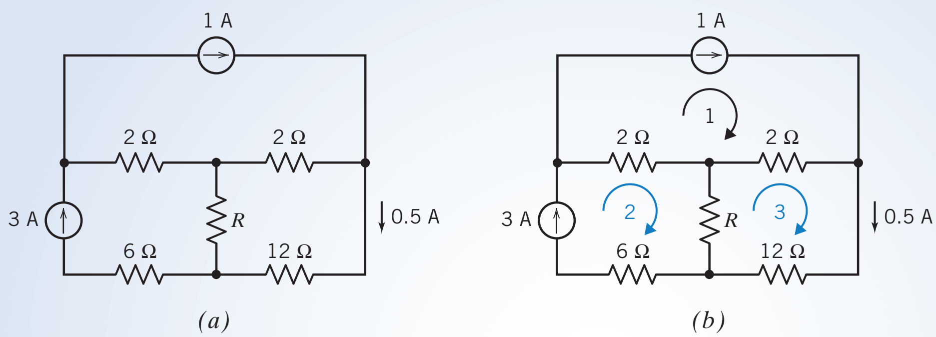


**FIGURE 4.8-2** (*a*) The circuit from Figure 4.8-1 after replacing the ammeter by a short circuit. (*b*) The circuit after labeling the meshes.

Figure 4.8-2*b* shows the circuit after numbering the meshes. Let $i_1$, $i_2$, and $i_3$ denote the mesh currents in meshes 1, 2, and 3, respectively. The mesh current $i_1$ is equal to the current in the 1-A current source, so

$$i_1 = 1 \text{ A}$$

The mesh current $i_2$ is equal to the current in the 3-A current source, so

$$i_2 = 3 \text{ A}$$

The mesh current $i_3$ is equal to the current in the short circuit that replaced the ammeter, so

$$i_3 = 0.5 \text{ A}$$

Apply KVL to mesh 3 to get

$$2(i_3 - i_1) + 12(i_3) + R(i_3 - i_2) = 0$$

Substituting the values of the mesh currents gives

$$2(0.5 - 1) + 12(0.5) + R(0.5 - 3) = 0 \quad \Rightarrow \quad R = 2\ \Omega$$

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## EXAMPLE 4.8-2 Node Equations

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 4.8-3. Find the value of the resistance, *R*.

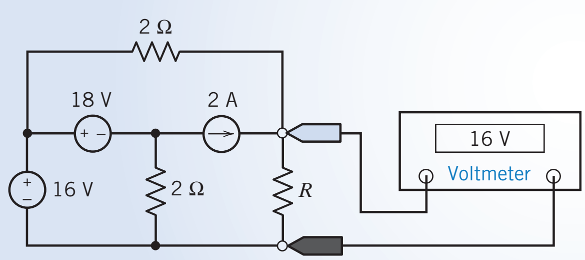



**FIGURE 4.8-3** The circuit considered in Example 4.8-2.

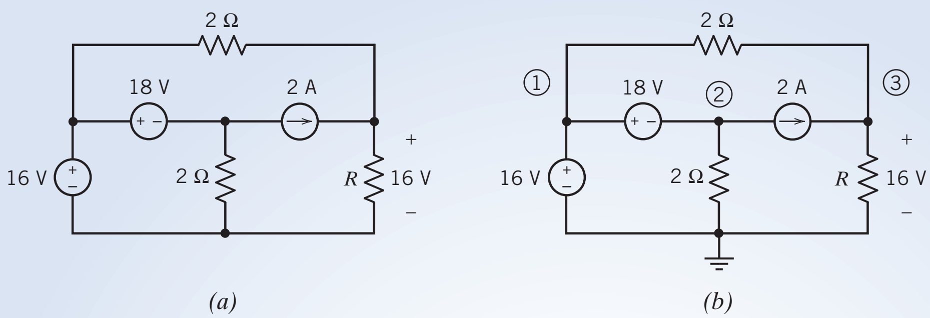



**FIGURE 4.8-4** (*a*) The circuit from Figure 4.8-3 after replacing the voltmeter by an open circuit. (*b*) The circuit after labeling the nodes.

## Solution

Figure 4.8-4*a* shows the circuit from Figure 4.8-3 after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter. This circuit can be analyzed using mesh equations or node equations. To decide which will be easier, we first count the nodes and meshes. This circuit has four nodes. Selecting a reference node and then applying KCL at the other three nodes will produce a set of three node equations. The circuit has three meshes. Applying KVL to these three meshes will produce a set of three mesh equations. Analyzing this circuit using mesh equations requires the same number of equations that are required to analyze the circuit using node equations. Notice that one of the three mesh currents can be determined directly from the current source current, but two of the three node voltages can be determined directly from the voltage source voltages. This makes the node equations easier to solve. We will analyze this circuit by writing and solving node equations.

Figure 4.8-4*b* shows the circuit after selecting a reference node and numbering the other nodes. Let $v_1$, $v_2$, and $v_3$ denote the node voltages at nodes 1, 2, and 3, respectively. The voltage of the 16-V voltage source can be expressed in terms of the node voltages as

$$16 = v_1 - 0 \quad \Rightarrow \quad v_1 = 16\,\text{V}$$

The voltage of the 18-V voltage source can be expressed in terms of the node voltages as

$$18 = v_1 - v_2 \quad \Rightarrow \quad 18 = 16 - v_2 \quad \Rightarrow \quad v_2 = -2\,\text{V}$$

The voltmeter measures the node voltage at node 3, so

$$v_3 = 16\,\text{V}$$

Applying KCL at node 3 to get

$$\frac{v_1 - v_3}{2} + 2 = \frac{v_3}{R}$$

Substituting the values of the node voltages gives

$$\frac{16 - 16}{2} + 2 = \frac{16}{R} \quad \Rightarrow \quad R = 8\,\Omega$$

If a circuit has both current sources and voltage sources, it can be analyzed by either method. One approach is to compare the number of equations required for each method. If the circuit has fewer nodes than meshes, it may be wise to select the node voltage method. If the circuit has fewer meshes than nodes, it may be easier to use the mesh current method.

Another point to consider when choosing between the two methods is what information is required. If you need to know several currents, it may be wise to proceed directly with mesh current analysis. Remember, mesh current analysis only works for planar networks.

It is often helpful to determine which method is more appropriate for the problem requirements and to consider both methods.

## 4.9 Circuit Analysis Using MATLAB

We have seen that circuits that contain resistors and independent or dependent sources can be analyzed in the following way:

1. Writing a set of node or mesh equations.
2. Solving those equations simultaneously.

In this section, we will use the MATLAB computer program to solve the equations.

Consider the circuit shown in Figure 4.9-1*a*. This circuit contains a potentiometer. In Figure 4.9-1*b*, the potentiometer has been replaced by a model of a potentiometer. $R_p$ is the resistance of

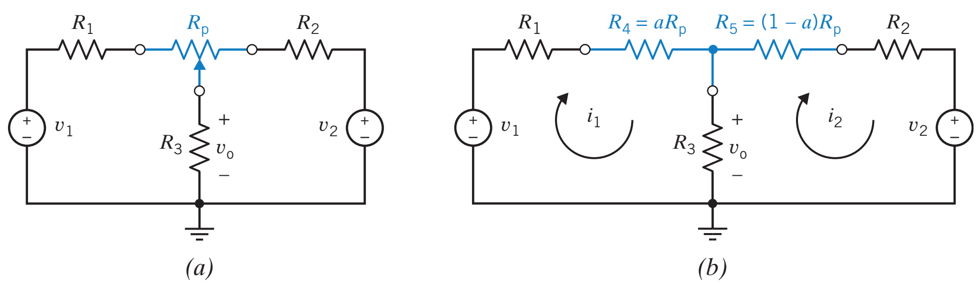



**FIGURE 4.9-1** (*a*) A circuit that contains a potentiometer and (*b*) an equivalent circuit formed by replacing the potentiometer with a model of a potentiometer $(0 < a < 1)$.

the potentiometer. The parameter $a$ varies from 0 to 1 as the wiper of the potentiometer is moved from one end of the potentiometer to the other. The resistances $R_4$ and $R_5$ are described by the equations

$$R_4 = aR_p \tag{4.9-1}$$

and

$$R_5 = (1 - a)R_p \tag{4.9-2}$$

Our objective is to analyze this circuit to determine how the output voltage changes as the position of the potentiometer wiper is changed.

The circuit in Figure 4.9-1*b* can be represented by mesh equations as

$$\begin{aligned} R_1 i_1 + R_4 i_1 + R_3(i_1 - i_2) - v_1 &= 0 \\ R_5 i_2 + R_2 i_2 + [v_2 - R_3(i_1 - i_2)] &= 0 \end{aligned} \tag{4.9-3}$$

These mesh equations can be rearranged as

$$\begin{aligned} (R_1 + R_4 + R_3)i_1 - R_3 i_2 &= \quad v_1 \\ -R_3 i_1 + (R_5 + R_2 + R_3)i_2 &= -v_2 \end{aligned} \tag{4.9-4}$$

Substituting Eqs. 4.9-1 and 4.9-2 into Eq. 4.9-4 gives

$$\begin{aligned} \left(R_1 + aR_p + R_3\right)i_1 - R_3 i_2 &= \quad v_1 \\ -R_3 i_1 + \left[(1 - a)R_p + R_2 + R_3\right]i_2 &= -v_2 \end{aligned} \tag{4.9-5}$$

```
% mesh.m solves mesh equations

%----------------------------------------------------------
% Enter values of the parameters that describe the circuit.
%----------------------------------------------------------
               % circuit parameters
R1=1000;       % ohms
R2=1000;       % ohms
R3=5000;       % ohms
V1= 15;        % volts
V2=-15;        % volts

               % potentiometer parameters
Rp=20e3;       % ohms

%---------------------------------------------------------
% the parameter a varies from 0 to 1 in 0.05 increments.
%---------------------------------------------------------

a=0:0.05:1;    % dimensionless

for k=1:length(a)
    %------------------------------------------------------
    % Here is the mesh equation, RV=I:
    %------------------------------------------------------

    R = [R1+a(k)*Rp+R3        -R3;              % ------
             -R3       (1-a(k))*Rp+R2+R3];      %  eqn.
    V = [ V1;                                   %  4-9-6
         -V2];                                  % ------
    %------------------------------------------------------
    % Tell MATLAB to solve the mesh equation:
    %------------------------------------------------------
    I = R\V;
    %------------------------------------------------------
    % Calculate the output voltage from the mesh currents.
    %------------------------------------------------------

    Vo(k) = R3*(I(1)-I(2));    % eqn. 4.9-7

end

%-----------------------------------------------------------
% Plot Vo versus a
%-----------------------------------------------------------
plot(a, Vo)
axis([0 1 -15 15])
xlabel('a, dimensionless')
ylabel('Vo, V')
```

**FIGURE 4.9-2** MATLAB input file used to analyze the circuit shown in Figure 4.9-1.

Equation 4.9-5 can be written using matrices as

$$\begin{bmatrix} R_1 + aR_P + R_3 & -R_3 \\ -R_3 & (1-a)R_P + R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \tag{4.9-6}$$

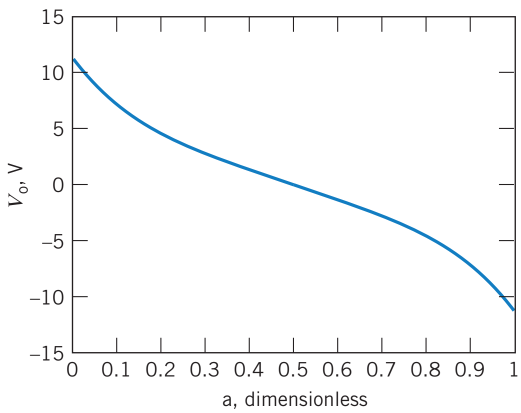



**FIGURE 4.9-3** Plot of $v_o$ versus $a$ for the circuit shown in Figure 4.9-1.

Next, $i_1$ and $i_2$ are calculated by using MATLAB to solve the mesh equation, Eq. 4.9-6. Then the output voltage is calculated as

$$v_o = R_3(i_1 - i_2) \tag{4.9-7}$$

Figure 4.9-2 shows the MATLAB input file. The parameter $a$ varies from 0 to 1 in increments of 0.05. At each value of $a$, MATLAB solves Eq. 4.9-6 and then uses Eq. 4.9-7 to calculate the output voltage. Finally, MATLAB produces the plot of $v_o$ versus $a$ that is shown in Figure 4.9-3.

## 4.10 *Using PSpice to Determine Node Voltages and Mesh Currents*

To determine the node voltages of a dc circuit using PSpice, we

1. Draw the circuit in the OrCAD Capture workspace.
2. Specify a "Bias Point" simulation.
3. Run the simulation.

PSpice will label the nodes with the values of the node voltages.

### EXAMPLE 4.10-1 Using PSpice to Find Node Voltages and Mesh Currents

Use PSpice to determine the values of the node voltages and mesh currents for the circuit shown in Figure 4.10-1.

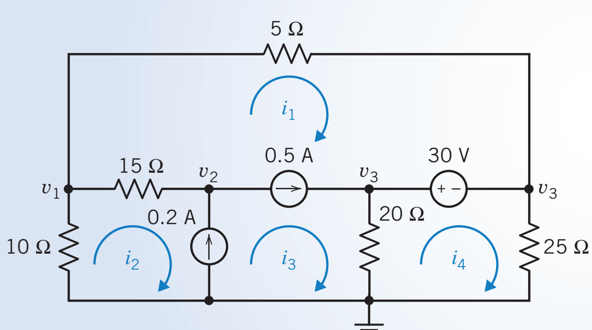



**FIGURE 4.10-1** A circuit having node voltages $v_1$, $v_2$, $v_3$, and $v_4$ and mesh currents $i_1$, $i_2$, $i_3$, and $i_4$.

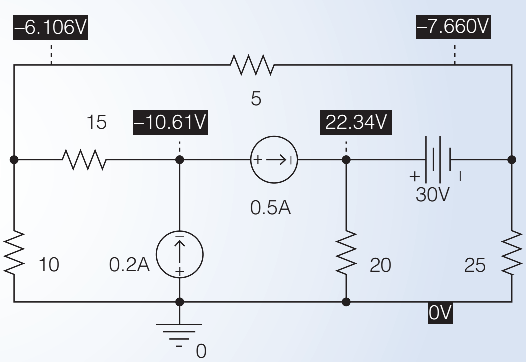



**FIGURE 4.10-2** The circuit from Figure 4.10-1 drawn in the OrCAD workspace. The white numbers shown on black backgrounds are the values of the node voltages.

## Solution

Figure 4.10-2 shows the result of drawing the circuit in the OrCAD workspace (see Appendix A) and performing a Bias Point simulation. (Select PSpice\New Simulation Profile from the OrCAD Capture menu bar; then choose Bias Point from the Analysis Type drop-down list in the Simulation Settings dialog box to specify a Bias Point simulation. Select PSpice\Run Simulation Profile from the OrCAD Capture menu bar to run the simulation.) PSpice labels the nodes with the values of the node voltages using white numbers shown on black backgrounds. Comparing Figures 4.10-1 and 4.10-2, we see that the node voltages are

$$v_1 = -6.106\text{ V}, v_2 = -10.61\text{ V}, v_3 = 22.34\text{ V}, \text{ and } v_4 = -7.660\text{ V}.$$

Figure 4.10-3 shows the circuit from Figure 4.10-2 after inserting a 0-V current source on the outside of each mesh. The currents in these 0-V sources will be the mesh currents shown in Figure 4.10-1. In particular, source V2

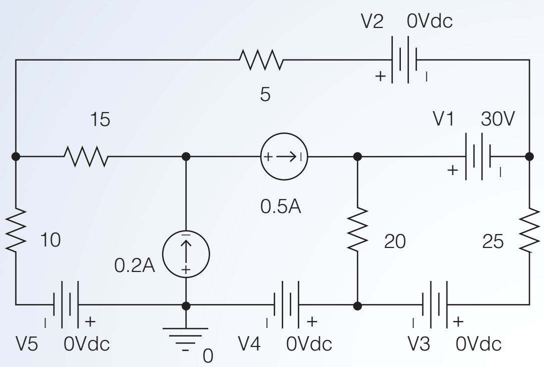



**FIGURE 4.10-3** The circuit from Figure 4.10-1 drawn in the OrCAD workspace with 0-V voltage sources added to measure the mesh currents.

measures mesh current $i_1$, source V3 measures mesh current $i_2$, source V4 measures mesh current $i_3$, and source V5 measures mesh current $i_4$.

After we rerun the simulation (Select PSpice\Run from the OrCAD Capture menu bar), OrCAD Capture will open a Schematics window. Select View\Output File from the menu bar in the Schematics window. Scroll down through the output file to find the currents in the voltage sources:

```
VOLTAGE SOURCE CURRENTS
NAME           CURRENT

V_V1          − 6.170E − 01
V_V2            3.106E − 01
V_V3          − 3.064E − 01
V_V4            8.106E − 01
V_V5            6.106E − 01

TOTAL POWER DISSIPATION   1.85E + 01  WATTS

          JOB CONCLUDED
```

PSpice uses the passive convention for the current and voltage of all circuit elements, including voltage sources. Noticing the small + and − signs on the voltage source symbols in Figure 4.10-3, we see that the currents provided by PSpice are directed form left to right in sources VI and V2 and are directed from right to left in sources V3, V4, and V5. In particular, the mesh currents are

$$i_1 = 0.3106\text{ A}, i_2 = 0.6106\text{ A}, i_3 = 0.8106\text{ A}, \text{ and } i_4 = -0.3064\text{ A}.$$

An extra step is needed to use PSpice to determine the mesh currents. PSpice does not label the values of the mesh currents, but it does provide the value of the current in each voltage source. Recall that a 0-V voltage source is equivalent to a short circuit. Consequently, we can insert 0-V current sources into the circuit without altering the values of the mesh currents. We will insert those sources into the circuit in such a way that their currents are also the mesh currents. To determine the mesh currents of a dc circuit using PSpice, we

1. Draw the circuit in the OrCAD Capture workspace.
2. Add 0-V voltage sources to measure the mesh currents.
3. Specify a Bias Point simulation.
4. Run the simulation.

PSpice will write the voltage source currents in the output file.

## 4.11 *How Can We Check . . . ?*

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able quickly to identify those solutions that need more work.

The following examples illustrate techniques useful for checking the solutions of the sort of problem discussed in this chapter.

### Example 4.11-1 How Can We Check Node Voltages?

The circuit shown in Figure 4.11-1*a* was analyzed using PSpice. The PSpice output file, Figure 4.11-1*b*, includes the node voltages of the circuit. **How can we check** that these node voltages are correct?

### Solution

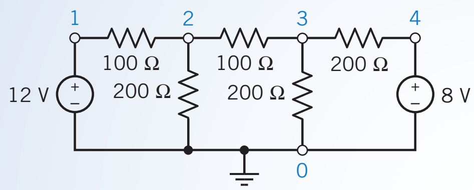
The node equation corresponding to node 2 is

$$\frac{V(2) - V(1)}{100} + \frac{V(2)}{200} + \frac{V(2) - V(3)}{100} = 0$$

where, for example, $V(2)$ is the node voltage at node 2. When the node voltages from Figure 4.11-1*b* are substituted into the left-hand side of this equation, the result is

$$\frac{7.2727 - 12}{100} + \frac{7.2727}{200} + \frac{7.2727 - 5.0909}{100} = 0.011$$

The right-hand side of this equation should be 0 instead of 0.011. It looks like something is wrong. Is a current of only 0.011 negligible? Probably not in this case. If the node voltages were correct, then the currents of the 100-Ω resistors would be 0.047 A and 0.022 A, respectively. The current of 0.011 A does not seem negligible when compared to currents of 0.047 A and 0.022 A.




(*a*)

```
Node Voltage Example

V1    1    0    12
R1    1    2    100
R2    2    0    200
R3    2    3    200
R4    3    0    200
R5    3    4    200
V2    4    0    8

.END

  NODE VOLTAGES

  NODE VOLTAGE

  (1)    12.0000
  (2)     7.2727
  (3)     5.0909
  (4)     8.0000
```

(*b*)

**FIGURE 4.11-1** (*a*) A circuit and (*b*) the node voltages calculated using PSpice. The bottom node has been chosen as the reference node, which is indicated by the ground symbol and the node number 0. The voltages and resistors have units of voltages and ohms, respectively.

Is it possible that PSpice would calculate the node voltages incorrectly? Probably not, but the PSpice input file could easily contain errors. In this case, the value of the resistance connected between nodes 2 and 3 has been mistakenly specified to be 200 Ω. After changing this resistance to 100 Ω, PSpice calculates the node voltages to be

$$V(1) = 12.0, \; V(2) = 7.0, \; V(3) = 5.5, \; V(4) = 8.0$$

Substituting these voltages into the node equation gives

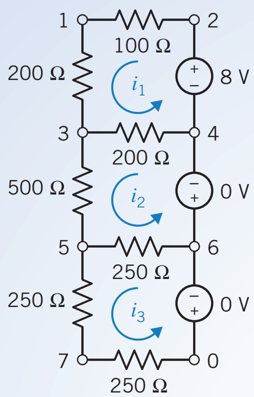
$$\frac{7.0 - 12.0}{100} + \frac{7.0}{200} + \frac{7.0 - 5.5}{100} = 0.0$$

so these node voltages do satisfy the node equation corresponding to node 2.

## EXAMPLE 4.11-2 How Can We Check Mesh Currents?

The circuit shown in Figure 4.11-2*a* was analyzed using PSpice. The PSpice output file, Figure 4.11-2*b*, includes the mesh currents of the circuit. **How can we check** that these mesh currents are correct?

(The PSpice output file will include the currents through the voltage sources. Recall that PSpice uses the passive convention, so the current in the 8-V source will be $-i_1$ instead of $i_1$. The two 0-V sources have been added to include mesh currents $i_2$ and $i_3$ in the PSpice output file.)




(*a*)

```
Mesh Current Example

R1   1   2   100
R2   1   3   200
V1   2   4   8
R3   3   4   200
R5   3   5   500
V2   4   6   0
R6   5   6   250
R7   5   7   250
V3   6   0   0
R8   7   0   250

.END

 MESH CURRENTS

 NAME     CURRENT

 I1    1.763E-02
 I2   -4.068E-03
 I3   -1.356E-03
```

(*b*)

**FIGURE 4.11–2** (*a*) A circuit and (*b*) the mesh currents calculated using PSpice. The voltages and resistances are given in volts and ohms, respectively.

## Solution

The mesh equation corresponding to mesh 2 is

$$200(i_2 - i_1) + 500i_2 + 250(i_2 - i_3) = 0$$

When the mesh currents from Figure 4.11-2*b* are substituted into the left-hand side of this equation, the result is

$$200(-0.004068 - 0.01763) + 500(-0.004068) + 250(-0.004068 - (-0.001356)) = 1.629$$

The right-hand side of this equation should be 0 instead of 1.629. It looks like something is wrong. Most likely, the PSpice input file contains an error. This is indeed the case. The nodes of both 0-V voltage sources have been entered in the wrong order. Recall that the first node should be the positive node of the voltage source. After correcting this error, PSpice gives

$$i_1 = 0.01763, \quad i_2 = 0.004068, \quad i_3 = 0.001356$$

Using these values in the mesh equation gives

$$200(0.004068 - 0.01763) + 500(0.004068) + 250(0.004068 - 0.001356) = 0.0$$

These mesh currents do indeed satisfy the mesh equation corresponding to mesh 2.

## 4.12 Design Example Potentiometer Angle Display

A circuit is needed to measure and display the angular position of a potentiometer shaft. The angular position, $\theta$, will vary from $-180°$ to $180°$.

Figure 4.12-1 illustrates a circuit that could do the job. The +15-V and –15-V power supplies, the potentiometer, and resistors $R_1$ and $R_2$ are used to obtain a voltage, $v_i$, that is proportional to $\theta$. The amplifier is used to change the constant of proportionality to obtain a simple relationship between $\theta$ and the voltage, $v_o$, displayed by the voltmeter. In this example, the amplifier will be used to obtain the relationship

$$v_o = k \cdot \theta \text{ where } k = 0.1 \frac{\text{volt}}{\text{degree}} \tag{4.12-1}$$

so that $\theta$ can be determined by multiplying the meter reading by 10. For example, a meter reading of $-7.32$ V indicates that $\theta = -73.2°$.

### Describe the Situation and the Assumptions

The circuit diagram in Figure 4.12-2 is obtained by modeling the power supplies as ideal voltage sources, the voltmeter as an open circuit, and the potentiometer by two resistors. The parameter $a$ in the model of the potentiometer varies from 0 to 1 as $\theta$ varies from $-180°$ to $180°$. That means

$$a = \frac{\theta}{360°} + \frac{1}{2} \tag{4.12-2}$$

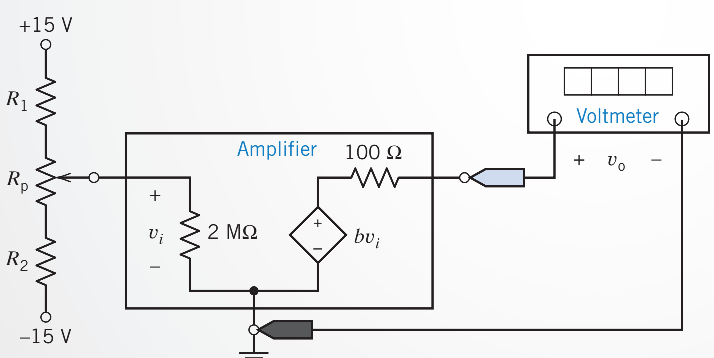



**FIGURE 4.12-1** Proposed circuit for measuring and displaying the angular position of the potentiometer shaft.

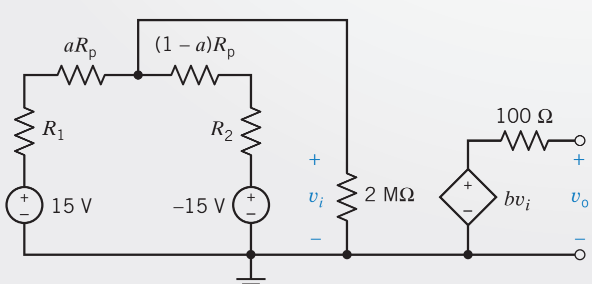



**FIGURE 4.12-2** Circuit diagram containing models of the power supplies, voltmeter, and potentiometer.

Solving for $\theta$ gives

$$\theta = \left(a - \frac{1}{2}\right) \cdot 360^\circ \tag{4.12-3}$$

## State the Goal

Specify values of resistors $R_1$ and $R_2$, the potentiometer resistance $R_P$, and the amplifier gain $b$ that will cause the meter voltage $v_o$ to be related to the angle $\theta$ by Eq. 4.12-1.

## Generate a Plan

Analyze the circuit shown in Figure 4.12-2 to determine the relationship between $v_i$ and $\theta$. Select values of $R_1$, $R_2$, and $R_p$. Use these values to simplify the relationship between $v_i$ and $\theta$. If possible, calculate the value of $b$ that will cause the meter voltage $v_o$ to be related to the angle $\theta$ by Eq. 4.12-1. If this isn't possible, adjust the values of $R_1$, $R_2$, and $R_p$ and try again.

## Act on the Plan

The circuit has been redrawn in Figure 4.12-3. A single node equation will provide the relationship between between $v_i$ and $\theta$:

$$\frac{v_i}{2\,\text{M}\Omega} + \frac{v_i - 15}{R_1 + aR_p} + \frac{v_i - (-15)}{R_2 + (1-a)R_p} = 0$$

Solving for $v_i$ *gives*

$$v_i = \frac{2\,\text{M}\Omega\left(R_p(2a-1) + R_1 - R_2\right)15}{\left(R_1 + aR_p\right)\left(R_2 + (1-a)R_p\right) + 2\,\text{M}\Omega\left(R_1 + R_2 + R_p\right)} \tag{4.12-4}$$

This equation is quite complicated. Let's put some restrictions on $R_1$, $R_2$, and $R_p$ that will make it possible to simplify this equation. First, let $R_1 = R_2 = R$. Second, require that both $R$ and $R_p$ be much smaller than 2 MΩ (for example, $R < 20\,\text{k}\Omega$). Then,

$$\left(R + aR_p\right)\left(R + (1-a)R_p\right) \ll 2\,\text{M}\Omega\left(2R + R_p\right)$$

That is, the first term in the denominator of the left side of Eq. 4.12-4 is negligible compared to the second term. Equation 4.12-4 can be simplified to

$$v_i = \frac{R_p(2a-1)15}{2R + R_p}$$

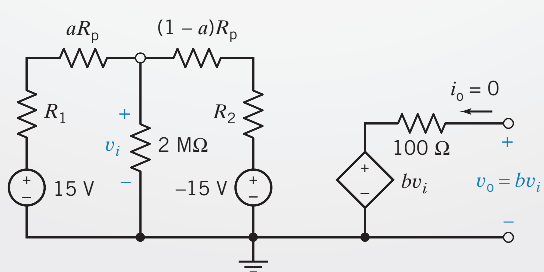



**FIGURE 4.12-3** The redrawn circuit showing the mode $v_i$.

Next, using Eq. 4.12-3,

$$v_i = \left(\frac{R_p}{2R + R_p}\right)\left(\frac{15\text{ V}}{180^\circ}\right)\theta$$

It is time to pick values for $R$ and $R_p$. Let $R = 5\text{ k}\Omega$ and $R_p = 10\text{ k}\Omega$; then

$$v_i = \left(\frac{7.5\text{ V}}{180^\circ}\right)$$

Referring to Figure 4.12-2, the amplifier output is given by

$$v_o = bv_i \tag{4.12-5}$$

so

$$v_o = b\left(\frac{7.5\text{ V}}{180^\circ}\right)\theta$$

Comparing this equation to Eq. 4.12-1 gives

$$b\left(\frac{7.5\text{ V}}{180^\circ}\right) = 0.1\,\frac{\text{volt}}{\text{degree}}$$

or

$$b = \frac{180}{7.5}(0.1) = 2.4$$

The final circuit is shown in Figure 4.12-4.

## Verify the Proposed Solution

As a check, suppose $\theta = 150^\circ$. From Eq. 4.12-2, we see that

$$a = \frac{150^\circ}{360^\circ} + \frac{1}{2} = 0.9167$$

Using Eq. 4.12-4, we calculate

$$v_i = \frac{2\text{ M}\Omega(10\text{ k}\Omega(2 \times 0.9167 - 1))15}{(5\text{ k}\Omega + 0.9167 \times 10\text{ k}\Omega)(5\text{ k}\Omega + (1 - 0.9167)10\text{ k}\Omega) + 2\text{ M}\Omega(2 \times 5\text{ k}\Omega + 10\text{ k}\Omega)} = 6.24$$

Finally, Eq. 4.12-5 indicates that the meter voltage will be

$$v_o \times 2.4 \cdot 6.24 = 14.98$$

This voltage will be interpreted to mean that the angle was

$$\theta = 10 \cdot v_o = 149.8^\circ$$

which is correct to three significant digits.

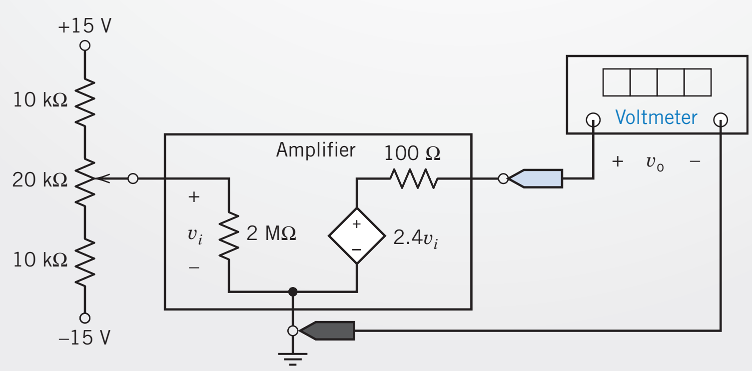



**FIGURE 4.12-4** The final designed circuit.

## 4.13 SUMMARY

- The node voltage method of circuit analysis identifies the nodes of a circuit where two or more elements are connected. When the circuit consists of only resistors and current sources, the following procedure is used to obtain the node equations.
  1. We choose one node as the reference node. Label the node voltages at the other nodes.
  2. Express element currents as functions of the node voltages. Figure 4.13-1*a* illustrates the relationship between the current in a resistor and the voltages at the nodes of the resistor.
  3. Apply KCL at all nodes except for the reference node.

  Solution of the simultaneous equations results in knowledge of the node voltages. All the voltages and currents in the circuit can be determined when the node voltages are known.
- When a circuit has voltage sources as well as current sources, we can still use the node voltage method by using the concept of a supernode. A supernode is a large node that includes two nodes connected by a known voltage source. If the voltage source is directly connected between a node $q$ and the reference node, we may set $v_q = v_s$ and write the KCL equations at the remaining nodes.
- If the circuit contains a dependent source, we first express the controlling voltage or current of the dependent source as a function of the node voltages. Next, we express the controlled voltage or current as a function of the node voltages. Finally, we apply KCL to nodes and supernodes.
- Mesh current analysis is accomplished by applying KVL to the meshes of a planar circuit. When the circuit consists of only resistors and voltage sources, the following procedure is used to obtain the mesh equations.
  1. Label the mesh currents.
  2. Express element voltages as functions of the mesh currents. Figure 4.13-1*b* illustrates the relationship between the voltage across a resistor and the currents of the meshes that include the resistor.
  3. Apply KVL to all meshes.

  Solution of the simultaneous equations results in knowledge of the mesh currents. All the voltages and currents in the circuit can be determined when the mesh currents are known.
- If a current source is common to two adjoining meshes, we define the interior of the two meshes as a supermesh. We then write the mesh current equation around the periphery of the supermesh. If a current source appears at the periphery of only one mesh, we may define that mesh current as equal to the current of the source, accounting for the direction of the current source.
- If the circuit contains a dependent source, we first express the controlling voltage or current of the dependent source as a function of the mesh currents. Next, we express the controlled voltage or current as a function of the mesh currents. Finally, we apply KVL to meshes and supermeshes.
- In general, either node voltage or mesh current analysis can be used to obtain the currents or voltages in a circuit. However, a circuit with fewer node equations than mesh current equations may require that we select the node voltage method. Conversely, mesh current analysis is readily applicable for a circuit with fewer mesh current equations than node voltage equations.
- MATLAB greatly reduces the drudgery of solving node or mesh equations.

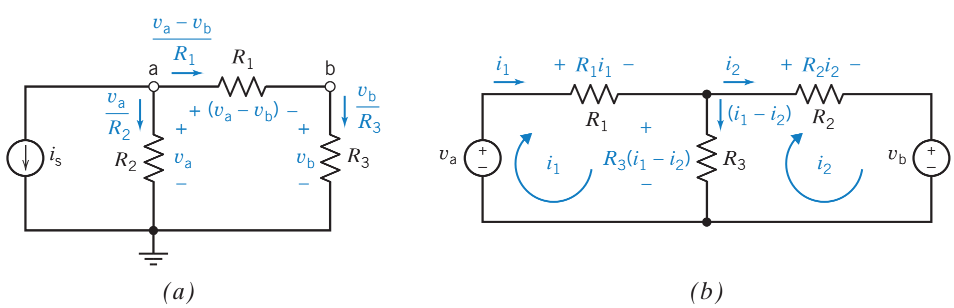



**FIGURE 4.13-1** Expressing resistor currents and voltages in terms of (*a*) node voltage or (*b*) mesh currents.

## PROBLEMS

⊕ Problem available in WileyPLUS at instructor's discretion.

### Section 4.2 Node Voltage Analysis of Circuits with Current Sources

**P 4.2-1** ⊕ The node voltages in the circuit of Figure P 4.2-1 are $v_1 = -4$ V and $v_2 = 2$ V. Determine $i$, the current of the current source.

*Answer:* $i = 1.5$ A

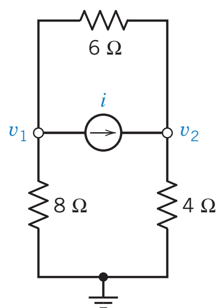



**Figure P 4.2-1**

**P 4.2-2** Determine the node voltages for the circuit of Figure P 4.2-2.

*Answer:* $v_1 = 2$ V, $v_2 = 30$ V, and $v_3 = 24$ V

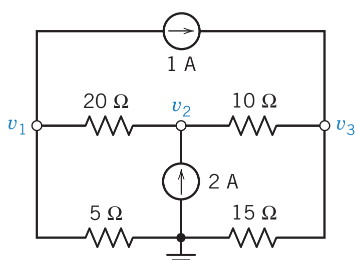



**Figure P 4.2-2**

**P 4.2-3** The encircled numbers in the circuit shown in Figure P 4.2-3 are node numbers. Determine the values of the corresponding node voltages $v_1$ and $v_2$.

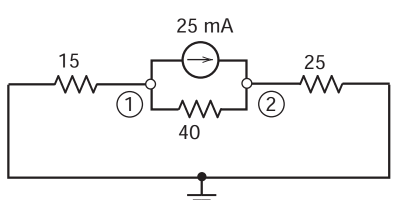



**Figure P 4.2-3**

**P 4.2-4** ⊕ Consider the circuit shown in Figure P 4.2-4. Find values of the resistances $R_1$ and $R_2$ that cause the voltages $v_1$ and $v_2$ to be $v_1 = 1$V and $v_2 = 2$ V.

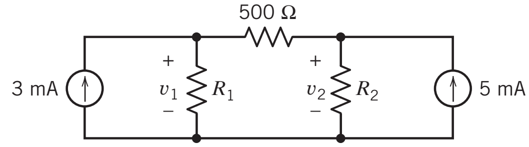



**Figure P 4.2-4**

**P 4.2-5** Find the voltage $v$ for the circuit shown in Figure P 4.2-5.

*Answer:* $v = 21.7$ mV

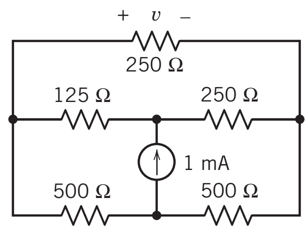



**Figure P 4.2-5**

**P 4.2-6** Simplify the circuit shown in Figure P 4.2-6 by replacing series and parallel resistors with equivalent resistors; then analyze the simplified circuit by writing and solving node equations. (*a*) Determine the power supplied by each current source. (*b*) Determine the power received by the 12-Ω resistor.

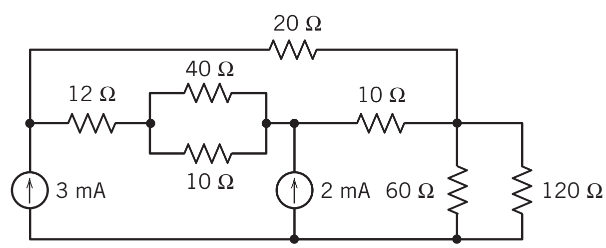



**Figure P 4.2-6**

**P 4.2-7** The node voltages in the circuit shown in Figure P 4.2-7 are $v_a = 7$ V and $v_b = 10$ V. Determine values of the current source current, $i_s$, and the resistance, $R$.

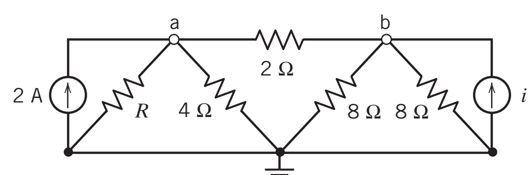



**Figure P 4.2-7**

**P 4.2-8** The encircled numbers in the circuit shown in Figure P 4.2-8 are node numbers. The corresponding node voltages are $v_1$ and $v_2$. The node equation representing this circuit is

$$\begin{bmatrix} 0.225 & -0.125 \\ -0.125 & 0.125 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

**(a)** Determine the values of $R$ and $I_s$ in Figure P 4.2-8.

**(b)** Determine the value of the power supplied by the 3-A current source.

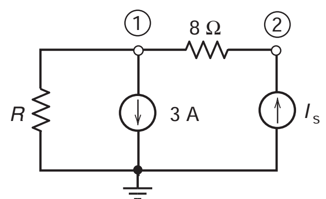



**Figure P 4.2-8**

### Section 4.3 Node Voltage Analysis of Circuits with Current and Voltage Sources

**P 4.3-1** ⊕ The voltmeter in Figure P 4.3-1 measures $v_c$, the node voltage at node c. Determine the value of $v_c$.

*Answer:* $v_c = 2$ V

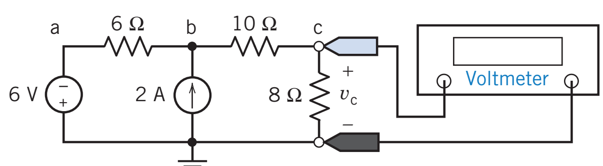



**Figure P 4.3-1**

**P 4.3-2** ⊕ The voltages $v_a$, $v_b$, $v_c$, and $v_d$ in Figure P 4.3-2 are the node voltages corresponding to nodes a, b, c, and d. The current $i$ is the current in a short circuit connected between nodes b and c. Determine the values of $v_a$, $v_b$, $v_c$, and $v_d$ and of $i$.

*Answer:* $v_a = -12$ V, $v_b = v_c = 4$ V, $v_d = -4$ V, $i = 2$ mA

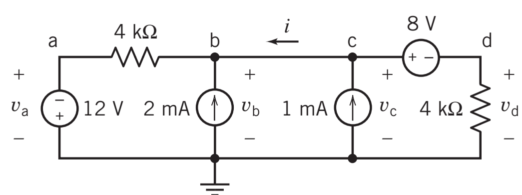



**Figure P 4.3-2**

**P 4.3-3** Determine the values of the power supplied by each of the sources in the circuit shown in Figure P 4.3-3.

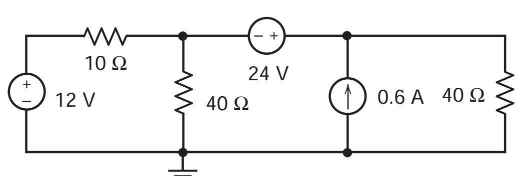



**Figure P 4.3-3**

**P 4.3-4** Determine the values of the node voltages $v_1$, $v_2$, and $v_3$ in the circuit shown in Figure P 4.3-4.

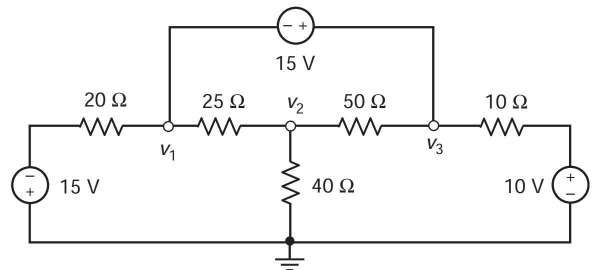



**Figure P 4.3-4**

**P 4.3-5** ⊕ The voltages $v_a$, $v_b$, and $v_c$ in Figure P 4.3-5 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 12\text{ V},\ v_b = 9.882\text{ V},\ \text{and}\ \ v_c = 5.294\text{ V}$$

Determine the power supplied by the voltage source.

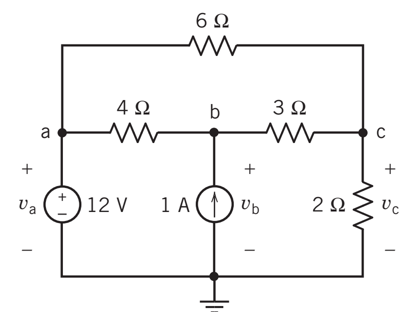



**Figure P 4.3-5**

**P 4.3-6** ⊕ The voltmeter in the circuit of Figure P 4.3-6 measures a node voltage. The value of that node voltage depends on the value of the resistance $R$.

**(a)** Determine the value of the resistance $R$ that will cause the voltage measured by the voltmeter to be 4 V.

**(b)** Determine the voltage measured by the voltmeter when $R = 1.2\text{ k}\Omega = 1200\ \Omega$.

*Answers:* **(a)** 6 kΩ **(b)** 2V

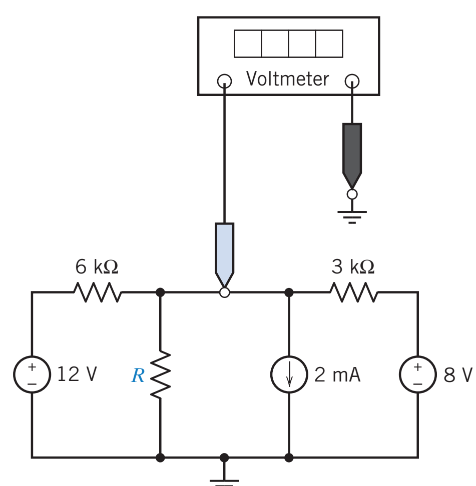



**Figure P 4.3-6**

**P 4.3-7** Determine the values of the node voltages $v_1$ and $v_2$ in Figure P 4.3-7. Determine the values of the currents $i_a$ and $i_b$.

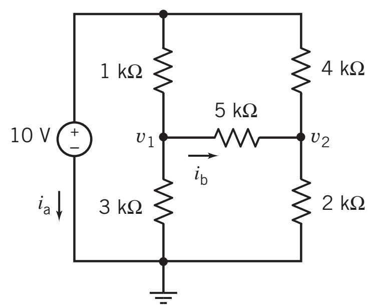



**Figure P 4.3-7**

**P 4.3-8** The circuit shown in Figure P 4.3-8 has two inputs, $v_1$ and $v_2$, and one output, $v_o$. The output is related to the input by the equation

$$v_o = av_1 + bv_2$$

where $a$ and $b$ are constants that depend on $R_1$, $R_2$, and $R_3$.

**(a)** Determine the values of the coefficients $a$ and $b$ when $R_1 = 10\ \Omega, R_2 = 40\ \Omega,$ and $R_3 = 8\ \Omega$.

**(b)** Determine the values of the coefficients $a$ and $b$ when $R_1 = R_2$ and $R_3 = R_1||R_2$.

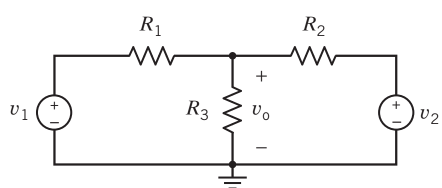



**Figure P 4.3-8**

**P 4.3-9** Determine the values of the node voltages of the circuit shown in Figure P 4.3-9.

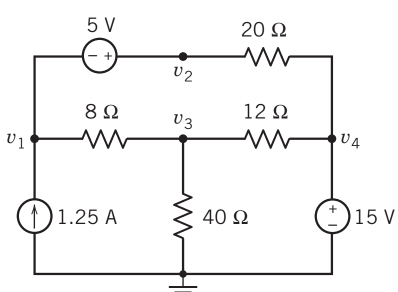



**Figure P 4.3-9**

**P 4.3-10** Figure P 4.3-10 shows a measurement made in the laboratory. Your lab partner forgot to record the values of $R_1$, $R_2$, and $R_3$. He thinks that the two resistors were 10-kΩ resistors and the other was a 5-kΩ resistor. Is this possible? Which resistor is the 5-kΩ resistor?

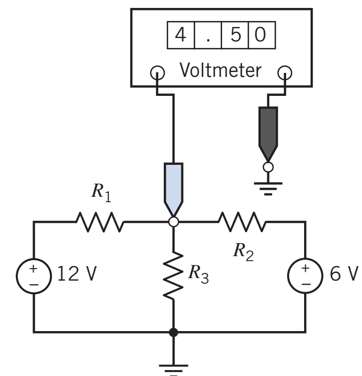



**Figure P 4.3-10**

**P 4.3-11** Determine the values of the power supplied by each of the sources in the circuit shown in Figure P 4.3-11.

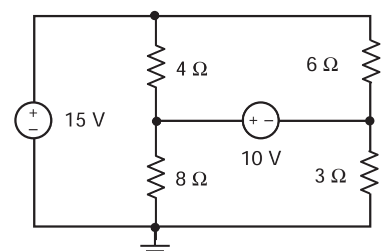



**Figure P 4.3-11**

**P 4.3-12** Determine the values of the node voltages of the circuit shown in Figure P 4.3-12.

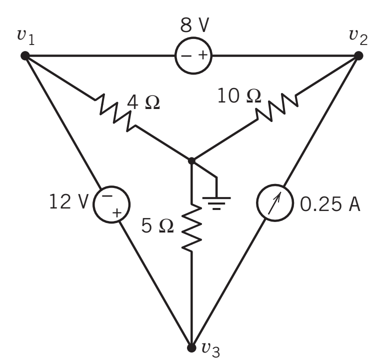



**Figure P 4.3-12**

**P 4.3-13** Determine the values of node voltages $v_1$ and $v_2$ in the circuit shown in Figure P 4.3-13.

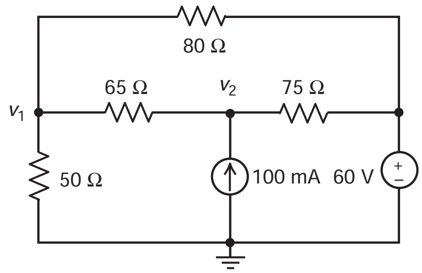



Figure P 4.3-13

**P 4.3-14** The voltage source in the circuit shown in Figure P 4.3-14 supplies 83.802 W. The current source supplies 17.572 W. Determine the values of the node voltages $v_1$ and $v_2$.

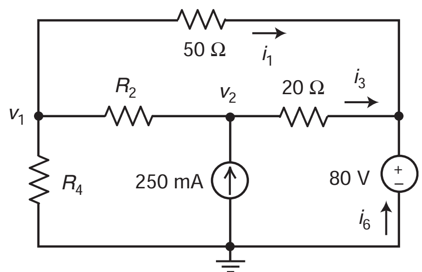



Figure P 4.3-14

## Section 4.4 Node Voltage Analysis with Dependent Sources

**P 4.4-1** The voltages $v_a$, $v_b$, and $v_c$ in Figure P 4.4-1 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 8.667\text{ V}, v_b = 2\text{ V}, \text{ and } v_c = 10\text{ V}$$

Determine the value of $A$, the gain of the dependent source.

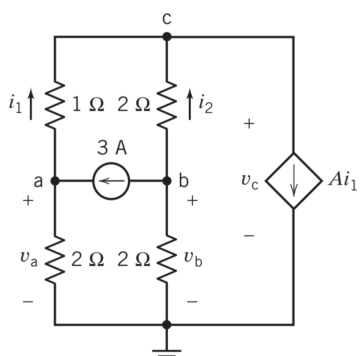



Figure P 4.4-1

**P 4.4-2** Find $i_b$ for the circuit shown in Figure P 4.4-2.

*Answer:* $i_b = -12$ mA

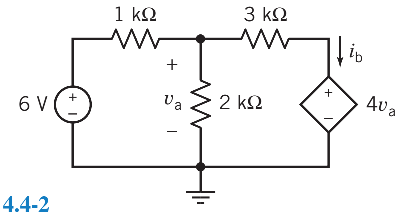



Figure P 4.4-2

**P 4.4-3** Determine the node voltage $v_b$ for the circuit of Figure P 4.4-3.

*Answer:* $v_b = 1.5$ V

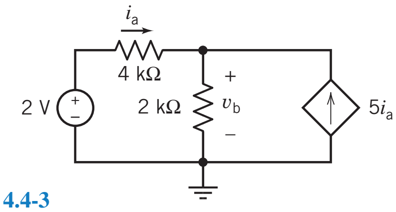



Figure P 4.4-3

**P 4.4-4** The circled numbers in Figure P 4.4-4 are node numbers. The node voltages of this circuit are $v_1 = 10$ V, $v_2 = 14$ V, and $v_3 = 12$ V.

**(a)** Determine the value of the current $i_b$.
**(b)** Determine the value of $r$, the gain of the CCVS.

*Answers:* **(a)** −2 A **(b)** 4 V/A

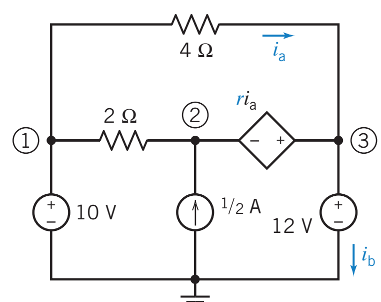



Figure P 4.4-4

**P 4.4-5** Determine the value of the current $i_x$ in the circuit of Figure P 4.4-5.

*Answer:* $i_x = 2.4$ A

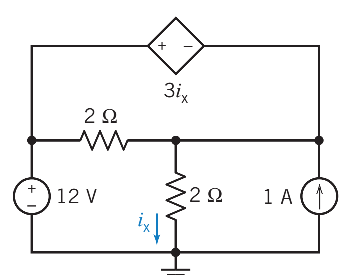



Figure P 4.4-5

**P 4.4-6** The encircled numbers in the circuit shown in Figure P 4.4-6 are node numbers. Determine the value of the power supplied by the CCVS.

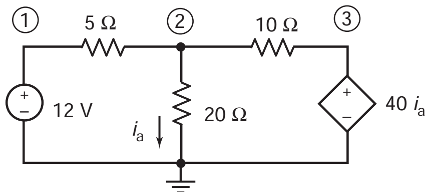



Figure P 4.4-6

**P 4.4-7** The encircled numbers in the circuit shown in Figure P 4.4-7 are node numbers. The corresponding node voltages are:

$$v_1 = 9.74\ \text{V and } v_2 = 6.09\ \text{V}$$

Determine the values of the gains of the dependent sources, $r$ and $g$.

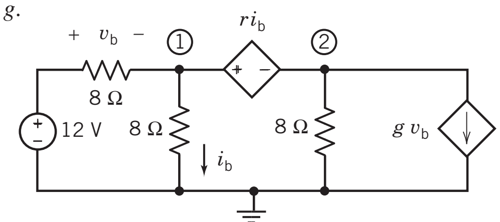



Figure P 4.4-7

**P 4.4-8** Determine the value of the power supplied by the dependent source in Figure P 4.4-8.

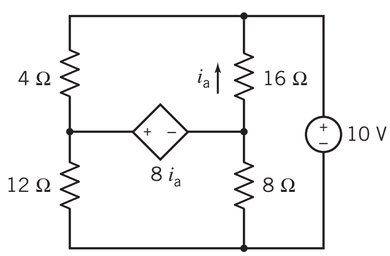



Figure P 4.4-8

**P 4.4-9** The node voltages in the circuit shown in Figure P 4.4-9 are

$$v_1 = 4\ \text{V}, v_2 = 0\ \text{V, and } v_3 = -6\ \text{V}$$

Determine the values of the resistance $R$ and of the gain $b$ of the CCCS.

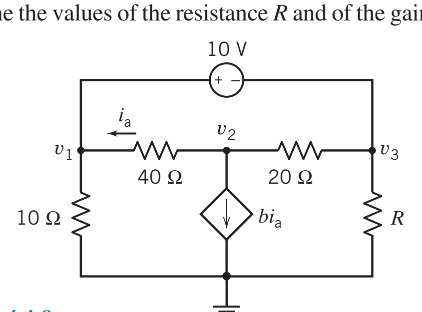



Figure P 4.4-9

**P 4.4-10** The value of the node voltage at node $b$ in the circuit shown in Figure P 4.4-10 is $v_b = 18$ V.

**(a)** Determine the value of $A$, the gain of the dependent source.
**(b)** Determine the power supplied by the dependent source.

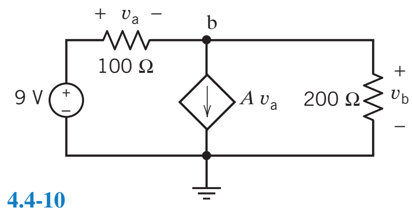



Figure P 4.4-10

***P 4.4-11** Determine the power supplied by the dependent source in the circuit shown in Figure P 4.4-11.

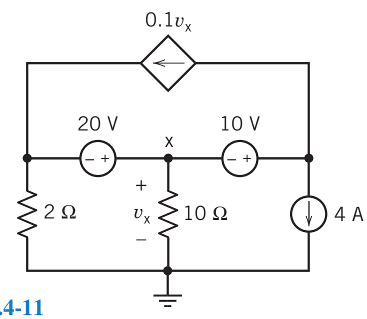



Figure P 4.4-11

***P 4.4-12** Determine values of the node voltages $v_1$, $v_2$, $v_3$, $v_4$, and $v_5$ in the circuit shown in Figure P 4.4-12.

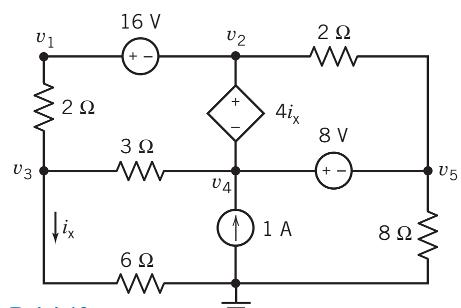



Figure P 4.4-12

***P 4.4-13** Determine values of the node voltages $v_1$, $v_2$, $v_3$, $v_4$, and $v_5$ in the circuit shown in Figure P 4.4-13.

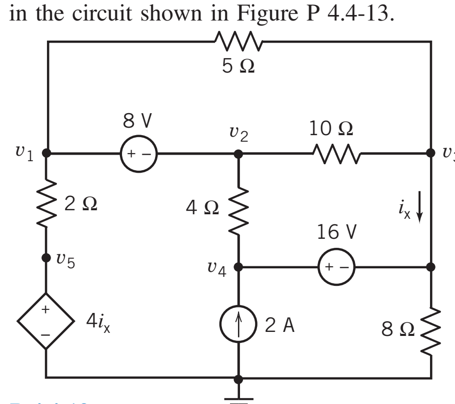



Figure P 4.4-13

**P 4.4-14** The voltages $v_1$, $v_2$, $v_3$, and $v_4$ are the node voltages corresponding to nodes 1, 2, 3, and 4 in Figure P 4.4-14. Determine the values of these node voltages.

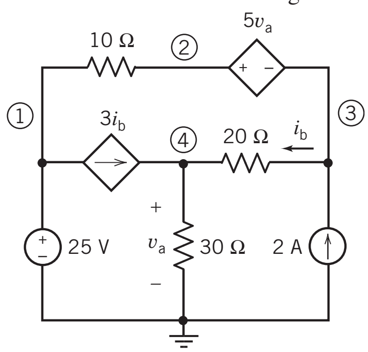



**Figure P 4.4-14**

**P 4.4-15** The voltages $v_1$, $v_2$, $v_3$, and $v_4$ in Figure P 4.4-15 are the node voltages corresponding to nodes 1, 2, 3, and 4. The values of these voltages are

$$v_1 = 10\ \text{V}, v_2 = 75\ \text{V}, v_3 = -15\ \text{V},\ \text{and}\ \ v_4 = 22.5\ \text{V}$$

Determine the values of the gains of the dependent sources, $A$ and $B$, and of the resistance $R_1$.

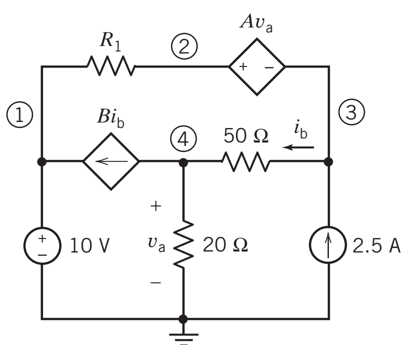



**Figure P 4.4-15**

**P 4.4-16** The voltages $v_1$, $v_2$, and $v_3$ in Figure P 4.4-16 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12\ \text{V}, v_2 = 21\ \text{V},\ \text{and}\ v_3 = -3\ \text{V}$$

**(a)** Determine the values of the resistances $R_1$ and $R_2$.
**(b)** Determine the power supplied by each source.

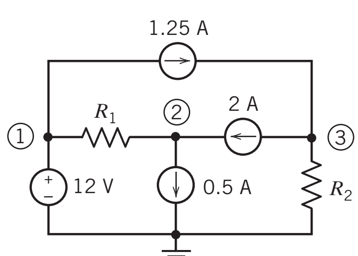



**Figure P 4.4-16**

**P 4.4-17** ⊕ The voltages $v_1$, $v_2$, and $v_3$ in Figure P 4.4-17 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12\ \text{V}, v_2 = 9.6\ \text{V},\ \text{and}\ v_3 = -1.33\ \text{V}$$

**(a)** Determine the values of the resistances $R_1$ and $R_2$.
**(b)** Determine the power supplied by each source.

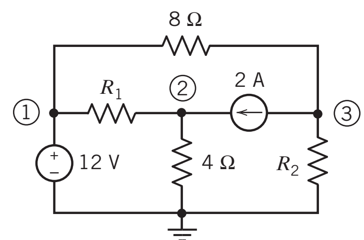



**Figure P 4.4-17**

**P 4.4-18** The voltages $v_2$, $v_3$, and $v_4$ for the circuit shown in Figure P 4.4-18 are:

$$v_2 = 16\ \text{V}, v_3 = 8\ \text{V},\ \text{and}\ v_4 = 6\ \text{V}$$

Determine the values of the following:

**(a)** The gain, $A$, of the VCVS
**(b)** The resistance $R_5$
**(c)** The currents $i_b$ and $i_c$
**(d)** The power received by resistor $R_4$

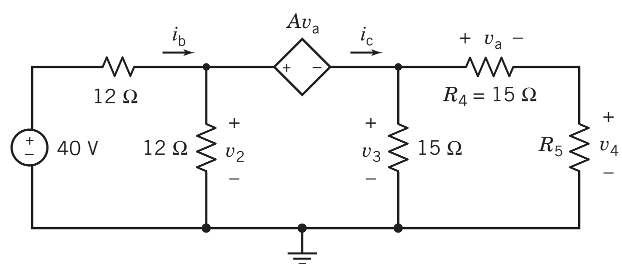



**Figure P 4.4-18**

**P 4.4-19** Determine the values of the node voltages $v_1$ and $v_2$ for the circuit shown in Figure P 4.4-19.

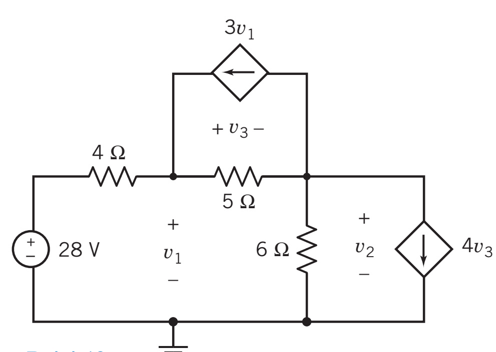



**Figure P 4.4-19**

**P 4.4-20** The encircled numbers in Figure P 4.4-20 are node numbers. Determine the values of $v_1$, $v_2$, and $v_3$, the node voltages corresponding to nodes 1, 2, and 3.

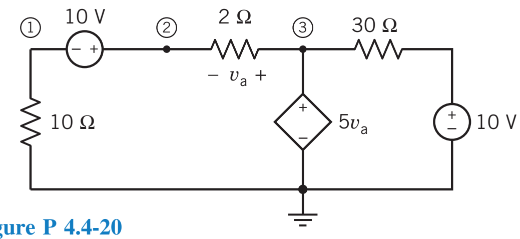



Figure P 4.4-20

**P 4.4-21** Determine the values of the node voltages $v_1$, $v_2$, and $v_3$ for the circuit shown in Figure P 4.4-21.

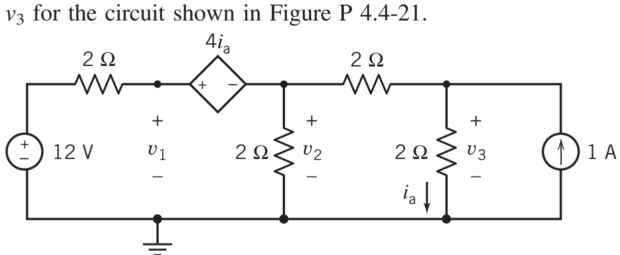



Figure P 4.4-21

**P 4.4-22** Determine the values of the node voltages $v_1$, $v_2$, and $v_3$ for the circuit shown in Figure P 4.4-22.

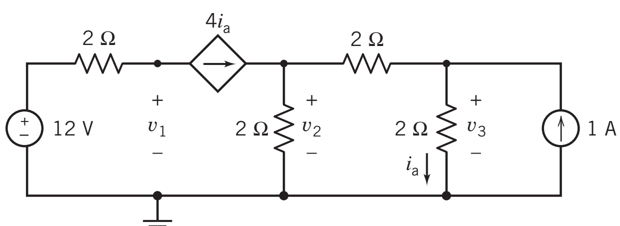



Figure P 4.4-22

## Section 4.5 Mesh Current Analysis with Independent Voltage Sources

**P 4.5-1** Determine the mesh currents $i_1$, $i_2$, and $i_3$ for the circuit shown in Figure P 4.5-1.

*Answers:* $i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 4$ A

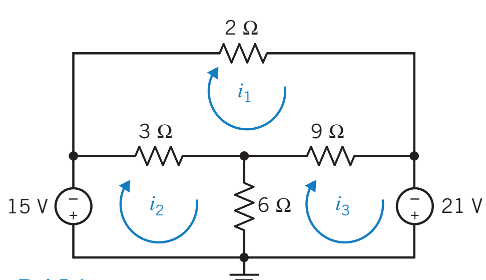



Figure P 4.5-1

**P 4.5-2** The values of the mesh currents in the circuit shown in Figure P 4.5-2 are $i_1 = 2$ A, $i_2 = 3$ A, and $i_3 = 4$ A. Determine the values of the resistance $R$ and of the voltages $v_1$ and $v_2$ of the voltage sources.

*Answers:* $R = 12\ \Omega$, $v_1 = -4$ V, and $v_2 = -28$ V

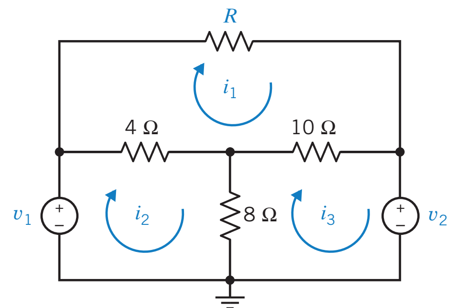



Figure P 4.5-2

**P 4.5-3** The currents $i_1$ and $i_2$ in Figure P 4.5-3 are the mesh currents. Determine the value of the resistance $R$ required to cause $v_a = -6$ V.

*Answer:* $R = 4\ \Omega$

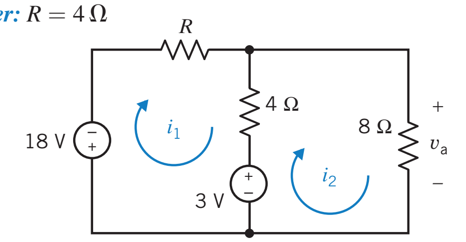



Figure P 4.5-3

**P 4.5-4** Determine the mesh currents $i_a$ and $i_b$ in the circuit shown in Figure P 4.5-4.

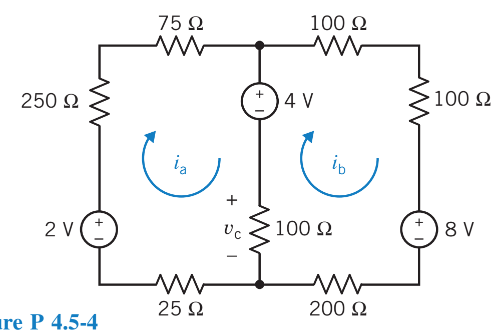



Figure P 4.5-4

**P 4.5-5** Find the current $i$ for the circuit of Figure P 4.5-5.

*Hint:* A short circuit can be treated as a 0-V voltage source.

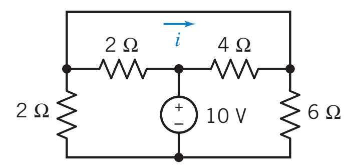



Figure P 4.5-5

**P 4.5-6** Simplify the circuit shown in Figure P 4.5-6 by replacing series and parallel resistors by equivalent resistors. Next, analyze the simplified circuit by writing and solving mesh equations.

**(a)** Determine the power supplied by each source,
**(b)** Determine the power absorbed by the 30-Ω resistor.

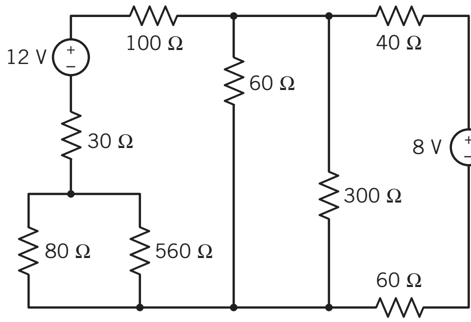



**Figure P 4.5-6**

## Section 4.6 Mesh Current Analysis with Current and Voltage Sources

**P 4.6-1** Find $i_b$ for the circuit shown in Figure P 4.6-1.

*Answer:* $i_b = 0.6$ A

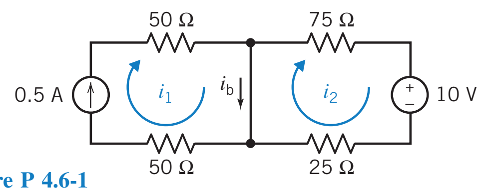



**Figure P 4.6-1**

**P 4.6-2** Find $v_c$ for the circuit shown in Figure P 4.6-2.

*Answer:* $v_c = 15$ V

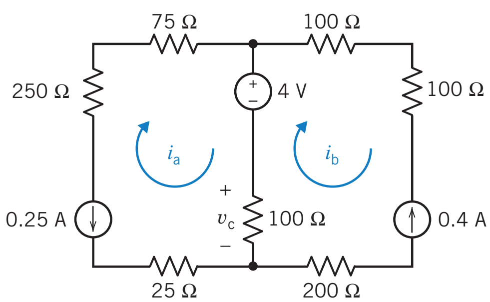



**Figure P 4.6-2**

**P 4.6-3** Find $v_2$ for the circuit shown in Figure P 4.6-3.

*Answer:* $v_2 = 2$ V

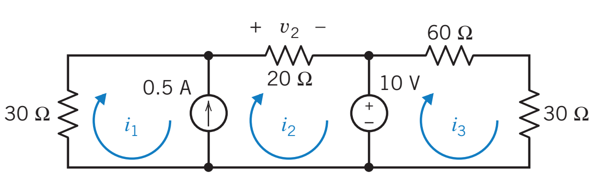



**Figure P 4.6-3**

**P 4.6-4** Find $v_c$ for the circuit shown in Figure P 4.6-4.

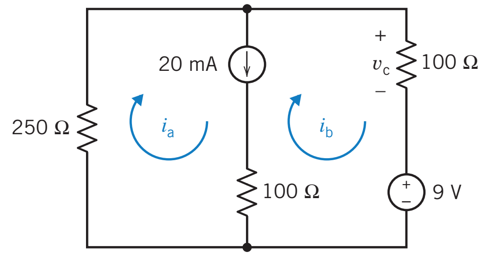



**Figure P 4.6-4**

**P 4.6-5** Determine the value of the voltage measured by the voltmeter in Figure P 4.6-5.

*Answer:* 8 V

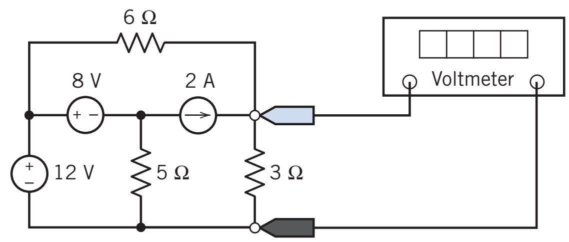



**Figure P 4.6-5**

**P 4.6-6** Determine the value of the current measured by the ammeter in Figure P 4.6-6.

*Hint:* Write and solve a single mesh equation.

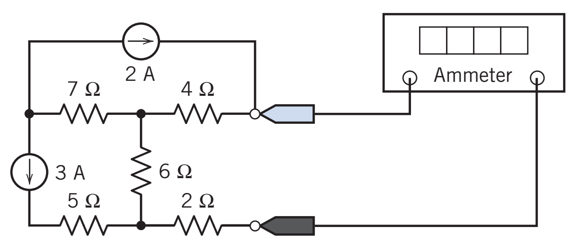



**Figure P 4.6-6**

**P 4.6-7** The mesh currents are labeled in the circuit shown in Figure P 4.6-7. The values of these mesh currents are:

$$i_1 = -1.1014 \text{ A}, i_2 = 0.8986 \text{ A and } i_3 = -0.2899 \text{ A}$$

**(a)** Determine the values of the resistances $R_1$ and $R_3$.
**(b)** Determine the value of the current source current.
**(c)** Determine the value of the power supplied by the 12-V voltage source.

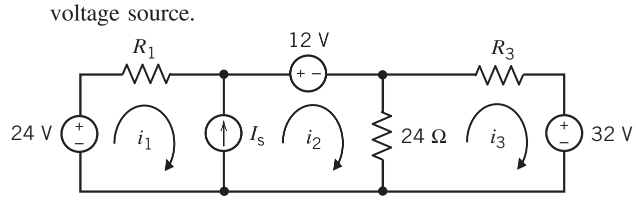



**Figure P 4.6-7**

**P 4.6-8** Determine values of the mesh currents $i_1$, $i_2$, and $i_3$ in the circuit shown in Figure P 4.6-8.

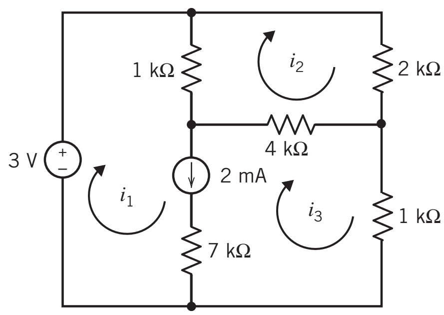



**Figure P 4.6-8**

**P 4.6-9** The mesh currents are labeled in the circuit shown in Figure P 4.6-9. Determine the value of the mesh currents $i_1$, and $i_2$.

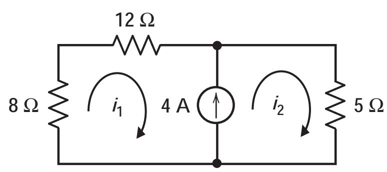



**Figure P 4.6-9**

**P 4.6-10** ⊕ The mesh currents in the circuit shown in Figure P 4.6-10 are

$$i_1 = -2.2213 \text{ A}, i_2 = 0.7787 \text{ A}, \text{ and } i_3 = 0.0770 \text{ A}$$

**(a)** Determine the values of the resistances $R_1$ and $R_3$.

**(b)** Determine the value of the power supplied by the current source.

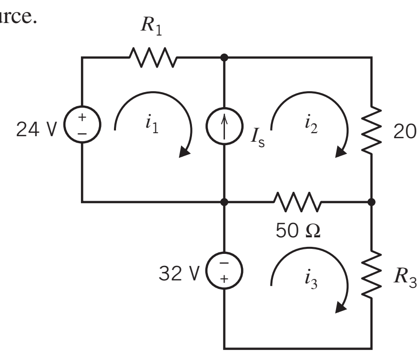



**Figure P 4.6-10**

**P 4.6-11** ⊕ Determine the value of the voltage measured by the voltmeter in Figure P 4.6-11.

*Hint:* Apply KVL to a supermesh to determine the current in the 2-Ω resistor.

*Answer:* 4/3 V

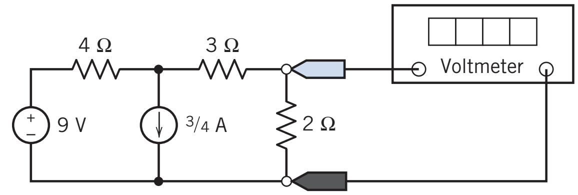



**Figure P 4.6-11**

**P 4.6-12** ⊕ Determine the value of the current measured by the ammeter in Figure P 4.6-12.

*Hint:* Apply KVL to a supermesh.

*Answer:* −0.333 A

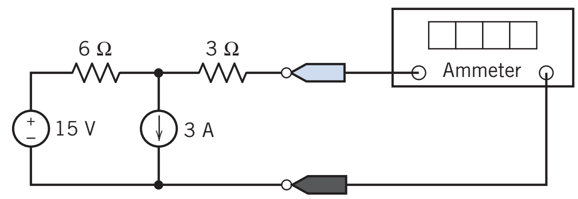



**Figure P 4.6-12**

**P 4.6-13** Determine the values of the mesh currents $i_1$, $i_2$, and $i_3$ and the output voltage $v_0$ in the circuit shown in Figure P 4.6-13.

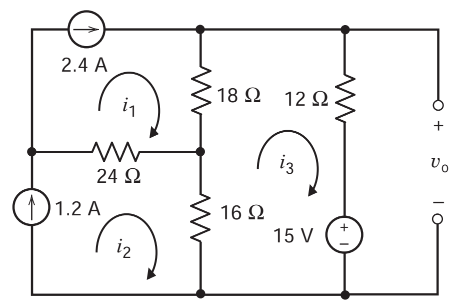



**Figure P 4.6-13**

**P 4.6-14** Determine the values of the power supplied by the sources in the circuit shown in Figure P 4.6-14.

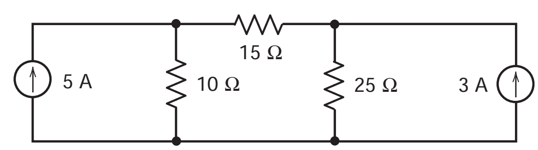



**Figure P 4.6-14**

**P 4.6-15** Determine the values of the resistance $R$ and of the power supplied by the 6-A current source in the circuit shown in Figure P 4.6-15.

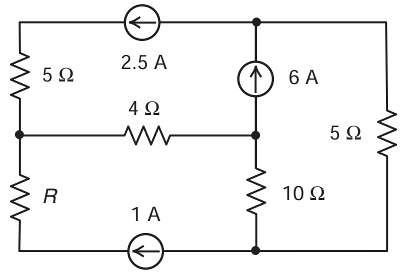



Figure P 4.6-15

### Section 4.7 Mesh Current Analysis with Dependent Sources

**P 4.7-1** Find $v_2$ for the circuit shown in Figure P 4.7-1.

*Answer:* $v_2 = 10$ V

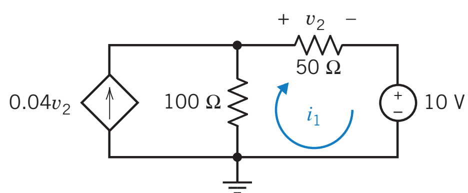



Figure P 4.7-1

**P 4.7-2** Determine the values of the power supplied by the voltage source and by the CCCS in the circuit shown in Figure P 4.7-2.

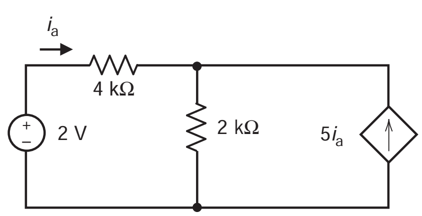



Figure P 4.7-2

**P 4.7-3** Find $v_o$ for the circuit shown in Figure P 4.7-3.

*Answer:* $v_o = 2.5$ V

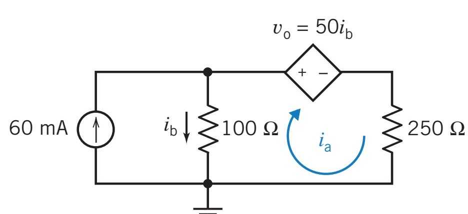



Figure P 4.7-3

**P 4.7-4** Determine the mesh current $i_a$ for the circuit shown in Figure P 4.7-4.

*Answer:* $i_a = -24$ mA

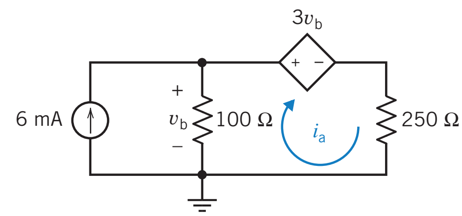



Figure P 4.7-4

**P 4.7-5** Although scientists continue to debate exactly why and how it works, the process of using electricity to aid in the repair and growth of bones—which has been used mainly with fractures—may soon be extended to an array of other problems, ranging from osteoporosis and osteoarthritis to spinal fusions and skin ulcers.

An electric current is applied to bone fractures that have not healed in the normal period of time. The process seeks to imitate natural electrical forces within the body. It takes only a small amount of electric stimulation to accelerate bone recovery. The direct current method uses an electrode that is implanted at the bone. This method has a success rate approaching 80 percent.

The implant is shown in Figure P 4.7-5*a*, and the circuit model is shown in Figure P 4.7-5*b*. Find the energy delivered to the cathode during a 24-hour period. The cathode is represented by the dependent voltage source and the 100-kΩ resistor.

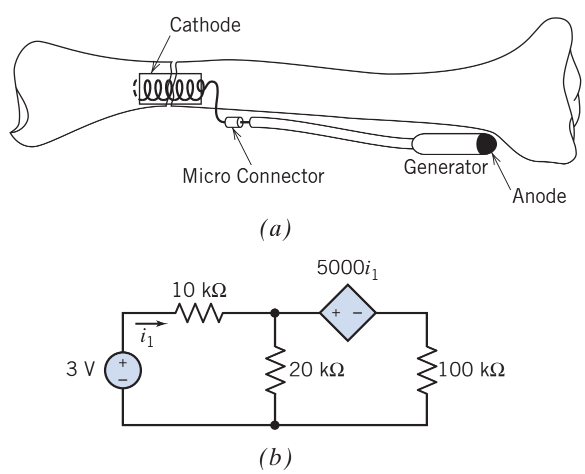



Figure P 4.7-5 (*a*) Electric aid to bone repair. (*b*) Circuit model.

**P 4.7-6** Determine the value of the power supplied by the VCCS in the circuit shown in Figure P 4.7-6.

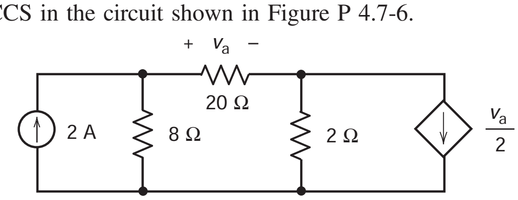



Figure P 4.7-6

**P 4.7-7** The currents $i_1$, $i_2$, and $i_3$ are the mesh currents of the circuit shown in Figure P 4.7-7. Determine the values of $i_1$, $i_2$, and $i_3$.

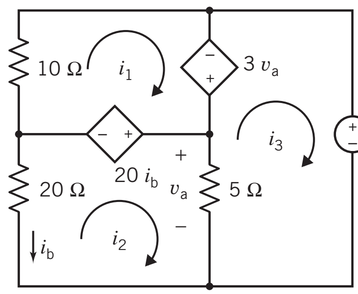



Figure P 4.7-7

**P 4.7-8** Determine the value of the power supplied by the dependent source in Figure P 4.7-8.

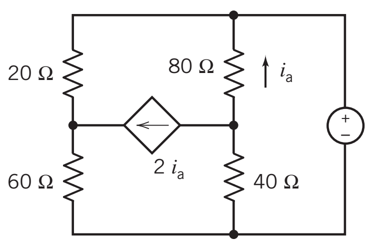



Figure P 4.7-8

**P 4.7-9** Determine the value of the resistance $R$ in the circuit shown in Figure P 4.7-9.

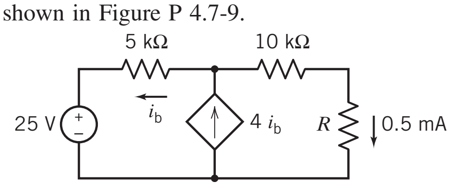



Figure P 4.7-9

**P 4.7-10** The circuit shown in Figure P 4.7-10 is the small signal model of an amplifier. The input to the amplifier is the voltage source voltage $v_s$. The output of the amplifier is the voltage $v_o$.

**(a)** The ratio of the output to the input, $v_o/v_s$, is called the gain of the amplifier. Determine the gain of the amplifier.

**(b)** The ratio of the current of the input source to the input voltage $i_b/v_s$ is called the input resistance of the amplifier. Determine the input resistance.

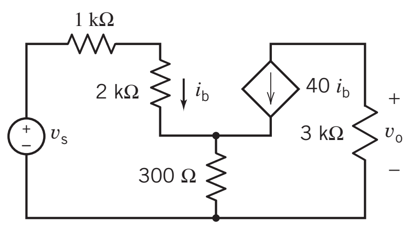



Figure P 4.7-10

**P 4.7-11** Determine the values of the mesh currents of the circuit shown in Figure P 4.7-11.

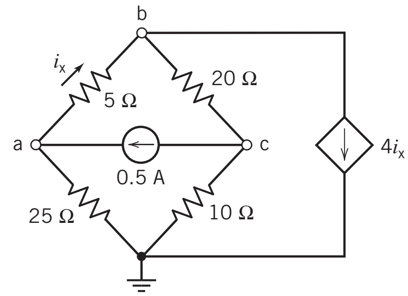



Figure P 4.7-11

**P 4.7-12** The currents $i_1$, $i_2$, and $i_3$ are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-12. Determine the values of these mesh currents.

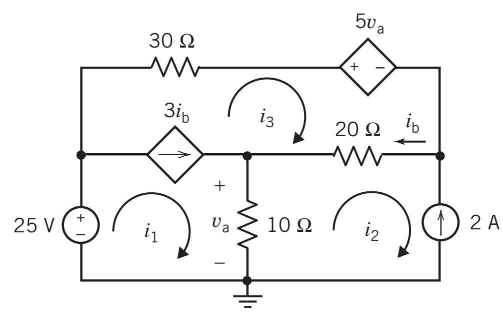



Figure P 4.7-12

**P 4.7-13** The currents $i_1$, $i_2$, and $i_3$ are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-13. The values of these currents are

$$i_1 = -1.375 \text{ A}, i_2 = -2.5 \text{ A and } i_3 = -3.25 \text{ A}$$

Determine the values of the gains of the dependent sources, $A$ and $B$.

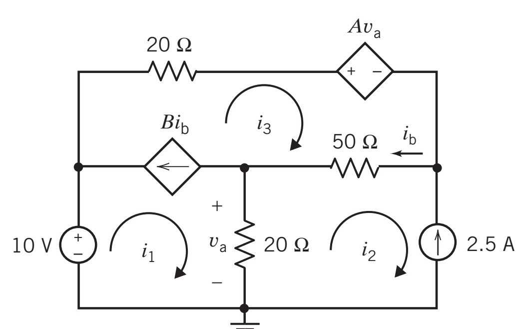



Figure P 4.7-13

**P 4.7-14** Determine the current $i$ in the circuit shown in Figure P 4.7-14.

*Answer:* $i = 3$ A

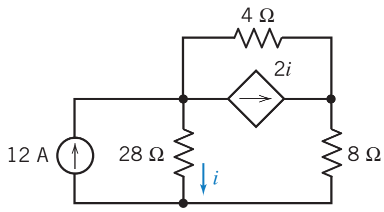



**Figure P 4.7-14**

**P 4.7-15** Determine the values of the mesh currents $i_1$ and $i_2$ for the circuit shown in Figure P 4.7-15.

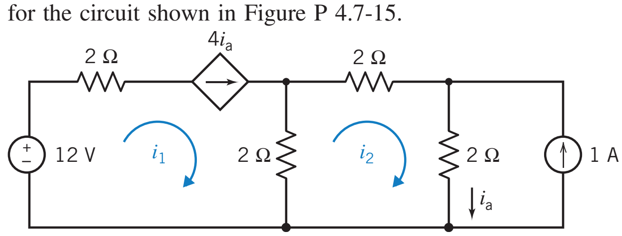



**Figure P 4.7-15**

**P 4.7-16** Determine the values of the mesh currents $i_1$ and $i_2$ for the circuit shown in Figure P 4.7-16.

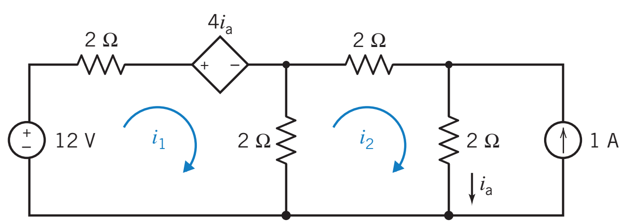



**Figure P 4.7-16**

### Section 4.8 The Node Voltage Method and Mesh Current Method Compared

**P 4.8-1** The circuit shown in Figure P 4.8-1 has two inputs, $v_s$ and $i_s$, and one output, $v_o$. The output is related to the inputs by the equation

$$v_o = ai_s + bv_s$$

where $a$ and $b$ are constants to be determined. Determine the values $a$ and $b$ by (a) writing and solving mesh equations and (b) writing and solving node equations.

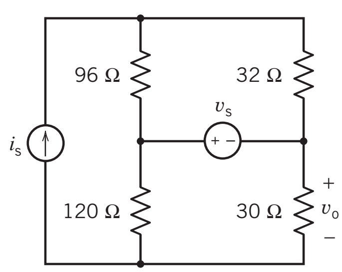



**Figure P 4.8-1**

**P 4.8-2** Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-2 by writing and solving (a) node equations and (b) mesh equations.

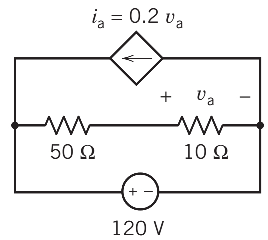



**Figure P 4.8-2**

### Section 4.9 Circuit Analysis Using MATLAB

**P 4.9-1** The encircled numbers in the circuit shown Figure P 4.9-1 are node numbers. Determine the values of the corresponding node voltages $v_1$, $v_2$, and $v_3$.

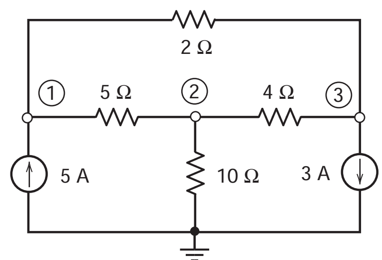



**Figure P 4.9-1**

**P 4.9-2** Determine the values of the node voltages $v_1$ and $v_2$ in the circuit shown in Figure P 4.9-2.

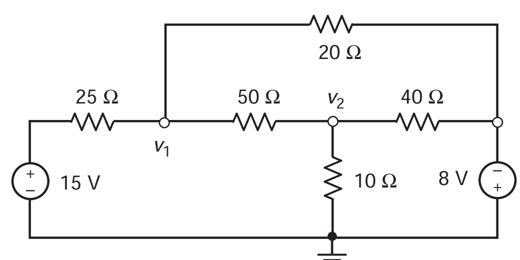



**Figure P 4.9-2**

**P 4.9-3** Determine the values of the node voltages $v_1$, $v_2$, and $v_3$ in the circuit shown in Figure P 4.9-3.

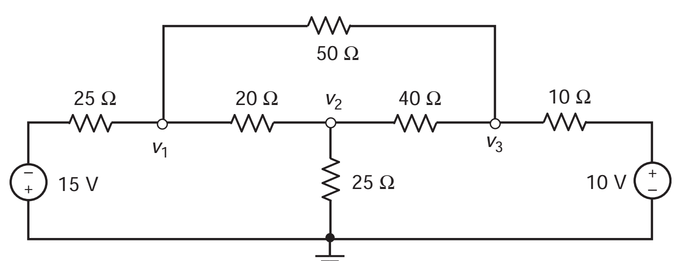



**Figure P 4.9-3**

**P 4.9-4** Determine the node voltages $v_1$ and $v_2$ for the circuit shown in Figure P 4.9-4.

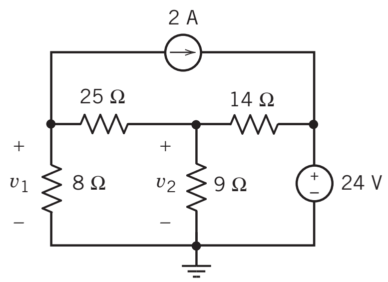



**Figure P 4.9-4**

**P 4.9-5** Determine the mesh currents $i_1$ and $i_2$ for the circuit shown in Figure P 4.9-5.

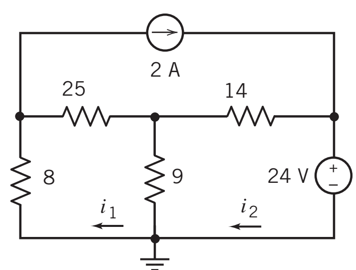



**Figure P 4.9-5**

**P 4.9-6** Represent the circuit shown in Figure P 4.9-6 by the matrix equation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -40 \\ -228 \end{bmatrix}$$

Determine the values of the coefficients $a_{11}$, $a_{12}$, $a_{21}$, and $a_{22}$.

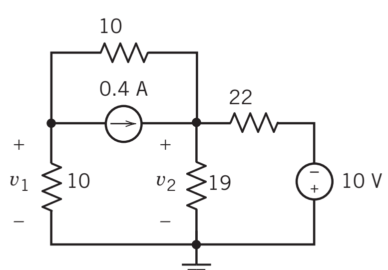



**Figure P 4.9-6**

**P 4.9-7** Represent the circuit shown in Figure P 4.9-7 by the matrix equation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

Determine the values of the coefficients $a_{11}$, $a_{12}$, $a_{21}$, and $a_{22}$.

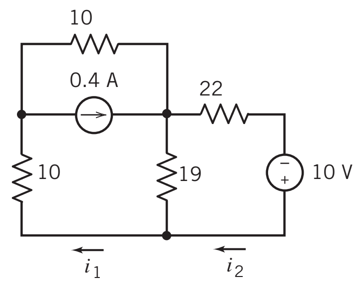



**Figure P 4.9-7**

**P 4.9-8** Determine the values of the power supplied by each of the sources for the circuit shown in Figure P 4.9-8.

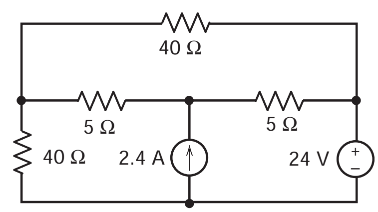



**Figure P 4.9-8**

**P 4.9-9** The mesh currents are labeled in the circuit shown in Figure P 4.9-9. Determine the value of the mesh currents $i_1$ and $i_2$.

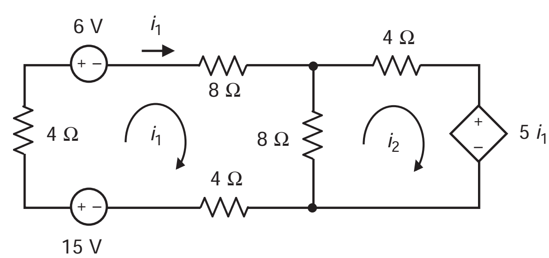



**Figure P 4.9-9**

**P 4.9-10** The encircled numbers in the circuit shown in Figure P 4.9-10 are node numbers. Determine the values of the corresponding node voltages $v_1$ and $v_2$.

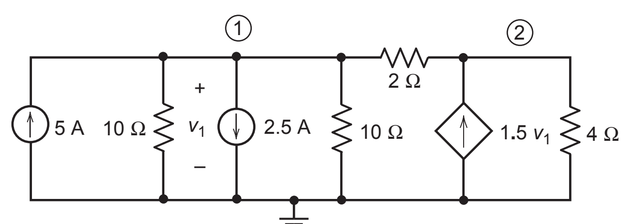



**Figure P 4.9-10**

### Section 4.11 How Can We Check . . . ?

**P 4.11-1** Computer analysis of the circuit shown in Figure P 4.11-1 indicates that the node voltages are $v_a = 5.2$ V, $v_b = -4.8$ V, and $v_c = 3.0$ V. Is this analysis correct?

***Hint:*** Use the node voltages to calculate all the element currents. Check to see that KCL is satisfied at each node.

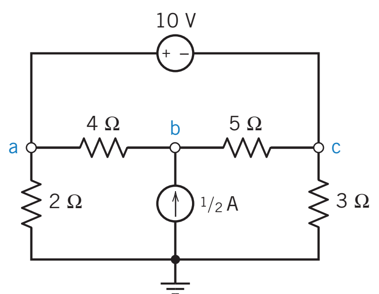



**Figure P 4.11-1**

**P 4.11-2** An old lab report asserts that the node voltages of the circuit of Figure P 4.11-2 are $v_a = 4$ V, $v_b = 20$ V, and $v_c = 12$ V. Are these correct?

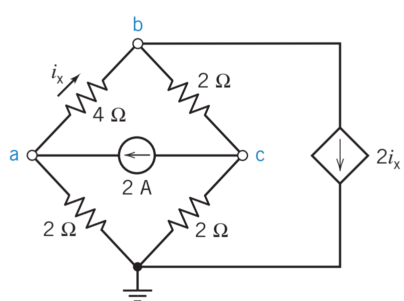



**Figure P 4.11-2**

**P 4.11-3** Your lab partner forgot to record the values of $R_1$, $R_2$, and $R_3$. He thinks that two of the resistors in Figure P 4.11-3 had values of 10 kΩ and that the other had a value of 5 kΩ. Is this possible? Which resistor is the 5-kΩ resistor?

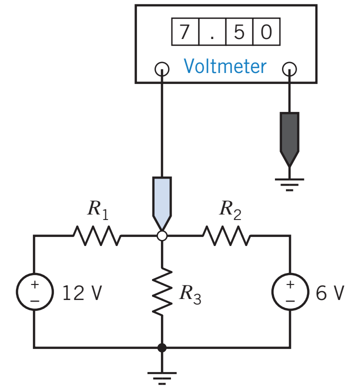



**Figure P 4.11-3**

**P 4.11-4** Computer analysis of the circuit shown in Figure P 4.11-4 indicates that the mesh currents are $i_1 = 2$ A, $i_2 = 4$ A, and $i_3 = 3$ A. Verify that this analysis is correct.

***Hint:*** Use the mesh currents to calculate the element voltages. Verify that KVL is satisfied for each mesh.

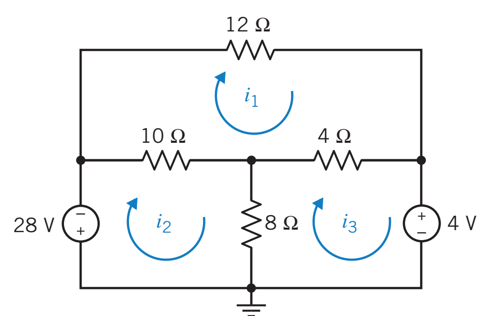



**Figure P 4.11-4**

## PSpice Problems

**SP 4-1** Use PSpice to determine the node voltages of the circuit shown in Figure SP 4-1.

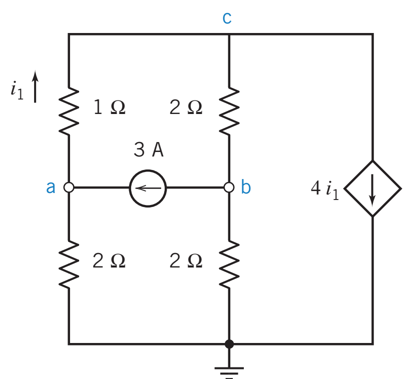



**Figure SP 4-1**

**SP 4-2** Use PSpice to determine the mesh currents of the circuit shown in Figure SP 4-2 when $R = 4\ \Omega$.

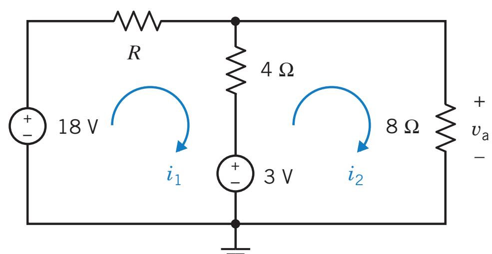



**Figure SP 4-2**

**SP 4-3** The voltages $v_a$, $v_b$, $v_c$, and $v_d$ in Figure SP 4-3 are the node voltages corresponding to nodes a, b, c, and d. The current $i$ is the current in a short circuit connected between nodes b and c. Use PSpice to determine the values of $v_a$, $v_b$, $v_c$, and $v_d$ and of $i$.

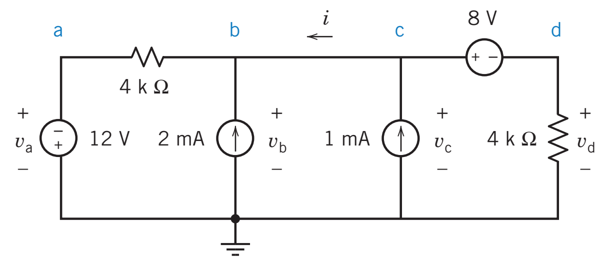



**Figure SP 4-3**

**SP 4-4** Determine the current $i$ shown in Figure SP 4-4.

***Answer:*** $i = 0.56$ A

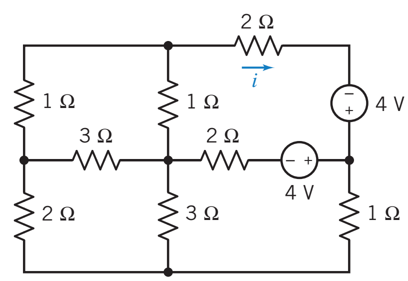



**Figure SP 4-4**

## Design Problems

**DP 4-1** An electronic instrument incorporates a 15-V power supply. A digital display is added that requires a 5-V power supply. Unfortunately, the project is over budget, and you are instructed to use the existing power supply. Using a voltage divider, as shown in Figure DP 4-1, you are able to obtain 5 V. The specification sheet for the digital display shows that the display will operate properly over a supply voltage range of 4.8 V to 5.4 V. Furthermore, the display will draw 300 mA ($I$) when the display is active and 100 mA when quiescent (no activity).

**(a)** Select values of $R_1$ and $R_2$ so that the display will be supplied with 4.8 V to 5.4 V under all conditions of current $I$.

**(b)** Calculate the maximum power dissipated by each resistor, $R_1$ and $R_2$, and the maximum current drawn from the 15-V supply.

**(c)** Is the use of the voltage divider a good engineering solution? If not, why? What problems might arise?

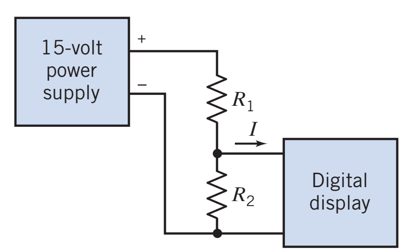



**Figure DP 4-1**

**DP 4-2** For the circuit shown in Figure DP 4-2, it is desired to set the voltage at node a equal to 0 V control an electric motor. Select voltages $v_1$ and $v_2$ to achieve $v_a = 0$ V when $v_1$ and $v_2$ are less than 20 V and greater than zero and $R = 2\ \Omega$.

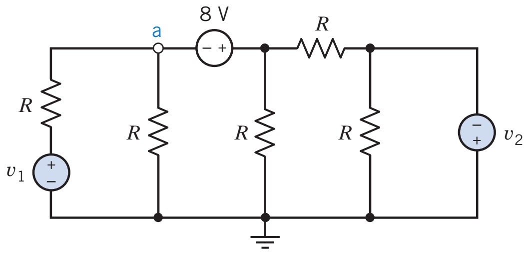



**Figure DP 4-2**

**DP 4-3** A wiring circuit for a special lamp in a home is shown in Figure DP 4-3. The lamp has a resistance of 2 Ω, and the designer selects $R = 100\,\Omega$. The lamp will light when $I \geq 50$ mA but will burn out when $I > 75$ mA.

**(a)** Determine the current in the lamp and whether it will light for $R = 100\,\Omega$.

**(b)** Select $R$ so that the lamp will light but will not burn out if $R$ changes by ±10 percent because of temperature changes in the home.

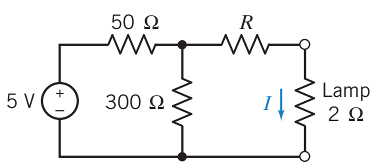



**Figure DP 4-3** A lamp circuit.

**DP 4-4** To control a device using the circuit shown in Figure DP 4-4, it is necessary that $v_{ab} = 10$ V. Select the resistors when it is required that all resistors be greater than 1 Ω and $R_3 + R_4 = 20\,\Omega$.

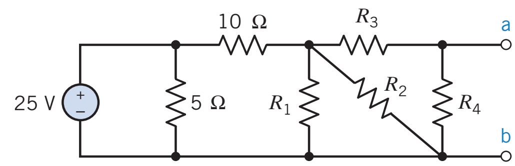



**Figure DP 4-4**

**DP 4-5** The current $i$ shown in the circuit of Figure DP 4-5 is used to measure the stress between two sides of an earth fault line. Voltage $v_1$ is obtained from one side of the fault, and $v_2$ is obtained from the other side of the fault. Select the resistances $R_1$, $R_2$, and $R_3$ so that the magnitude of the current $i$ will remain in the range between 0.5 mA and 2 mA when $v_1$ and $v_2$ may each vary independently between +1 V and +2 V ($1\text{ V} \leq v_n \leq 2\text{ V}$).

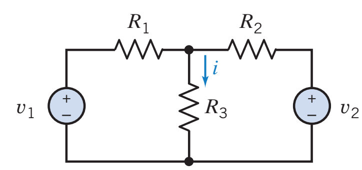



**Figure DP 4-5** A circuit for earth fault-line stress measurement.

# CHAPTER 5 Circuit Theorems

## IN THIS CHAPTER



## 5.1 *Introduction*

In this chapter, we consider five circuit theorems:

- A **source transformation** allows us to replace a voltage source and series resistor by a current source and parallel resistor. Doing so does not change the element current or voltage of any other element of the circuit.
- **Superposition** says that the response of a linear circuit to several inputs working together is equal to the sum of the responses to each of the inputs working separately.
- **Thévenin's theorem** allows us to replace part of a circuit by a voltage source and series resistor. Doing so does not change the element current or voltage of any element in the rest of the circuit.
- **Norton's theorem** allows us to replace part of a circuit by a current source and parallel resistor. Doing so does not change the element current or voltage of any element in the rest of the circuit.
- The **maximum power transfer theorem** describes the condition under which one circuit transfers as much power as possible to another circuit.

Each of these circuit theorems can be thought of as a shortcut, a way to reduce the complexity of an electric circuit so that it can be analyzed more easily. More important, these theorems provide insight into the nature of linear electric circuits.

## 5.2 *Source Transformations*

The ideal voltage source is the simplest model of a voltage source, but occasionally we need a more accurate model. Figure 5.2-1*a* shows a more accurate but more complicated model of a voltage source. The circuit shown in Figure 5.2-1 is sometimes called a nonideal voltage source. (The voltage of a practical voltage source decreases as the voltage source supplies more power. The nonideal voltage source models this behavior, whereas the ideal voltage source does not. The nonideal voltage source is a more accurate model of a practical voltage source than the ideal voltage source, but it is also more complicated. We will usually use ideal voltage sources to model practical voltage sources but will occasionally need to use a nonideal voltage source.) Figure 5.2-1*b* shows a nonideal current source. It is a more accurate but more complicated model of a practical current source.

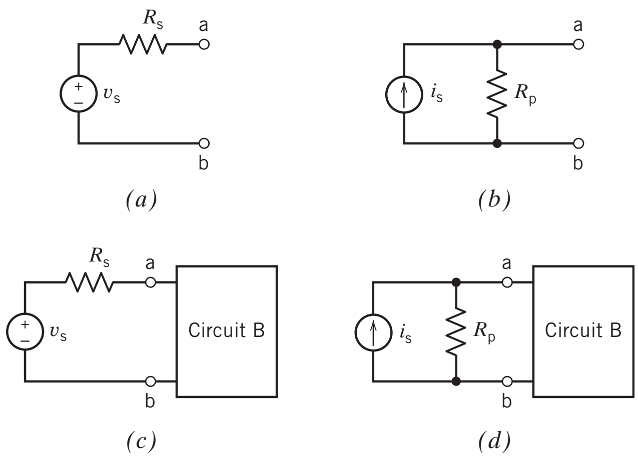



**FIGURE 5.2-1** (*a*) A nonideal voltage source. (*b*) A nonideal current source. (*c*) Circuit B connected to the nonideal voltage source. (*d*) Circuit B connected to the nonideal current source.

Under certain conditions ($R_p = R_s$ and $v_s = R_s i_s$), the nonideal voltage source and the nonideal current source are equivalent to each other. Figure 5.2-1 illustrates the meaning of "equivalent." In Figure 5.2-1*c*, a nonideal voltage source is connected to circuit B. In Figure 5.2-1*d*, a nonideal current source is connected to that same circuit B. Perhaps Figure 5.2-1*d* was obtained from Figure 5.2-1*c*, by replacing the nonideal voltage source with a nonideal current source. Replacing the nonideal voltage source by the *equivalent* nonideal current source does not change the voltage or current of any element in circuit B. That means that if you looked at a list of the values of the currents and voltages of all the circuit elements in circuit B, you could not tell whether circuit B was connected to a nonideal voltage source or to an equivalent nonideal current source. Similarly, we can imagine that Figure 5.2-1*c* was obtained from Figure 5.2-1*d* by replacing the nonideal current source with a nonideal voltage source. Replacing the nonideal current source by the *equivalent* nonideal voltage source does not change the voltage or current of any element in circuit B. The process of transforming Figure 5.2-1*c* into Figure 5.2-1*d*, or vice versa, is called a source transformation.

To see why the source transformation works, we will perform an experiment using the test circuit shown in Figure 5.2-2. This test circuit contains a device called an "operational amplifier." We will learn about operational amplifiers in Chapter 6, so we aren't ready to analyze this circuit yet. Instead, imagine building the circuit and making some measurements to learn how it works.

Consider the following experiment. We connect a resistor having resistance $R$ to the terminals of the test circuit as shown in Figure 5.2-2 and measure the resistor voltage $v$ and resistor current $i$. Next, we change the resistor and measure the new values of the resistor voltage and current. After some trial and error, we collect the following data:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $R$, kΩ | 0 | 1 | 2 | 5 | 10 | 20 | 50 | ∞ |
| $i$, mA | 3 | 2.667 | 2.4 | 1.846 | 1.33 | 0.857 | 0.414 | 0 |
| $v$, V | 0 | 2.667 | 4.8 | 9.231 | 13.33 | 17.143 | 20.69 | 24 |

Two of these data points deserve special attention. The resistor acts like an open circuit when $R = \infty$ so we connect an open circuit across the terminals of the test circuit in this case. As expected, $i = 0$. The resistor voltage is referred to as the "open circuit voltage," denoted as $v_{oc}$. We have measured $v_{oc} = 24$ V. The resistor acts like a short circuit when $R = 0$, so we connect a short circuit across the terminals of the test circuit. As expected, $v = 0$. The resistor current is referred to as the "short-circuit current," denoted as $i_{sc}$. We have measured $i_{sc} = 3$ mA.

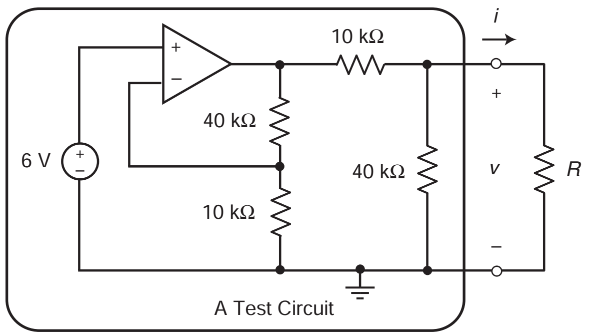



**FIGURE 5.2-2** A test circuit.

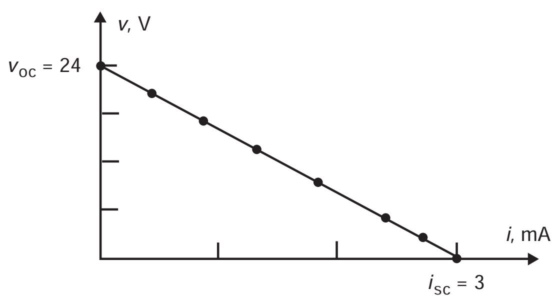



**FIGURE 5.2-3** A plot of the data collected from the test circuit.

Figure 5.2-3 shows a plot of the data. All of the data points lie on the straight line segment that connects the points $(i_{sc},0)$ and $(0,v_{oc})$! The slope of the straight line is

$$slope = -\frac{v_{oc}}{i_{sc}}$$

This slope has units of Ω. It's convenient to define $R_t$ as

$$R_t = \frac{v_{oc}}{i_{sc}} \tag{5.2-1}$$

The equation of the straight line representing our data is

$$v = \left(-\frac{v_{oc}}{i_{sc}}\right) i + v_{oc}$$

or

$$v = -R_t i + v_{oc} \tag{5.2-2}$$

Our experiment has worked quite well. Equation 5.2-2 is a concise description of the test circuit. Now we are ready for a surprise. Consider the circuit shown in Figure 5.2-4

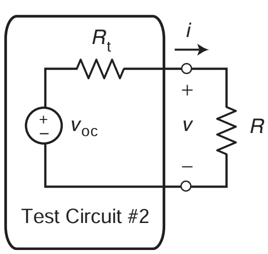



**FIGURE 5.2-4** Thévenin equivalent circuit.

The test circuit in Figure 5.2-4 consists of a voltage source connected in series with a resistor. The voltage of the voltage source in the second test circuit is equal to the open circuit voltage of the first test circuit. Also, the resistance of the resistor in the second test circuit is the parameter $R_t$ from the first test circuit, given by Eq. 5.2-1.

Apply KVL in Figure 5.2-4 to get

$$R_t i + v - v_{oc} = 0 \quad \Rightarrow \quad v = -R_t i + v_{oc} \tag{5.2-3}$$

Eq. 5.2-3 is the same equation as Eq. 5.2-2. The circuits in Figures 5.2-2 and 5.2-4 are both described by the same equation! There's more. Consider the circuit shown in Figure 5.2-5. The test circuit in Figure 5.2-5 consists of a current source connected in parallel with a resistor. The current of the current source in the third test circuit is equal to the short-circuit current of the first test circuit. Also, the resistance of the resistor in the third test circuit is the parameter $R_t$ from the first test circuit, again given by Eq. 5.2-1.

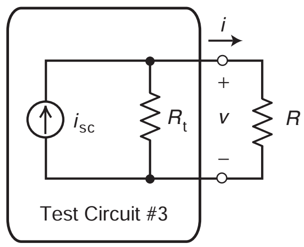



**FIGURE 5.2-5** Norton equivalent circuit.

Apply KCL in Figure 5.2-5 to get

$$i_{sc} = \frac{v}{R_t} + i = 0 \quad \Rightarrow \quad v = -R_t i + R_t i_{sc} \tag{5.2-4}$$

Equations 5.2-2, 5.2-3, and 5.2-4 are identical. The three test circuits are each represented by the equation that describes our data. Any one of them could have generated our data! It is in this sense that we say that the second and third test circuits are equivalent to the first test circuit.

The second and third test circuits have names. They are called the "Thévenin equivalent circuit" and "Norton equivalent circuit" of the first test circuit. Also, the parameter $R_t$ given by Eq. 5.2-1 is called the "Thévenin resistance" of the first test circuit.

The Thévenin and Norton equivalent circuits are equivalent to each other. The source transformation, described earlier in this section and summarized in Figure 5.2-6, may be preformed by replacing a Thévenin equivalent circuit with a Norton equivalent circuit or vice versa.

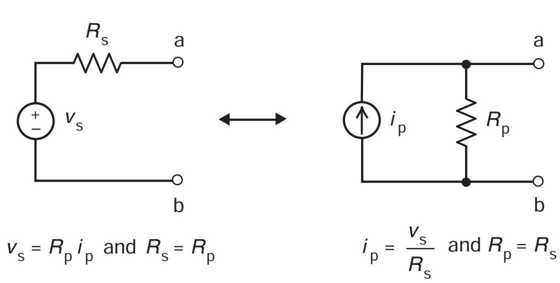
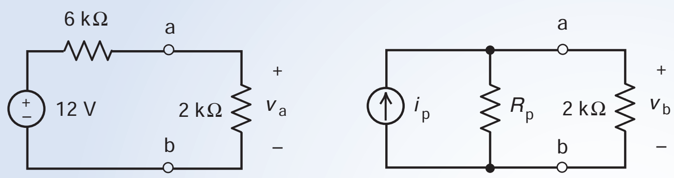



**FIGURE 5.2-6** Source Transformations.

## EXAMPLE 5.2-1 Source Transformations

First, determine the values of $i_p$ and $R_p$ that cause the part of the circuit connected to the 2-kΩ resistor in Figure 5.2-7*b* to be equivalent to part of the circuit connected to the 2-kΩ resistor in Figure 5.2-7*a*. Next, determine the values of $v_a$ and $v_b$.




(*a*) (*b*)

**FIGURE 5.2-7** The circuit considered in Example 5.2-1.

### Solution

We can use a source transformation to determine the required values of $i_p$ and $R_p$. Referring to Figure 5.2-6 we get

$$i_p = \frac{12}{6000} = 0.002 \text{ A} = 2 \text{ mA and } R_p = 6 \text{ k}\Omega$$

Using voltage division in Figure 5.2-7*a*, we calculate

$$v_a = \frac{2000}{2000 + 6000}(12) = 3 \text{ V}$$

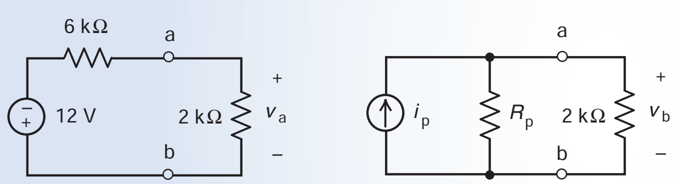
The voltage across the parallel resistors in Figure 5.2-7*b* is given by

$$v_b = \frac{2000 R_p}{2000 + R_p} i_p = \frac{2000(6000)}{2000 + 6000}(0.002) = 1500(0.002) = 3 \text{ V}$$

As expected, the source transformation did not change the value of the voltage across the 2-kΩ resistor.

## EXAMPLE 5.2-2 Source Transformations

First, determine the values of $i_p$ and $R_p$ that cause the part of the circuit connected to the 2-kΩ resistor in Figure 5.2-8*b* to be equivalent to part of the circuit connected to the 2-kΩ resistor in Figure 5.2-8*a*. Next, determine the values of $v_a$ and $v_b$.




(*a*) (*b*)

**FIGURE 5.2-8** The circuit considered in Example 5.2-2.

### Solution

This example is very similar to the previous example. The difference between these examples is the polarity of the voltage source in part (a) of the figures. Reversing both the polarity of voltage source and the sign of the source voltage produces an equivalent circuit. Consequently, we can redraw Figure 5.2-8 as shown in Figure 5.2-9.

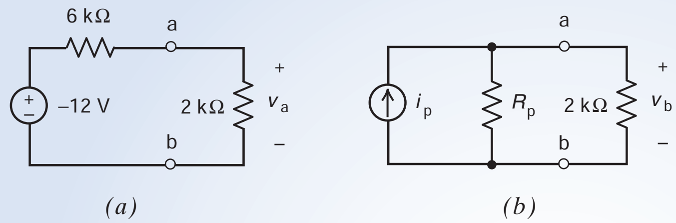



**FIGURE 5.2-9** The circuit from Figure 5.2-8 after changing the polarity of the voltage source.

Now we are ready use a source transformation to determine the required values of $i_p$ and $R_p$. Comparing Figure 5.2-9 to Figure 5.2-6, we write

$$i_p = \frac{-12}{6000} = -0.002 \text{ A} = -2 \text{ mA and } R_p = 6 \text{ k}\Omega$$

Using voltage division in Figure 5.2-9*a*, we calculate

$$v_a = -\frac{2000}{2000 + 6000}(12) = -3 \text{ V}$$

The voltage across the parallel resistors in Figure 5.2-9*b* is given by

$$v_b = \frac{2000\,R_p}{2000 + R_p}\,i_p = \frac{2000(6000)}{2000 + 6000}(-0.002) = 1500(-0.002) = -3 \text{ V}$$

As before, the source transformation did not change the value of the voltage across the 2-kΩ resistor.

## EXAMPLE 5.2-3 Application of Source Transformations

Use a source transformation to determine a relationship between the resistance $R$ and the resistor current $i$ in Figure 5.2-10.

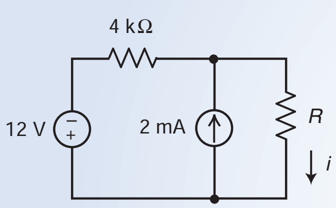



**FIGURE 5.2-10** The circuit considered in Example 5.2-3.

## Solution

We can use a source transformation to replace the 12-volt source in series with the 4-kΩ resistor by the parallel combination of a current source and resistor. The resulting circuit is shown in Figure 5.2-11.

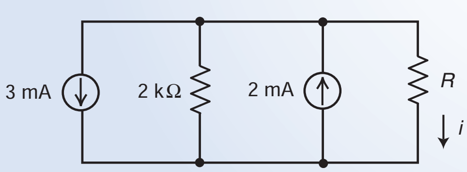



**FIGURE 5.2-11** The circuit from Figure 5.2-10 after a source transformation.

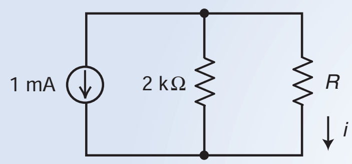



**FIGURE E 5.2-12** The circuit from Figure 5.2-11 replacing parallel current sources by an equivalent current source.

Now we will replace the parallel current sources by an equivalent current source. The resulting circuit is shown Figure 5.2-12. Using current division in Figure 5.2-12 gives

$$i = \frac{2000}{2000 + R}(0.001) = \frac{2}{2000 + R} \tag{5.2-5}$$

The source transformation did not change the value of the current in resistor $R$ and neither did replacing parallel current sources by an equivalent current source. The relationship between resistance $R$ and the resistor current $i$ is the same in Figure 5.2-10 as it is in Figure 5.2-12. Consequently, Equation 5.2-5 describes the relationship between resistance $R$ and the resistor current $i$ in Figure 5.2-11.

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***EXERCISE 5.2-1*** Determine values of $R$ and $i_s$ so that the circuits shown in Figures E 5.2-1$a$,$b$ are equivalent to each other due to a source transformation.

***Answer:*** $R = 10\ \Omega$ and $i_s = 1.2$ A

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***EXERCISE 5.2-2*** Determine values of $R$ and $i_s$ so that the circuits shown in Figures E 5.2-2$a$,$b$ are equivalent to each other due to a source transformation.

***Hint:*** Notice that the polarity of the voltage source in Figure E 5.2-2$a$ is not the same as in Figure E 5.2-1$a$.

***Answer:*** $R = 10\ \Omega$ and $i_s = -1.2$ A

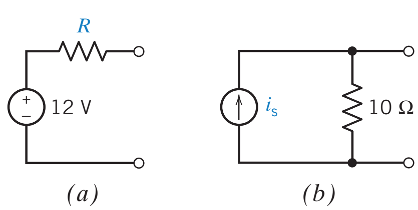


**FIGURE E 5.2-1**

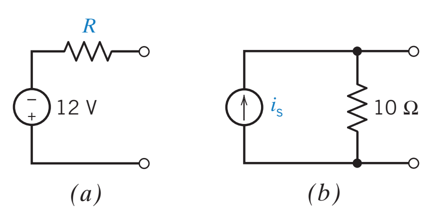


**FIGURE E 5.2-2**

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***EXERCISE 5.2-3*** Determine values of $R$ and $v_s$ so that the circuits shown in Figures E 5.2-3$a$,$b$ are equivalent to each other due to a source transformation.

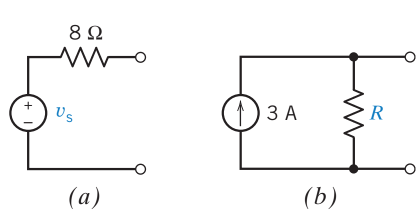


**FIGURE E 5.2-3**

***Answer:*** $R = 8\ \Omega$ and $v_s = 24$ V

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**EXERCISE 5.2-4** Determine values of $R$ and $v_s$ so that the circuits shown in Figures E 5.2-4*a*,*b* are equivalent to each other due to a source transformation.

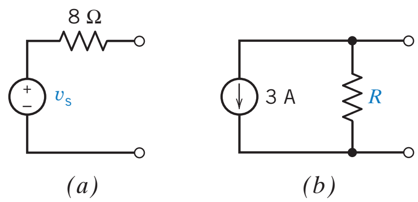


**FIGURE E 5.2-4**

***Hint:*** Notice that the reference direction of the current source in Figure E 5.2-4*b* is not the same as in Figure E 5.2-3*b*.

***Answer:*** $R = 8\ \Omega$ and $v_s = -24$ V

## 5.3 Superposition

The output of a linear circuit can be expressed as a linear combination of its inputs. For example, consider any circuit having the following three properties:

1. The circuit consists entirely of resistors and dependent and independent sources.
2. The circuit inputs are the voltages of all the independent voltage sources and the currents of all the independent current sources.
3. The output is the voltage or current of any element of the circuit.

Such a circuit is a linear circuit. Consequently, the circuit output can be expressed as a linear combination of the circuit inputs. For example,

$$v_o = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n \tag{5.3-1}$$

where $v_0$ is the output of the circuit (it could be a current instead of a voltage) and $v_1, v_2, \ldots, v_n$ are the inputs to the circuit (any or all the inputs could be currents instead of voltages). The coefficients $a_1, a_2, \ldots, a_n$ of the linear combination are real constants called gains.

Next, consider what would happen if we set all but one input to zero. Let $v_{oi}$ denote output when all inputs except the $i$th input have been set to zero. For example, suppose we set $v_2, v_3, \ldots, v_n$ to zero. Then

$$v_{o1} = a_1 v_1 \tag{5.3-2}$$

We can interpret $v_{o1} = a_1 v_1$ as the circuit output due to input $v_1$ acting separately. In contrast, the $v_o$ in Eq 5.3-1 is the circuit output due to all the inputs working together. We now have the following important interpretation of Eq. 5.3-1:

> The output of a linear circuit due to several inputs working together is equal to the sum of the outputs due to each input working separately.

The inputs to our circuit are voltages of independent voltage sources and the currents of independent current sources. When we set all but one input to zero, the other inputs become 0-V voltage sources and 0-A current sources. Because 0-V voltage sources are equivalent to short circuits and 0-A current sources are equivalent to open circuits, we replace the sources corresponding to the other inputs by short or open circuits.

Equation 5.3-2 suggests a method for determining the values of the coefficients $a_1, a_2, \ldots, a_n$ of the linear combination. For example, to determine $a_1$, set $v_2, v_3, \ldots, v_n$ to zero. Then, dividing both sides of Eq. 5.5-2 by $v_1$, we get

$$a_1 = \frac{v_{o1}}{v_1}$$

The other gains are determined similarly.

## EXAMPLE 5.3-1 Superposition

The circuit shown in Figure 5.3-1 has one output, $v_o$, and three inputs, $v_1$, $i_2$, and $v_3$. (As expected, the inputs are voltages of independent voltage sources and the currents of independent current sources.) Express the output as a linear combination of the inputs.

### Solution

Let's analyze the circuit using node equations. Label the node voltage at the top node of the current source and identify the supernode corresponding to the horizontal voltage source as shown in Figure 5.3-2.
Apply KCL to the supernode to get

$$\frac{v_1 - (v_3 + v_o)}{40} + i_2 = \frac{v_o}{10}$$

Multiply both sides of this equation by 40 to eliminate the fractions. Then we have

$$v_1 - (v_3 + v_o) + 40i_2 = 4v_o \quad \Rightarrow \quad v_1 + 40i_2 - v_3 = 5v_o$$

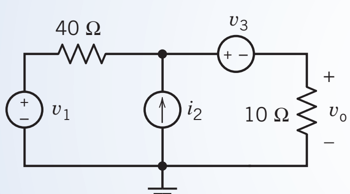


**FIGURE 5.3-1** The linear circuit for Example 5.3-1.

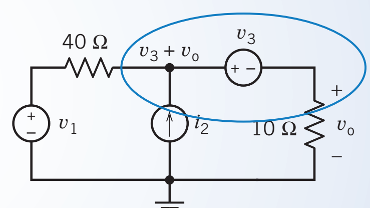


**FIGURE 5.3-2** A supernode.

Dividing both sides by 5 expresses the output as a linear combination of the inputs:

$$v_o = \frac{v_1}{5} + 8i_2 - \frac{v_3}{5}$$

Also, the coefficients of the linear combination can now be determined to be

$$a_1 = \frac{v_{o1}}{v_1} = \frac{1}{5}\text{V/V}, a_2 = \frac{v_{o2}}{i_2} = 8\text{V/A, and } a_3 = \frac{v_{o3}}{v_3} = -\frac{1}{5}\text{V/V}$$

**Alternate Solution**

Figure 5.3-3 shows the circuit from Figure 5.3-1 when $i_2 = 0$ A and $v_3 = 0$ V. (A zero current source is equivalent to an open circuit, and a zero voltage source is equivalent to a short circuit.)

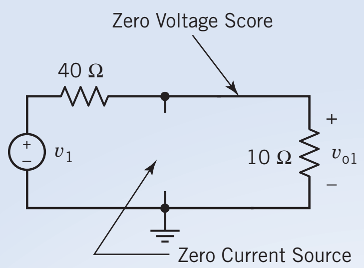


**FIGURE 5.3-3** Output due to the first input.

Using voltage division

$$v_{o1} = \frac{10}{40+10} v_1 = \frac{1}{5} v_1$$

In other words,

$$a_1 = \frac{v_{o1}}{v_1} = \frac{1}{5}\ \text{V/V}$$

Next, Figure 5.3-4 shows the circuit when $v_1 = 0$ V and $v_3 = 0$ V. The resistors are connected in parallel. Applying Ohm's law to the equivalent resistance gives

$$v_{o2} = \frac{40 \times 10}{40+10} i_2 = 8 i_2$$

In other words,

$$a_2 = \frac{v_{o2}}{i_2} = 8\ \text{V/A}$$

Finally, Figure 5.3-5 shows the circuit when $v_1 = 0$ V and $i_2 = 0$ A. Using voltage division,

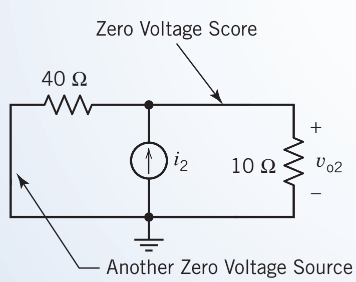


**FIGURE 5.3-4** Output due to the second input.

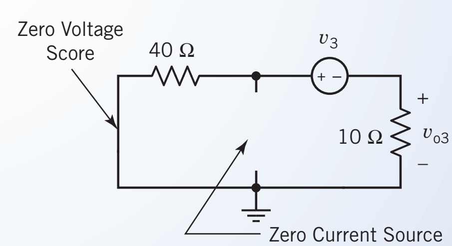


**FIGURE 5.3-5** Output due to the third input.

$$v_{o3} = \frac{10}{40+10}(-v_3) = -\frac{1}{5} v_3$$

In other words,

$$a_3 = \frac{v_{o3}}{v_3} = -\frac{1}{5}\text{V/V}$$

Now the output can be expressed as a linear combination of the inputs

$$v_o = a_1 v_1 + a_2 i_2 + a_3 v_3 = \frac{1}{5} v_1 + 8 i_2 + \left(-\frac{1}{5}\right) v_3$$

as before.

**EXAMPLE 5.3-2 Superposition**

Find the current $i$ for the circuit of Figure 5.3-6*a*.

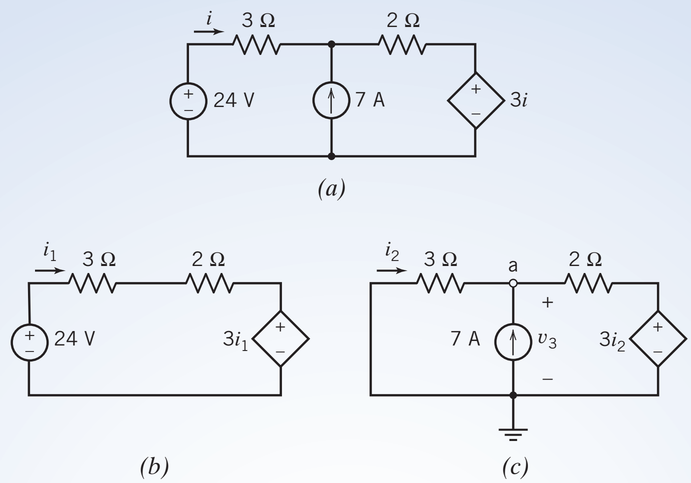


**FIGURE 5.3-6** (*a*) The circuit for Example 5.3-2. (*b*) The independent voltage source acting alone. (*c*) The independent current source acting alone.

## Solution

Independent sources provide the inputs to a circuit. The circuit in Figure 5.3-6*a* has two inputs: the voltage of the independent voltage source and the current of the independent current source. The current, $i$, caused by the two sources acting together is equal to the sum of the currents caused by each independent source acting separately.

**Step 1:** Figure 5.3-6*b* shows the circuit used to calculate the current caused by the independent voltage source acting alone. The current source current is set to zero for this calculation. (A zero current source is equivalent to an open circuit, so the current source has been replaced by an open circuit.) The current due to the voltage source alone has been labeled as $i_1$ in Figure 5.3-6*b*.

Apply Kirchhoff's voltage law to the loop in Figure 5.3-6*b* to get

$$-24 + (3 + 2)i_1 + 3i_1 + 0 \quad \Rightarrow \quad i_1 = 3\text{ A}$$

(Notice that we did not set the dependent source to zero. The inputs to a circuit are provided by the independent sources, not by the dependent sources. When we find the response to one input acting alone, we set the other inputs to zero. Hence, we set the other independent sources to zero, but there is no reason to set the dependent source to zero.)

**Step 2:** Figure 5.3-6*c* shows the circuit used to calculate the current caused by the current source acting alone. The voltage of the independent voltage is set to zero for this calculation. (A zero voltage source is equivalent to a short circuit, so the independent voltage source has been replaced by a short circuit.) The current due to the voltage source alone has been labeled as $i_2$ in Figure 5.3-6*c*.

First, express the controlling current of the dependent source in terms of the node voltage, $v_a$, using Ohm's law:

$$i_2 = -\frac{v_a}{3} \quad \Rightarrow \quad v_a = -3i_2$$

Next, apply Kirchhoff's current law at node a to get

$$i_2 + 7 = \frac{v_a - 3i_2}{2} \quad \Rightarrow \quad i_2 + 7 = \frac{-3i_2 - 3i_2}{2} \quad \Rightarrow \quad i_2 = -\frac{7}{4}\ \text{A}$$

**Step 3:** The current, $i$, caused by the two independent sources acting together is equal to the sum of the currents, $i_1$ and $i_2$, caused by each source acting separately:

$$i = i_1 + i_2 = 3 - \frac{7}{4} = \frac{5}{4}\ \text{A}$$

## 5.4 *Thévenin's Theorem*

In this section, we introduce the Thévenin equivalent circuit, based on a theorem developed by M. L. Thévenin, a French engineer, who first published the principle in 1883. Thévenin, who is credited with the theorem, probably based his work on earlier work by Hermann von Helmholtz (see Figure 5.4-1).

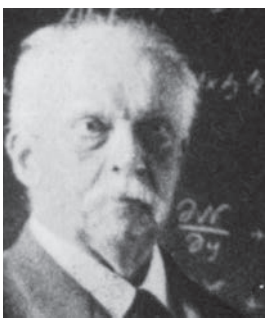

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**FIGURE 5.4-1** Hermann von Helmholtz (1821–1894), who is often credited with the basic work leading to Thévenin's theorem.

Figure 5.4-2 illustrates the use of the Thévenin equivalent circuit. In Figure 5.4-2*a*, a circuit is partitioned into two parts—circuit A and circuit B—that are connected at a single pair of terminals. (This is the only connection between circuits A and B. In particular, if the overall circuit contains a dependent source, then either both parts of that dependent source must be in circuit A or both parts must be in circuit B.) In Figure 5.4-2*b*, circuit A is replaced by its Thévenin equivalent circuit, which consists of an ideal voltage source in series with a resistor. Replacing circuit A by its Thévenin equivalent circuit does not change the voltage or current of any element in circuit B. This means that if you looked at a list of the values of the currents and voltages of all the circuit elements in circuit B, you could not tell whether circuit B was connected to circuit A or connected to its Thévenin equivalent circuit.

Finding the Thévenin equivalent circuit of circuit A involves three parameters: the open-circuit voltage, $v_{oc}$, the short-circuit current, $i_{sc}$, and the Thévenin resistance, $R_t$. Figure 5.4-3 illustrates the meaning of these three parameters. In Figure 5.4-3*a*, an open circuit is connected across the terminals of circuit A. The voltage across that open circuit is the open-circuit voltage, $v_{oc}$. In Figure 5.4-3*b*, a short circuit is connected across the terminals of circuit A. The current in that short circuit is the short-circuit current, $i_{sc}$.

Figure 5.4-3*c* indicates that the Thévenin resistance, $R_t$, is the equivalent resistance of circuit A*. Circuit A* is formed from circuit A by replacing all the *independent* voltage sources by short circuits and replacing all the *independent* current sources by open circuits. (*Dependent* current and voltage sources are not replaced with open circuits or short circuits.) Frequently, the Thévenin resistance, $R_t$, can be determined by repeatedly replacing series or parallel resistors by equivalent resistors. Sometimes, a more formal method is required. Figure 5.4-4 illustrates a formal method for determining the value of the Thévenin resistance. A current source having current $i_t$ is connected across the terminals of circuit A*. The voltage, $v_t$, across the current source is calculated or measured. The Thévenin

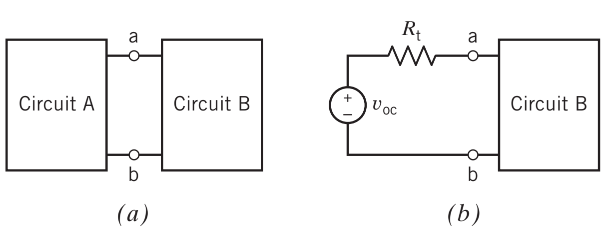


**FIGURE 5.4-2** (*a*) A circuit partitioned into two parts: circuit A and circuit B. (*b*) Replacing circuit A by its Thévenin equivalent circuit.

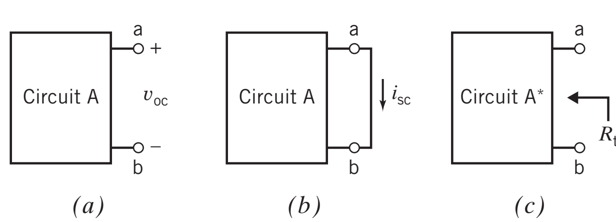


**FIGURE 5.4-3** The Thévenin equivalent circuit involves three parameters: (*a*) the open-circuit voltage, $v_{oc}$, (*b*) the short-circuit current, $i_{sc}$, and (*c*) the Thévenin resistance, $R_t$.

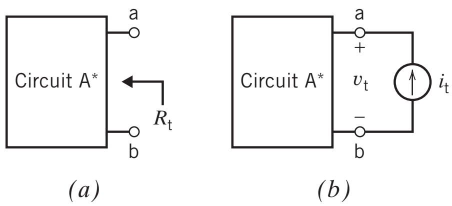


**FIGURE 5.4-4** (*a*) The Thévenin resistance, $R_t$, and (*b*) a method for measuring or calculating the Thévenin resistance, $R_t$.

resistance is determined from the values of $i_t$ and $v_t$, using

$$R_t = \frac{v_t}{i_t} \tag{5.4-1}$$

The open-circuit voltage, $v_{oc}$, the short-circuit current, $i_{sc}$, and the Thévenin resistance, $R_t$, are related by the equation

$$v_{oc} = R_t i_{sc} \tag{5.4-2}$$

Consequently, the Thévenin resistance can be calculated from the open-circuit voltage and the short-circuit current.

In summary, the Thévenin equivalent circuit for circuit A consists of an ideal voltage source, having voltage $v_{oc}$, in series with a resistor, having resistance $R_t$. Replacing circuit A by its Thévenin equivalent circuit does not change the voltage or current of any element in circuit B.

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## EXAMPLE 5.4-1 Thévenin Equivalent Circuit

Determine the Thévenin equivalent circuit for the circuit shown in Figure 5.4-5.

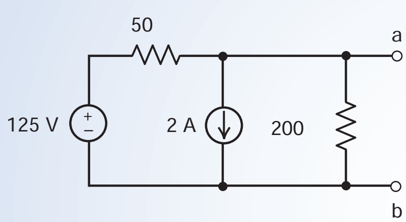


**FIGURE 5.4-5** The circuit considered in Example 5.4-1.

### First Solution

Referring to Figure 5.4-2, we see that we can draw a Thévenin equivalent circuit once we have found the open-circuit voltage $v_{oc}$ and Thévenin resistance, $R_t$. Figure 5.4-3 shows how to determine the open-circuit voltage, the Thévenin resistance, and also the short-circuit current $i_{sc}$. After we determine the values of $v_{oc}$, $R_t$, and $i_{sc}$ we will use Eq. 5.4-2 to check that our values are correct.

To determine the open-circuit voltage of the circuit shown in Figure 5.4-5, we connect an open circuit between terminals a and b as shown in Figure 5.4-6*a*. As the name suggests, the voltage across that open circuit is the open-circuit voltage, $v_{oc}$. After taking node b in Figure 5.4-6*a* to be the reference node, we see that the node voltage at node a is equal to $v_{oc}$. Applying KCL at node a, we obtain the node equation

$$\frac{125 - v_{oc}}{50} = 2 + \frac{v_{oc}}{200}$$

Solving for $v_{oc}$ gives $$v_{oc} = 20 \text{ V}$$

To determine the short-circuit current of the circuit shown in Figure 5.4-5, we connect a short circuit between terminals a and b as shown in Figure 5.4-6*b*. The current in that short circuit is $i_{sc}$. Due to the short circuit, the voltage across the 200-Ω resistor is 0 V. By Ohm's law, the current in the 200-Ω resistor is 0 A as shown in Figure 5.4-6*b*. Applying KVL to the loop consisting of the voltage source, 50-Ω resistor, and short circuit, we see that the voltage across the 50-Ω resistor is 125 V, also as shown in Figure 5.4-6*b*. Finally, apply KCL at node a in Figure 5.4-6*b* to get

$$\frac{125}{50} = 2 + 0 + i_{sc}$$

Solving for $i_{sc}$ gives $$i_{sc} = 0.5 \text{ A}$$

To determine the Thévenin resistance of the circuit shown in Figure 5.4-5, we set the voltage of the independent voltage source to zero and the current of the independent current source to zero. (Recall that a zero-volt voltage source is equivalent to a short circuit and a zero-amp current source is equivalent to an open circuit.) $R_t$ is the equivalent resistance connected to terminals a-b as shown in Figure 5.4-6*c*.

$$R_t = 50||200 = \frac{50(200)}{50 + 200} = 40 \ \Omega$$

Our values of $v_{oc}$, $R_t$, and $i_{sc}$ satisfy Eq. 5.4-2, so we're confident that they are correct. Finally, the Thévenin equivalent circuit is shown in Figure 5.4-6*d*.

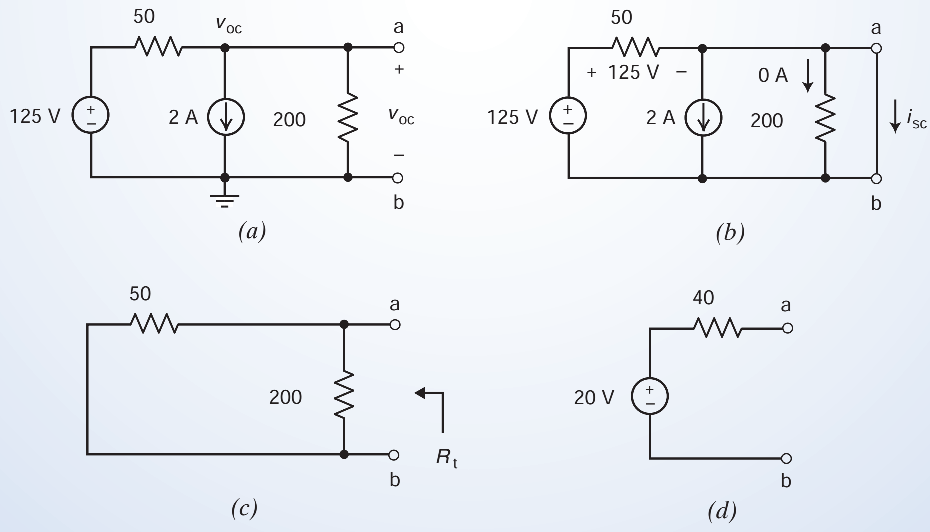


**FIGURE 5.4-6** Determining the (*a*) open-circuit voltage, (*b*) short-circuit current, and (*c*) Thévenin resistance of the circuit in Figure 5.4-5. (*d*) The Thévenin equivalent of the circuit in Figure 5.4-5.

Notice the important role of the terminals a-b in this problem. Those terminals are used to identify $v_{oc}$ in Figure 5.4-6*a*, $i_{sc}$ in Figure 5.4-6*b*, and $R_t$ in Figure 5.4-6*c*. Importantly, the Thévenin equivalent circuit in Figure 5.4-6*d* is connected to the same pair of terminals as the original circuit in Figure 5.4-5. Finally, notice that the orientation of $v_{oc}$ is the same, + near terminal a, in Figures 5.4-6*a* and *d*.

## Second Solution

Often we can simplify a circuit using source transformations and equivalent circuits. In this solution we will transform a circuit into an equivalent circuit repeatedly. We will start at the left side of the circuit in Figure 5.4-5, away from terminals a-b. If it is possible to continue these transformations until the equivalent circuit consists of the series connection of a voltage source and a resistor, connected between terminals a-b, then that series circuit is the Thévenin equivalent circuit. Figure 5.4-7 illustrates this procedure.

The circuit in Figure 5.4-6 contains a voltage source connected in series with a 50-Ω resistor. Using a source transformation, these circuit elements are replaced by the parallel connection of a 2.5-A current source and 50-Ω resistor in Figure 5.4-7*a*. The circuit in Figure 5.4-7*a* contains both parallel current sources and parallel resistors. In Figure 5.4-7*b* the parallel current sources are replaced by an equivalent current source and the parallel resistors are replaced by an equivalent resistor. A final source transformation converts the parallel connection of a current source and resistor in Figure 5.4-7*b* to the series connection of a voltage source and resistor in Figure 5.4-7*c*. We recognize Figure 5.4-7*c* as a Thévenin circuit that is equivalent to the circuit shown in Figure 5.4-5 and conclude that Figure 5.4-7*c* is the Thévenin equivalent of the circuit shown in Figure 5.4-5.

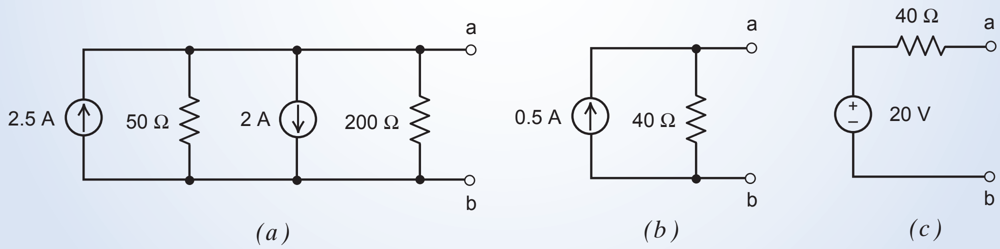


**FIGURE 5.4-7** Using source transformations and equivalent circuits to determine the Thévenin equivalent circuit of the circuit shown in Figure 5.4-5.

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## EXAMPLE 5.4-2 Thévenin Equivalent Circuit of a Circuit Containing a Dependent Source

Determine the Thévenin equivalent circuit for the circuit shown in Figure 5.4-8.

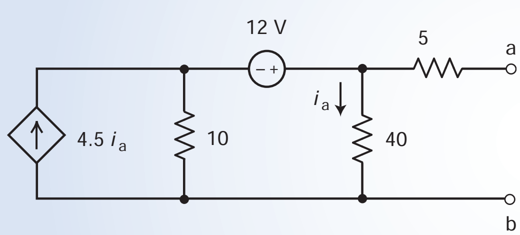


**FIGURE 5.4-8** The circuit considered in Example 5.4-2.

## Solution

We will determine the values of $v_{oc}$, $R_t$, and $i_{sc}$ and use Eq. 5.4-2 to check that our values are correct.

To determine the open-circuit voltage of the circuit shown in Figure 5.4-8, we connect an open circuit between terminals a and b and label the voltage across that open circuit as $v_{oc}$. Figure 5.4-9 shows the resulting circuit after using KCL to label the element currents.

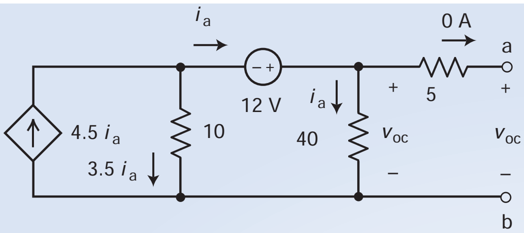


**FIGURE 5.4-9** The circuit used to find the open-circuit voltage.

The open circuit causes the current in the 5-Ω resistor to be zero. The voltage across that resistor is also zero, so the voltage across the 40-Ω resistor is $v_{oc}$ as labeled in Figure 5.4-9.

Using Ohm's law $$i_a = \frac{v_{oc}}{40}$$

Applying KVL to the loop consisting the 12-V source, 10-Ω resistor, and 40-Ω resistor gives

$$0 = -12 + v_{oc} - 10(3.5 i_a)$$

Solving these equations for $v_{oc}$ gives $$v_{oc} = 96 \text{ V}$$

To determine the short-circuit current of the circuit shown in Figure 5.4-8, we connect a short circuit between terminals a and b and label the current across that short circuit as $i_{sc}$. Figure 5.4-10 shows the resulting circuit after using KCL to label the element currents.

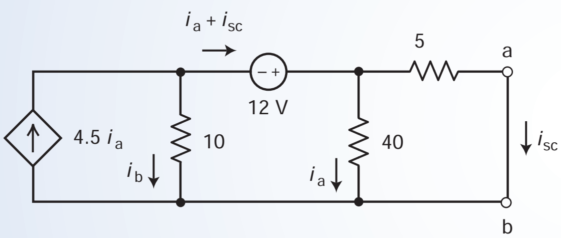


**FIGURE 5.4-10** The circuit used to find the short-circuit current.

Applying KVL to the loop consisting of the 5-Ω and 40-Ω resistors gives

$$5 i_{sc} - 40 i_a = 0 \quad \Rightarrow \quad i_a = \frac{i_{sc}}{8}$$

Apply KCL at the top node of the 10-Ω resistor to write

$$4.5 i_a = i_b + (i_a + i_{sc}) \quad \Rightarrow \quad i_b = 3.5 i_a - i_{sc} = -\frac{9}{16} i_{sc}$$

Apply KVL to the loop consisting of the voltage source and the 5-Ω and 10-Ω resistors to write

$$-12 + 5 i_{sc} - 10\left(-\frac{9}{16} i_{sc}\right) = 0$$

Solving this equation for $i_{sc}$ gives $$i_{sc} = \frac{12}{5 + \frac{90}{16}} = 1.1294 \text{ A}$$

Referring to Figure 5.4-4, we'll determine the Thévenin resistance of the circuit by replacing the independent voltage source by a short circuit and connecting a current source to terminal a-b as shown in Figure 5.4-11. (Circuit A* in Figure 5.4-4 is obtained from Circuit A by replacing the independent voltage sources by short circuits and the independent current sources by open circuits.)

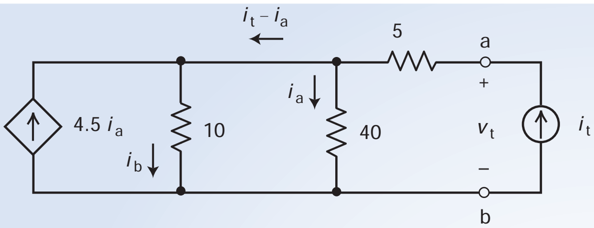


**FIGURE 5.4-11** The circuit used to find the Thévenin resistance.

Apply KCL at the top node of the 10-Ω resistor to write

$$4.5\,i_a + (i_t - i_a) = i_b \quad \Rightarrow \quad i_b = 3.5\,i_a + i_t$$

Applying KVL to the loop consisting of the 10-Ω and 40-Ω resistors gives

$$40\,i_a = 10\,i_b = 10\,(3.5\,i_a + i_t) \quad \Rightarrow \quad i_a = 2\,i_t$$

Applying KVL to the loop consisting of the independent current source and the 10-Ω and 5-Ω resistors gives

$$v_t = 5\,i_t + 10\,i_b = 5\,i_t + 10(3.5\,i_a + i_t) = 15\,i_t + 35\,i_a = 15\,i_t + 35\,(2\,i_t) = 85\,i_t$$

The Thévenin resistance is

$$R_t = \frac{v_t}{i_t} = 85\ \Omega$$

Our values of $v_{oc}$, $R_t$, and $i_{sc}$ satisfy Eq. 5.4-2, so we're confident that they are correct. Finally, the Thévenin equivalent circuit is shown in Figure 5.4-12.

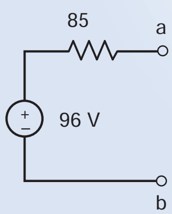


**FIGURE 5.4-12** The Thévenin equivalent circuit for the circuit shown in Figure 5.4-8.

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## EXAMPLE 5.4-3 An Application of the Thévenin Equivalent Circuit

Consider the circuit shown in Figure 5.4-13.

**(a)** Determine the current, $i$, when $R = 2\ \Omega$.
**(b)** Determine the value of the resistance $R$ required to cause $i = 5$ A.
**(c)** Determine the value of the resistance $R$ required to cause $i = 8$ A.

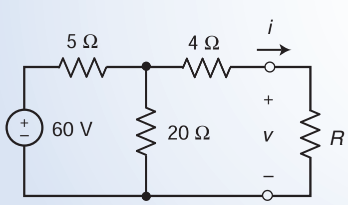


**FIGURE 5.4-13** The circuit considered in Example 5.4-3.

## Solution

The circuit shown in Figure 5.4-13 is an example of the situation shown in Figure 5.4-2*a* in which Circuit B is the resistor $R$ and Circuit A is the part of the circuit shown in Figure 5.4-13 that is connected to resistor $R$. Replacing the part of the circuit that is connected to resistor $R$ by its Thévenin equivalent circuit will not change the value of the current in resistor $R$.

In Figure 5.4-14 source transformations and equivalent resistances are used to determine the Thévenin equivalent of the part of the circuit that is connected to resistor $R$. That equivalent circuit is shown in Figure 5.4-14*e*. In Figure 5.4-15 the part of the circuit that is connected to resistor $R$ has been replaced by its Thévenin equivalent circuit. We readily determine that

$$i = \frac{48}{8 + R} \tag{5.4-3}$$

in Figure 5.4-15. Replacing the part of the circuit that is connected to resistor $R$ by its Thévenin equivalent circuit did not change the current in resistor $R$. Consequently, Eq. 5.4-3 also describes the relationship between $i$ and $R$ in Figure 5.4-13. We can now easily answer questions (a), (b) and (c).

**(a)** When $R = 2\ \Omega$ the resistor current is $i = \frac{48}{8+2} = 4.8$ A.

**(b)** To cause $i = 5$ A requires $R = \frac{48}{i} - 8 = \frac{48}{5} - 8 = 1.6\ \Omega$.

**(c)** To cause $i = 8$ A requires $R = \frac{48}{i} - 8 = \frac{48}{8} - 8 = -2\ \Omega$.

The answer in part (c) is probably not acceptable because we expect $0 < R < \infty$. Using Eq. 5.4-3 shows that when $0 < R < \infty$ the circuit in Figure 5.4-13 can only produce currents in the range $0 < i < 6$ A. The current specified in (c) is outside of this range and cannot be obtained using a positive resistance $R$.

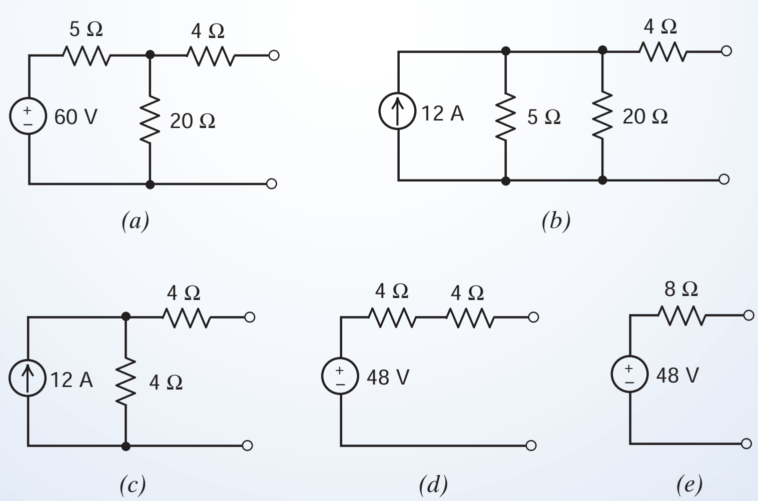


**FIGURE 5.4-14** Determining the Thévenin equivalent circuit using source transformations and equivalent resistance.

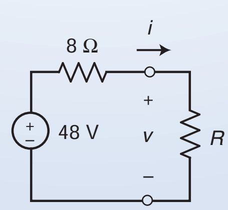


**FIGURE 5.4-15** The circuit obtained by replacing part of the circuit in Figure 5.4-13 by its Thévenin equivalent circuit.

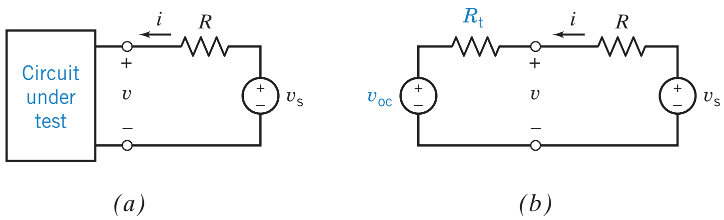


**FIGURE 5.4-16** (*a*) Circuit under test with laboratory source $v_s$ and resistor $R$. (*b*) Circuit of (*a*) with Thévenin equivalent circuit replacing the test circuit.

A laboratory procedure for determining the Thévenin equivalent of a black box circuit (see Figure 5.4-16*a*) is to measure $i$ and $v$ for two or more values of $v_s$ and a fixed value of $R$. For the circuit of Figure 5.4-16*b*, we replace the test circuit with its Thévenin equivalent, obtaining

$$v = v_{oc} + iR_t \tag{5.4-4}$$

The procedure is to measure $v$ and $i$ for a fixed $R$ and several values of $v_s$. For example, let $R = 10\ \Omega$ and consider the two measurement results

$$(1) \quad v_s = 49\text{ V}: \ i = 0.5\text{ A}, \ v = 44\text{ V}$$

and

$$(2) \quad v_s = 76\text{ V}: \ i = 2\text{ A}, \ v = 56\text{ V}$$

Then we have two simultaneous equations (using Eq. 5.4-4):

$$\begin{aligned} 44 &= v_{oc} + 0.5R_t \\ 56 &= v_{oc} + 2R_t \end{aligned}$$

Solving these simultaneous equations, we get $R_t = 8\ \Omega$ and $v_{oc} = 40$ V, thus obtaining the Thévenin equivalent of the black box circuit.

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**EXERCISE 5.4-1** Determine values of $R_t$ and $v_{oc}$ that cause the circuit shown in Figure E 5.4-1*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-1*a*.

***Answer:*** $R_t = 8\ \Omega$ and $v_{oc} = 2$ V

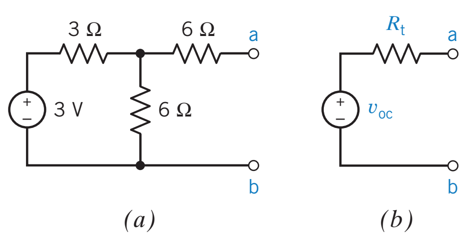


**FIGURE E 5.4-1**

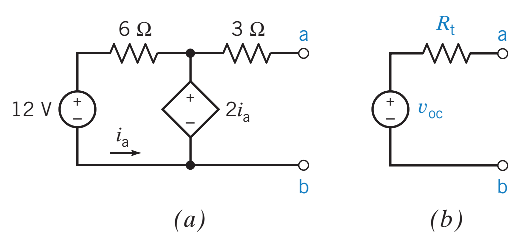


**FIGURE E 5.4-2**

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**EXERCISE 5.4-2** Determine values of $R_t$ and $v_{oc}$ that cause the circuit shown in Figure E 5.4-2*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-2*a*.

***Answer:*** $R_t = 3\ \Omega$ and $v_{oc} = -6$ V

## 5.5 Norton's Equivalent Circuit

An American engineer, E. L. Norton at Bell Telephone Laboratories, proposed an equivalent circuit for circuit A of Figure 5.4-2, using a current source and an equivalent resistance. The Norton equivalent circuit is related to the Thévenin equivalent circuit by a source transformation. In other words, a source transformation converts a Thévenin equivalent circuit into a Norton equivalent circuit or vice versa. Norton published his method in 1926, 43 years after Thévenin.

*Norton's theorem* may be stated as follows: Given any linear circuit, divide it into two circuits, A and B. If either A or B contains a dependent source, its controlling variable must be in the same circuit. Consider circuit A and determine its short-circuit current $i_{sc}$ at its terminals. Then the equivalent circuit of A is a current source $i_{sc}$ in parallel with a resistance $R_n$, where $R_n$ is the resistance looking into circuit A with all its independent sources deactivated.

We therefore have the Norton circuit for circuit A as shown in Figure 5.5-1. Finding the Thévenin equivalent circuit of the circuit in Figure 5.5-1 shows that $R_n = R_t$ and $v_{oc} = R_t i_{sc}$. The Norton equivalent is simply the source transformation of the Thévenin equivalent.

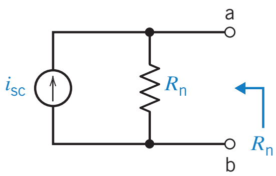


**FIGURE 5.5-1** Norton equivalent circuit for a linear circuit A.

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## EXAMPLE 5.5-1 Norton Equivalent Circuit

Determine the Norton equivalent circuit for the circuit shown in Figure 5.5-2.

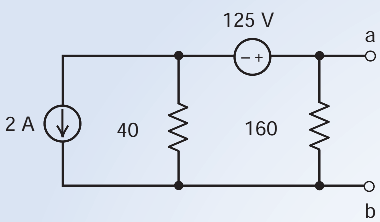


**FIGURE 5.5-2** The circuit considered in Example 5.5-1.

### Solution

In Figure 5.5-3, source transformations and equivalent circuits are used to simplify the circuit in Figure 5.5-2. These simplifications continue until the simplified circuit in Figure 5.5-3*d* consists of a single current source in parallel with a single resistor. The circuit in Figure 5.5-3*d* is the Norton equivalent circuit of the circuit in Figure 5.5-2. Consequently

$$i_{sc} = 1.125 \text{ A} \quad \text{and} \quad R_t = R_n = 32\ \Omega$$

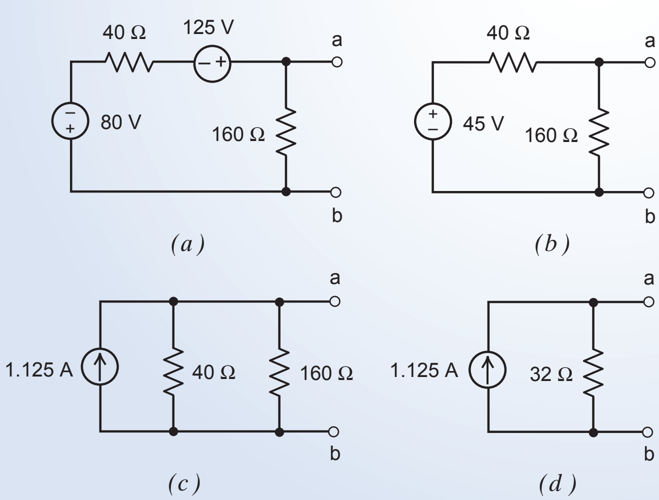


**FIGURE 5.5-3** Using source transformations and equivalent circuits to determine the Norton equivalent circuit of the circuit shown in Figure 5.5-2.

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## EXAMPLE 5.5-2 Norton Equivalent Circuit of a Circuit Containing a Dependent Source

Determine the Norton equivalent circuit for the circuit shown in Figure 5.5-4.

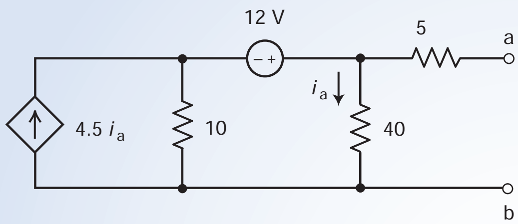


**FIGURE 5.5-4** The circuit considered in Example 5.5-2.

### Solution

We determined the Thévenin equivalent of the circuit shown in Figure 5.5-4 in Example 5.4-2. The procedure used to determine the Norton equivalent of a circuit is very similar to the procedure used to determine the Thévenin equivalent of that circuit. In particular the values of $v_{oc}$, $R_t$, and $i_{sc}$ for the Norton equivalent are determined in exactly the same way in which they were determined for the Thévenin equivalent in Example 5.4-2. Referring to Example 5.4-2 we have

$$v_{oc} = 96\text{ V}, \quad i_{sc} = 1.1294\text{ A} \quad \text{and} \quad R_n = R_t = 85\ \Omega$$

Our values of $v_{oc}$, $R_t$, and $i_{sc}$ satisfy Eq. 5.4-2, so we're confident that they are correct. Finally, the Norton equivalent circuit is shown in Figure 5.5-5.

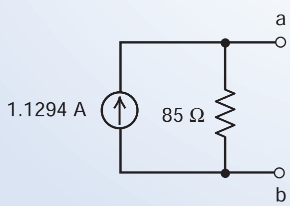


**FIGURE 5.5-5** The Norton equivalent circuit for the circuit shown in Figure 5.5-4.

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## EXAMPLE 5.5-3 An Application of the Norton Equivalent Circuit

Consider the circuit shown in Figure 5.5-6.

**(a)** Determine the voltage, $v$, when $R = 24\ \Omega$.
**(b)** Determine the value of the resistance $R$ required to cause $v = 40$ V.
**(c)** Determine the value of the resistance $R$ required to cause $v = 60$ V.

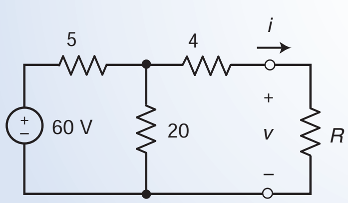



**FIGURE 5.5-6** The circuit considered in Example 5.5-3.

## Solution

We considered a similar problem in Example 5.4-3. In Example 5.4-3 we replaced the part of the circuit that is connected to resistor $R$ by its Thévenin equivalent circuit. In this example we will replace the part of the circuit that is connected to resistor $R$ by its Norton equivalent circuit. The Norton equivalent circuit can be obtained from the Thévenin equivalent using a source transformation. Referring to Figure 5.4-15, we obtain Figure 5.5-7 in which the part of the circuit that is connected to resistor $R$ has been replaced by its Norton equivalent circuit.

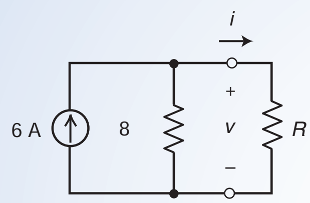



**FIGURE 5.5-7** The circuit obtained by replacing part of the circuit in Figure 5.5-6 by its Norton equivalent circuit.

We readily determine that

$$v = \frac{8R}{8+R}(6) = \frac{48R}{8+R} \tag{5.5-1}$$

in Figure 5.5-7. Replacing the part of the circuit that is connected to resistor $R$ by its Norton equivalent circuit did not change the current in resistor $R$. Consequently Eq. 5.5-1 describes the relationship between $v$ and $R$ in Figure 5.5-6! We can now easily answer questions (a), (b) and (c).

**(a)** When $R = 24\ \Omega$ the resistor current is $v = \frac{48(24)}{8+24} = 36$ V.

**(b)** To cause $v = 40$ V requires $R = \frac{8\,(40)}{48-40} = 40\ \Omega$.

**(c)** To cause $v = 60$ V requires $R = \frac{8\,(60)}{48-60} = -40\ \Omega$.

The answer in part (c) is probably not acceptable because we expect $0 < R < \infty$. Using Eq. 5.5-1 shows that the circuit in Figure 5.5-6 can only produce voltage in the range $0 < v < 48$ V. The voltage specified in (c) is outside of this range and cannot be obtained using a positive resistance $R$.

Try it yourself in WileyPLUS

**EXERCISE 5.5-1** Determine values of $R_t$ and $i_{sc}$ that cause the circuit shown in Figure E 5.5-1*b* to be the Norton equivalent circuit of the circuit in Figure E 5.5-1*a*.

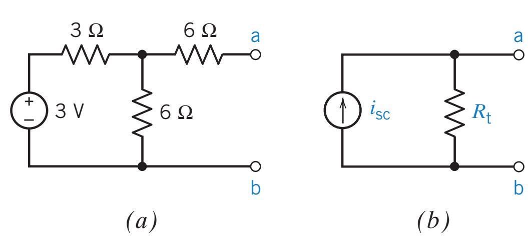



**FIGURE E 5.5-1**

***Answer:*** $R_t = 8\ \Omega$ and $i_{sc} = 0.25$ A

## 5.6 *Maximum Power Transfer*

Many applications of circuits require the maximum power available from a source to be transferred to a load resistor $R_L$. Consider the circuit A shown in Figure 5.6-1, terminated with a load $R_L$. As demonstrated in Section 5.4, circuit A can be reduced to its Thévenin equivalent, as shown in Figure 5.6-2.

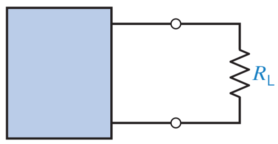



**FIGURE 5.6-1** Circuit A contains resistors and independent and dependent sources. The load is the resistor $R_L$.

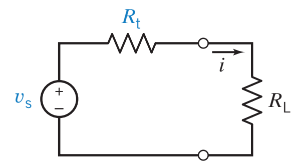



**FIGURE 5.6-2** The Thévenin equivalent is substituted for circuit A. Here we use $v_s$ for the Thévenin source voltage.

The general problem of power transfer can be discussed in terms of efficiency and effectiveness. Power utility systems are designed to transport the power to the load with the greatest efficiency by reducing the losses on the power lines. Thus, the effort is concentrated on reducing $R_t$, which would represent the resistance of the source plus the line resistance. Clearly, the idea of using superconducting lines that would exhibit no line resistance is exciting to power engineers.

In the case of signal transmission, as in the electronics and communications industries, the problem is to attain the maximum signal strength at the load. Consider the signal received at the antenna of an FM radio receiver from a distant station. It is the engineer's goal to design a receiver circuit so that the maximum power ultimately ends up at the output of the amplifier circuit connected to the antenna of your FM radio. Thus, we may represent the FM antenna and amplifier by the Thévenin equivalent circuit shown in Figure 5.6-2.

Let us consider the general circuit of Figure 5.6-2. We wish to find the value of the load resistance $R_L$ such that maximum power is delivered to it. First, we need to find the power from

$$p = i^2 R_L$$

Because the current $i$ is

$$i = \frac{v_s}{R_L + R_t}$$

we find that the power is

$$p = \left(\frac{v_s}{R_L + R_t}\right)^2 R_L \tag{5.6-1}$$

Assuming that $v_s$ and $R_t$ are fixed for a given source, the maximum power is a function of $R_L$. To find the value of $R_L$ that maximizes the power, we use the differential calculus to find where the derivative $dp/dR_L$ equals zero. Taking the derivative, we obtain

$$\frac{dp}{dR_L} = v_s^2 \frac{(R_t + R_L)^2 - 2(R_t + R_L)R_L}{(R_L + R_t)^4}$$

The derivative is zero when

$$(R_t + R_L)^2 - 2(R_t + R_L)R_L = 0 \tag{5.6-2}$$

or

$$(R_t + R_L)(R_t + R_L - 2R_L) = 0 \tag{5.6-3}$$

Solving Eq. 5.6-3, we obtain

$$R_L = R_t \tag{5.6-4}$$

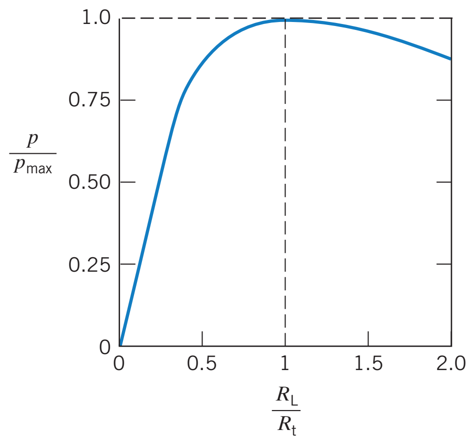



**FIGURE 5.6-3** Power actually attained as $R_L$ varies in relation to $R_t$.

To confirm that Eq. 5.6-4 corresponds to a maximum, it should be shown that $d^2p/dR_L{}^2 < 0$. Therefore, the maximum power is transferred to the load when $R_L$ is equal to the Thévenin equivalent resistance $R_t$.

The maximum power, when $R_L = R_t$, is then obtained by substituting $R_L = R_t$ in Eq. 5.6-1 to yield

$$p_{max} = \frac{v_s{}^2 R_t}{(2R_t)^2} = \frac{v_s{}^2}{4R_t}$$

The power delivered to the load will differ from the maximum attainable as the load resistance $R_L$ departs from $R_L = R_t$. The power attained as $R_L$ varies from $R_t$ is portrayed in Figure 5.6-3.

The **maximum power transfer** theorem states that the maximum power delivered to a load by a source is attained when the load resistance, $R_L$, is equal to the Thévenin resistance, $R_t$, of the source.

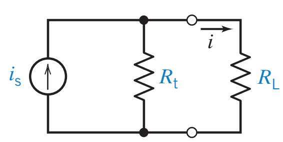



**FIGURE 5.6-4** Norton's equivalent circuit representing the source circuit and a load resistor $R_L$. Here we use $i_s$ as the Norton source current.

We may also use Norton's equivalent circuit to represent circuit A in Figure 5.6.1. We then have a circuit with a load resistor $R_L$ as shown in Figure 5.6-4. The current $i$ may be obtained from the current divider principle to yield

$$i = \frac{R_t}{R_t + R_L} i_s$$

Therefore, the power $p$ is

$$p = i^2 R_L = \frac{i_s{}^2 R_t{}^2 R_L}{(R_t + R_L)^2} \tag{5.6-5}$$

Using calculus, we can show that the maximum power occurs when

$$R_L = R_t \tag{5.6-6}$$

Then the maximum power delivered to the load is

$$p_{max} = \frac{R_t i_s{}^2}{4} \tag{5.6-7}$$

Try it yourself in WileyPLUS

## EXAMPLE 5.6-1 Maximum Power Transfer

Find the load resistance $R_L$ that will result in maximum power delivered to the load for the circuit of Figure 5.6-5. Also, determine the maximum power delivered to the load resistor.

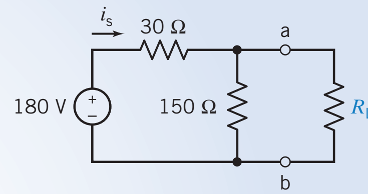



**FIGURE 5.6-5** Circuit for Example 5.6-1. Resistances in ohms.

### Solution

First, we determine the Thévenin equivalent circuit for the circuit to the left of terminals a–b. Disconnect the load resistor. The Thévenin voltage source $v_{oc}$ is

$$v_{oc} = \frac{150}{180} \times 180 = 150 \text{ V}$$

The Thévenin resistance $R_t$ is

$$R_t = \frac{30 \times 150}{30 + 150} = 25 \ \Omega$$

The Thévenin circuit connected to the load resistor is shown in Figure 5.6-6. Maximum power transfer is obtained when $R_L = R_t = 25 \ \Omega$.

Then the maximum power is

$$p_{max} = \frac{v_{oc}^2}{4R_L} = \frac{(150)^2}{4 \times 25} = 225 \text{ W}$$

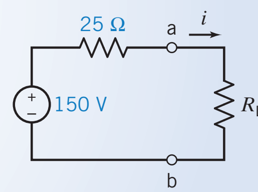



**FIGURE 5.6-6** Thévenin equivalent circuit connected to $R_L$ for Example 5.6-1.

Try it yourself in WileyPLUS

## EXAMPLE 5.6-2 Maximum Power Transfer

Find the load $R_L$ that will result in maximum power delivered to the load of the circuit of Figure 5.6-7*a*. Also, determine $p_{max}$ delivered.

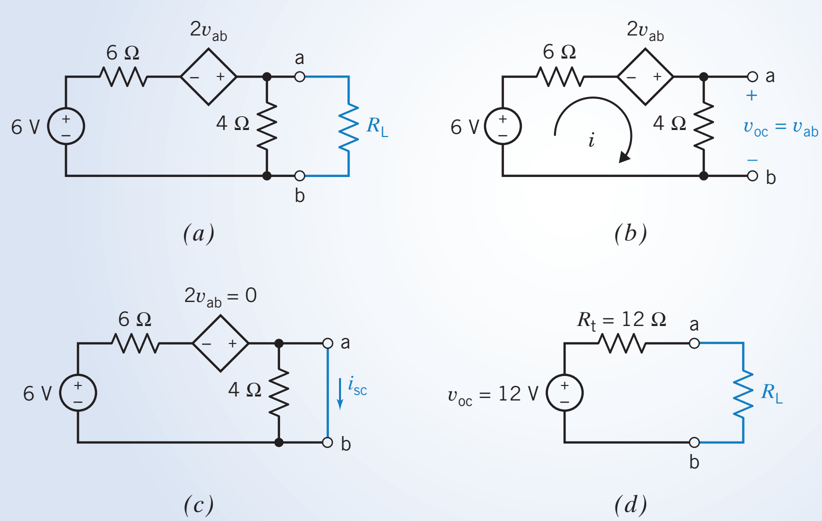



**FIGURE 5.6-7** Determination of maximum power transfer to a load $R_L$.

## Solution

We will obtain the Thévenin equivalent circuit for the part of the circuit to the left of terminals a,b in Figure 5.6-7*a*. First, we find $v_{oc}$ as shown in Figure 5.6-7*b*. The KVL gives

$$-6 + 10i - 2v_{ab} = 0$$

Also, we note that $v_{ab} = v_{oc} = 4i$. Therefore,

$$10i - 8i = 6$$

or $i = 3$ A. Therefore, $v_{oc} = 4i = 12$ V.

To determine the short-circuit current, we add a short circuit as shown in Figure 5.6-7*c*. The 4-Ω resistor is short circuited and can be ignored. Writing KVL, we have

$$-6 + 6i_{sc} = 0$$

Hence, $i_{sc} = 1$ A.

Therefore, $R_t = v_{oc}/i_{sc} = 12\ \Omega$. The Thévenin equivalent circuit is shown in Figure 5.6-7*d* with the load resistor. Maximum load power is achieved when $R_L = R_t = 12\ \Omega$. Then,

$$p_{max} = \frac{v_{oc}^2}{4R_L} = \frac{12^2}{4(12)} = 3\text{ W}$$

Try it yourself in WileyPLUS

***EXERCISE 5.6-1*** Find the maximum power that can be delivered to $R_L$ for the circuit of Figure E 5.6-1, using a Thévenin equivalent circuit.

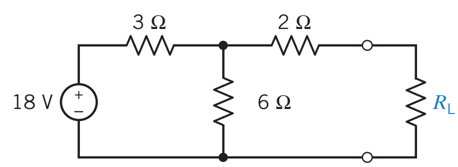



**FIGURE E 5.6-1**

***Answer:*** 9 W when $R_L = 4\ \Omega$

## 5.7 Using MATLAB to Determine the Thévenin Equivalent Circuit

MATLAB can be used to reduce the work required to determine the Thévenin equivalent of a circuit such as the one shown in Figure 5.7-1*a*. First, connect a resistor, $R$, across the terminals of the network, as shown in Figure 5.7-1*b*. Next, write node or mesh equations to describe the circuit with the resistor connected across its terminals. In this case, the circuit in Figure 5.7-1*b* is represented by the mesh equations

$$\begin{aligned} 12 &= 28i_1 - 10i_2 - 8i_3 \\ 12 &= -10i_1 + 28i_2 - 8i_3 \\ 0 &= -8i_1 - 8i_2 + (16 + R)i_3 \end{aligned} \qquad (5.7\text{-}1)$$

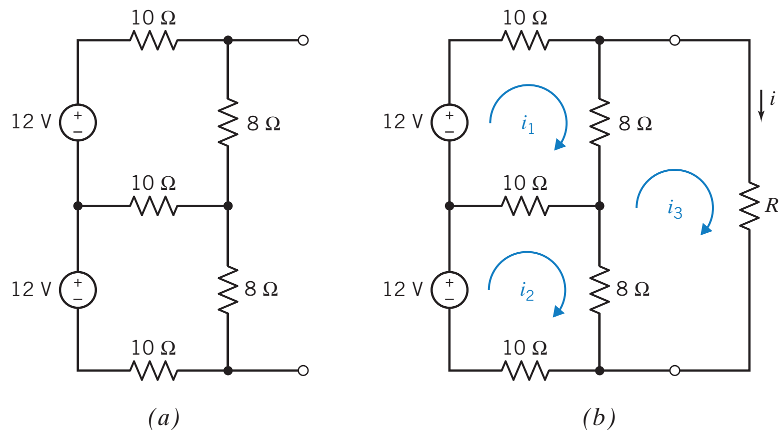



**FIGURE 5.7-1** The circuit in (*b*) is obtained by connecting a resistor, *R*, across the terminals of the circuit in (*a*).

The current $i$ in the resistor $R$ is equal to the mesh current in the third mesh, that is,

$$i = i_3 \tag{5.7-2}$$

The mesh equations can be written using matrices such as

$$\begin{bmatrix} 28 & -10 & -8 \\ -10 & 28 & -8 \\ -8 & -8 & 16+R \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 0 \end{bmatrix} \tag{5.7-3}$$

Notice that $i = i_3$ in Figure 5.7-1*b*.

Figure 5.7-2 shows a MATLAB file named ch5ex.m that solves Eq. 5.7-1. Figure 5.7-3 illustrates the use of this MATLAB file and shows that when $R = 6\ \Omega$, then $i = 0.7164$ A, and that when $R = 12\ \Omega$, then $i = 0.5106$ A.

Next, consider Figure 5.7-4, which shows a resistor $R$ connected across the terminals of a Thévenin equivalent circuit. The circuit in Figure 5.7-4 is represented by the mesh equation

$$V_t = R_t i + Ri \tag{5.7-4}$$

```
% ch5ex.m  -  MATLAB input file for Section 5-7

z  = [ 28     -10      -8;           %
      -10      28      -8;           %  Mesh Equation
       -8      -8    16+R];          %
                                     %  Equation 5.7-3
v  = [ 12;                           %
       12;                           %
        0];                          %


Im  = Z\V;           %  Calculate the mesh currents.

I  = Im(3)           %  Equation 5.7-2
```

**FIGURE 5.7-2** MATLAB file used to solve the mesh equation representing the circuit shown in Figure 5.7-1*b*.

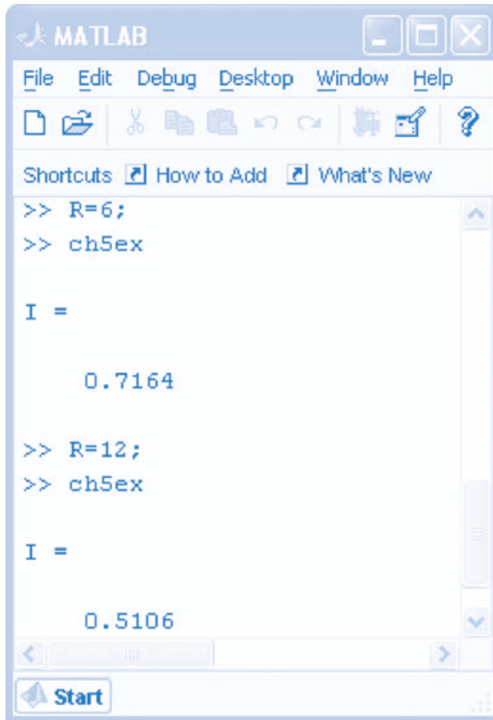



**FIGURE 5.7-3** Computer screen showing the use of MATLAB to analyze the circuit shown in Figure 5.7-1.

As a matter of notation, let $i = i_a$ when $R = R_a$. Similarly, let $i = i_b$ when $R = R_b$. Equation 5.7-4 indicates that

$$\begin{aligned} V_t &= R_t i_a + R_a i_a \\ V_t &= R_t i_b + R_b i_b \end{aligned} \tag{5.7-5}$$

Equation 5.7-5 can be written using matrices as

$$\begin{bmatrix} R_a i_a \\ R_b i_b \end{bmatrix} = \begin{bmatrix} 1 & -i_a \\ 1 & -i_b \end{bmatrix} \begin{bmatrix} V_t \\ R_t \end{bmatrix} \tag{5.7-6}$$

Given $i_a$, $R_a$, $i_b$, and $R_b$, this matrix equation can be solved for $V_t$ and $R_t$, the parameters of the Thévenin equivalent circuit. Figure 5.7-5 shows a MATLAB file that solves Eq. 5.7-6, using the values $i_b = 0.7164$ A, $R_b = 6\ \Omega$, $i_a = 0.5106$ A, and $R_a = 12\ \Omega$. The resulting values of $V_t$ and $R_t$ are

$$V_t = 10.664 \text{ V} \quad \text{and} \quad R_t = 8.8863\ \Omega$$

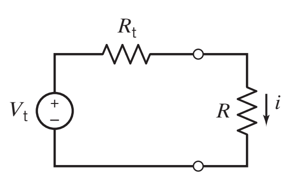



**FIGURE 5.7-4** The circuit obtained by connecting a resistor, $R$, across the terminals of a Thévenin equivalent circuit.

```
% Find the Thevenin equivalent of the circuit
% connected to the resister R.

Ra = 12;  ia = 0.5106;     % When R=Ra then i=ia

Rb = 6;   ib = 0.7164;     % When R=Rb then i=ib

A = [1 -ia;   %
     1 -ib];  %
              % Eqn 5.7-6
b = [Ra*ia;   %
     Rb*ib];  %

X = A\b;

Vt = X(1)  % Open-Circuit Voltage

Rt = X(2)  % Thevenin Resistance
```

**FIGURE 5.7-5** MATLAB file used to calculate the open-circuit voltage and Thévenin resistance.

## 5.8 *Using PSpice to Determine the Thévenin Equivalent Circuit*

We can use the computer program PSpice to find the Thévenin or Norton equivalent circuit for circuits even though they are quite complicated. Figure 5.8-1 illustrates this method. We calculate the Thévenin equivalent of the circuit shown in Figure 5.8-1*a* by calculating its open-circuit voltage $v_{oc}$ and its short-circuit current $i_{sc}$. To do so, we connect a resistor across its terminals as shown in Figure 5.8-1*b*. When the resistance of this resistor is infinite, the resistor voltage will be equal to the open-circuit voltage $v_{oc}$, as shown m Figure 5.8-1*b*. On the other hand, when the resistance of this resistor is zero, the resistor current will be equal to the short-circuit current $i_{sc}$, as shown in Figure 5.8-1*c*.

We can't use either infinite or zero resistances in PSpice, so we will approximate the infinite resistance by a resistance that is several orders of magnitude larger than the largest resistance in circuit A. We can check whether our resistance is large enough by doubling it and rerunning the PSpice simulation. If the computed value of $v_{oc}$ does not change, our large resistance is effectively infinite. Similarly, we can approximate a zero resistance by a resistance that is several orders of magnitude smaller than the smallest resistance in circuit A. Our small resistance is effectively zero when halving it does not change the computed value of $i_{sc}$.

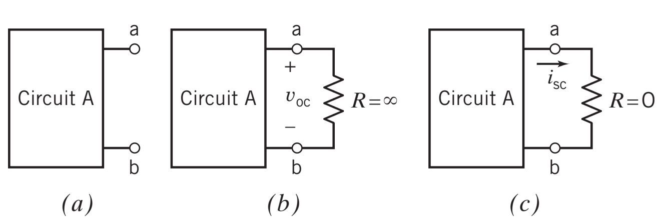



**FIGURE 5.8-1** A method for computing the values of $v_{oc}$ and $i_{sc}$, using PSpice.

## EXAMPLE 5.8-1 Using PSpice to find a Thévenin Equivalent Circuit

Use PSpice to determine the values of the open-circuit voltage $v_{oc}$ and the short-circuit current $i_{sc}$ for the circuit shown in Figure 5.8-2.

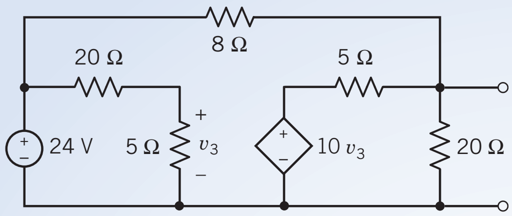



FIGURE 5.8-2 The circuit considered in Example 5.8-1.

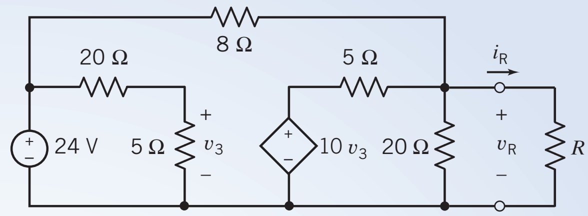



FIGURE 5.8-3 The circuit from Figure 5.8-2 after adding a resistor across its terminals.

## Solution

Following our method, we add a resistor across the terminals of the circuit as shown in Figure 5.8-3. Noticing that the largest resistance in our circuit is 20 Ω and the smallest is 5 Ω, we will determine $v_{oc}$ and $i_{sc}$, using

$$v_{oc} \approx v_R \quad \text{when} \quad R \gg 20\ \Omega$$

and

$$v_{sc} \approx i_R = \frac{v_R}{R} \quad \text{when} \quad R \ll 5\ \Omega$$

Using PSpice begins with drawing the circuit in the OrCAD Capture workspace as shown in Figure 5.8-4 (see Appendix A). The VCVS in Figure 5.8-3 is represented by a PSpice "Part E" in Figure 5.8-4. Figure 5.8-5 illustrates the correspondence between the VCVS and the PSpice "Part E."

To determine the open circuit voltage, we set the resistance $R$ to a very large value and perform a `Bias Point' simulation (see Appendix A). Figure 5.8-6 shows the simulation results when $R = 20\ \text{M}\Omega$. The voltage across the resistor $R$ is 33.6 V, so $v_{oc} = 33.6$ V. (Doubling the value of $R$ and rerunning the simulation did not change the value of the voltage across $R$, so we are confident that $v_{oc} = 33.6$ V.)

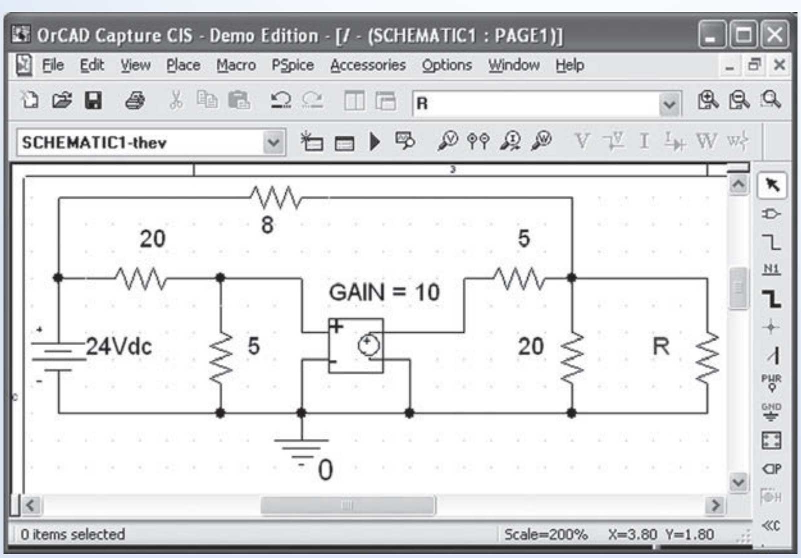
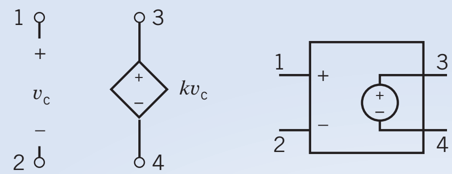



FIGURE 5.8-4 The circuit from Figure 5.8-3 drawn in the OrCAD Capture workspace.




(a) (b)

**FIGURE 5.8-5** A VCVS (*a*) and the corresponding PSpice "Part E" (*b*).

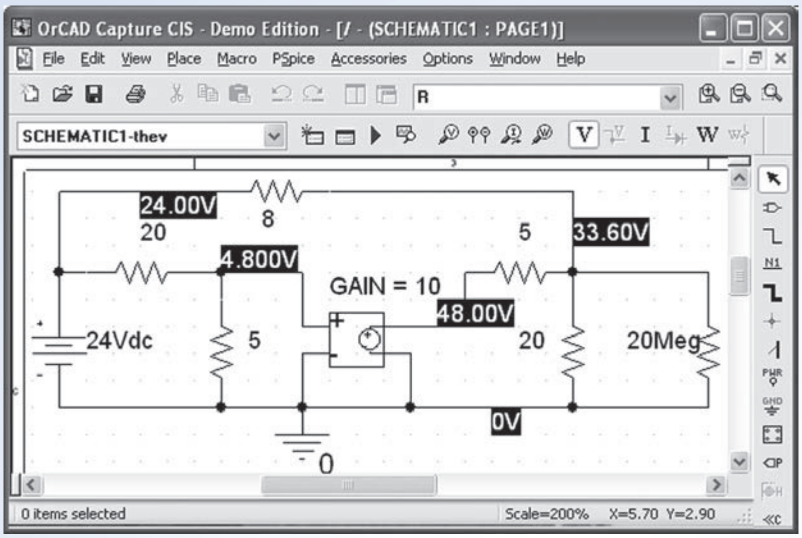



**FIGURE 5.8-6** Simulation results for $R = 20\ \text{M}\Omega$.

To determine the short-circuit current, we set the resistance $R$ to a very small value and perform a `Bias Point' simulation (see Appendix A). Figure 5.8-7 shows the simulation results when $R = 1\ \text{m}\Omega$. The voltage across the resistor $R$ is 12.6 mV. Using Ohm's law, the value of the short-circuit current is

$$i_{sc} = \frac{12.6 \times 10^{-3}}{1 \times 10^{-3}} = 12.6\ \text{A}$$

(Halving the value of $R$ and rerunning the simulation did not change the value of the voltage across $R$, so we are confident that $i_{sc} = 12.6$ A.)

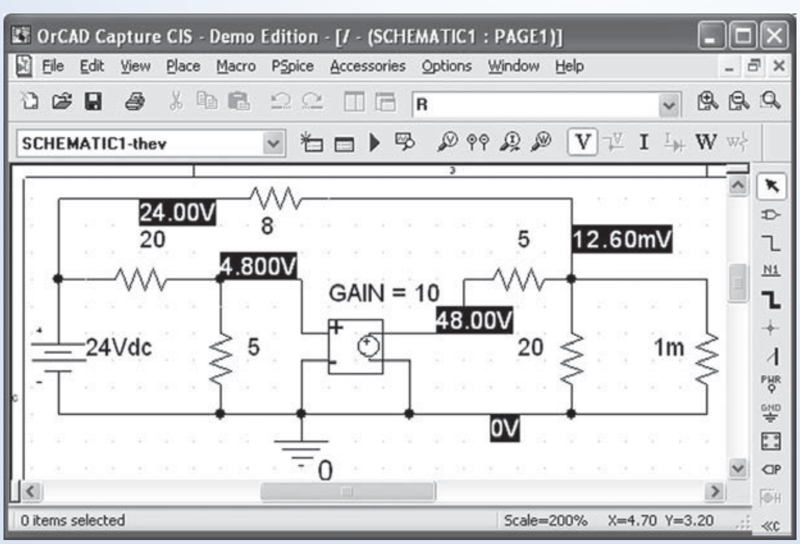



**FIGURE 5.8-7** Simulation results for $R = 1\ \text{M}\Omega = 0.001\ \Omega$.

## 5.9 *How Can We Check . . . ?*

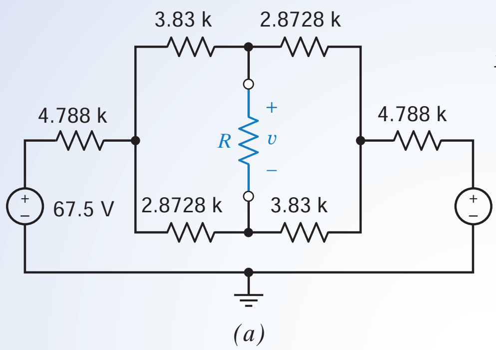
Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able to quickly identify those solutions that need more work.

The following example illustrates techniques useful for checking the solutions of the sort of problem discussed in this chapter.

### EXAMPLE 5.9-1 How Can We Check Thévenin Equivalent Circuits?

Suppose that the circuit shown in Figure 5.9-1*a* was built in the lab, using $R = 2$ kΩ, and that the voltage labeled $v$ was measured to be $v = -1.87$ V. Next, the resistor labeled $R$ was changed to $R = 5$ kΩ, and the voltage $v$ was measured to be $v = -3.0$ V. Finally, the resistor was changed to $R = 10$ kΩ, and the voltage was measured to be $v = -3.75$ V. **How can we check** that these measurements are consistent?




| $R$, k | $v$, V |
|---|---|
| 2 | –1.87 |
| 5 | –3.0 |
| 10 | –3.75 |

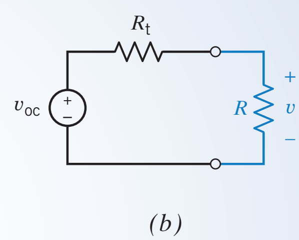



**FIGURE 5.9-1** (*a*) A circuit with data obtained by measuring the voltage across the resistor $R$, and (*b*) the circuit obtained by replacing the part of the circuit connected to $R$ by its Thévenin equivalent circuit.

## Solution

Let's replace the part of the circuit connected to the resistor $R$ by its Thévenin equivalent circuit. Figure 5.9-1*b* shows the resulting circuit. Applying the voltage division principle to the circuit in Figure 5.9-1*b* gives

$$v = \frac{R}{R + R_t} v_{oc} \tag{5.9-1}$$

When $R = 2$ kΩ, then $v = -1.87$ V, and Eq. 5.9-1 becomes

$$-1.87 = \frac{2000}{2000 + R_t} v_{oc} \tag{5.9-2}$$

Similarly, when $R = 5$ kΩ, then $v = -3.0$ V, and Eq. 5.9-1 becomes

$$-3.0 = \frac{5000}{5000 + R_t} v_{oc} \tag{5.9-3}$$

Equations 5.9-2 and 5.9-3 constitute a set of two equations in two unknowns, $v_{oc}$ and $R_t$. Solving these equations gives $v_{oc} = -5$ V and $R_t = 3333$ Ω. Substituting these values into Eq. 5.9-1 gives

$$v = \frac{R}{R + 3333}(-5) \tag{5.9-4}$$

Equation 5.9-4 can be used to predict the voltage that would be measured if $R = 10\,\text{k}\Omega$. If the value of $v$ obtained using Eq. 5.9-4 agrees with the measured value of $v$, then the measured data are consistent. Letting $R = 10\,\text{k}\Omega$ in Eq. 5.9-4 gives

$$v = \frac{10{,}000}{10{,}000 + 3333}(-5) = -3.75\text{ V} \tag{5.9-5}$$

Because this value agrees with the measured value of $v$, the measured data are indeed consistent.

## 5.10 DESIGN EXAMPLE Strain Gauge Bridge

Strain gauges are transducers that measure mechanical strain. Electrically, the strain gauges are resistors. The strain causes a change in resistance that is proportional to the strain.

Figure 5.10-1 shows four strain gauges connected in a configuration called a bridge. Strain gauge bridges measure force or pressure (Doebelin, 1966).

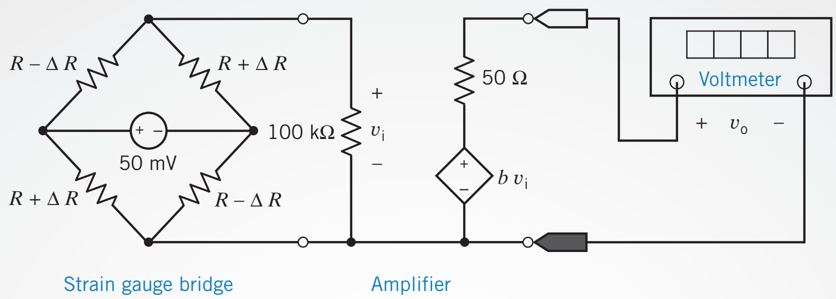



**FIGURE 5.10-1** Design problem involving a strain gauge bridge.

The bridge output is usually a small voltage. In Figure 5.10-1, an amplifier multiplies the bridge output, $v_i$, by a gain to obtain a larger voltage, $v_o$, which is displayed by the voltmeter.

### Describe the Situation and the Assumptions

A strain gauge bridge is used to measure force. The strain gauges have been positioned so that the force will increase the resistance of two of the strain gauges while, at the same time, decreasing the resistance of the other two strain gauges.

The strain gauges used in the bridge have nominal resistances of $R = 120\ \Omega$. (The nominal resistance is the resistance when the strain is zero.) This resistance is expected to increase or decrease by no more than 2 Ω due to strain. This means that

$$-2\,\Omega \le \Delta R \le 2\,\Omega \tag{5.10-1}$$

The output voltage $v_o$ is required to vary from −10 V to +10 V as $\Delta R$ varies from −2 Ω to 2 Ω.

### State the Goal

Determine the amplifier gain $b$ needed to cause $v_o$ to be related to $\Delta R$ by

$$v_o = 5\,\frac{\text{volt}}{\text{ohm}} \cdot \Delta R \tag{5.10-2}$$

### Generate a Plan

Use Thévenin's theorem to analyze the circuit shown in Figure 5.10-1 to determine the relationship between $v_i$ and $\Delta R$. Calculate the amplifier gain needed to satisfy Eq. 5.10-2.

## Act on the Plan

We begin by finding the Thévenin equivalent of the strain gauge bridge. This requires two calculations: one to find the open-circuit voltage, $v_t$, and the other to find the Thévenin resistance $R_t$. Figure 5.10-2*a* shows the circuit used to calculate $v_t$. Begin by finding the currents $i_1$ and $i_2$.

$$i_1 = \frac{50\text{ mV}}{(R - \Delta R) + (R + \Delta R)} = \frac{50\text{ mV}}{2R}$$

Similarly

$$i_2 = \frac{50\text{ mV}}{(R + \Delta R) + (R - \Delta R)} = \frac{50\text{ mV}}{2R}$$

Then

$$\begin{aligned} v_t &= (R + \Delta R)i_1 - (R - \Delta R)i_2 \\ &= (2\Delta R)\frac{50\text{ mV}}{2R} \\ &= \frac{\Delta R}{R}50\text{ mV} = \frac{50\text{ mV}}{120\ \Omega}\Delta R = \left(0.4167 \times 10^{-3}\right)\Delta R \end{aligned} \qquad (5.10\text{-}3)$$

Figure 5.10-2*b* shows the circuit used to calculate $R_t$. This figure shows that $R_t$ is composed of a series connection of two resistances, each of which is a parallel connection of two strain gauge resistances

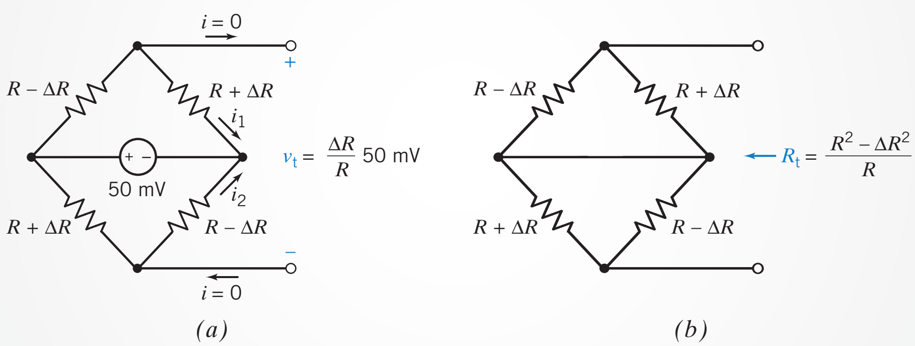



**FIGURE 5.10-2** Calculating (*a*) the open-circuit voltage, and (*b*) the Thévenin resistance of the strain gauge bridge.

$$R_t = \frac{(R - \Delta R)(R + \Delta R)}{(R - \Delta R) + (R + \Delta R)} + \frac{(R + \Delta R)(R - \Delta R)}{(R + \Delta R) + (R - \Delta R)} = 2\frac{R^2 - \Delta R^2}{2R}$$

Because $R$ is much larger than $\Delta R$, this equation can be simplified to

$$R_t = R$$

In Figure 5.10-3 the strain gauge bridge has been replaced by its Thévenin equivalent circuit. This simplification allows us to calculate $v_i$ using voltage division

$$v_i = \frac{100\text{ k}\Omega}{100\text{ k}\Omega + R_t}v_t = 0.9988v_t = \left(0.4162 \times 10^{-3}\right)\Delta R \qquad (5.10\text{-}4)$$

Model the voltmeter as an ideal voltmeter. Then the voltmeter current is $i = 0$ as shown in Figure 5.10-3. Applying KVL to the right-hand mesh gives

$$v_o + 50(0) - bv_i = 0$$

or

$$v_o = bv_i = b\left(0.4162 \times 10^{-3}\right)\Delta R \qquad (5.10\text{-}5)$$

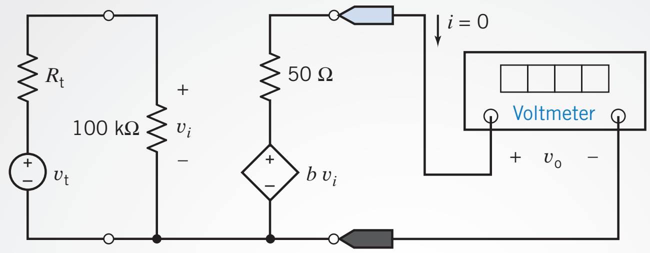



**FIGURE 5.10-3** Solution to the design problem.

Comparing Eq. 5.10-5 to Eq. 5.10-2 shows that the amplifier gain $b$ must satisfy

$$b\left(0.4162 \times 10^{-3}\right) = 5$$

Hence, the amplifier gain is

$$b = 12{,}013$$

### Verify the Proposed Solution

Substituting $b = 12{,}013$ into Eq. 5.10-5 gives

$$v_o = (12{,}013)\left(0.4162 \times 10^{-3}\right)\Delta R = 4.9998\,\Delta R \qquad (5.10\text{-}6)$$

which agrees with Eq. 5.10-2.

## 5.11 SUMMARY

- Source transformations, summarized in Table 5.11-1, are used to transform a circuit into an equivalent circuit. A voltage source $v_{oc}$ in series with a resistor $R_t$ can be transformed into a current source $i_{sc} = v_{oc}/R_t$ and a parallel resistor $R_t$. Conversely, a current source $i_{sc}$ in parallel with a resistor $R_t$ can be transformed into a voltage source $v_{oc} = R_t i_{sc}$ in series with a resistor $R_t$. The circuits in Table 5.11-1 are equivalent in the sense that the voltage and current of all circuit elements in circuit B are unchanged by the source transformation.
- The superposition theorem permits us to determine the total response of a linear circuit to several independent sources by finding the response to each independent source separately and then adding the separate responses algebraically.
- Thévenin and Norton equivalent circuits, summarized in Table 5.11-2, are used to transform a circuit into a smaller, yet equivalent, circuit. First the circuit is separated into two parts, circuit A and circuit B, in Table 5.11-2. Circuit A can be replaced by either its Thévenin equivalent circuit or its Norton equivalent circuit. The circuits in Table 5.11-2 are equivalent in the sense that the voltage and current of all circuit elements in circuit B are unchanged by replacing circuit A with either its Thévenin equivalent circuit or its Norton equivalent circuit.
- Procedures for calculating the parameters $v_{oc}$, $i_{sc}$, and $R_t$ of the Thévenin and Norton equivalent circuits are summarized in Figures 5.4-3 and 5.4-4.
- The goal of many electronic and communications circuits is to deliver maximum power to a load resistor $R_L$. Maximum power is attained when $R_L$ is set equal to the Thévenin resistance $R_t$ of the circuit connected to $R_L$. This results in maximum power at the load when the series resistance $R_t$ cannot be reduced.
- The computer programs MATLAB and SPICE can be used to reduce the computational burden of calculating the parameters $v_{oc}$, $i_{sc}$, and $R_t$ of the Thévenin and Norton equivalent circuits.

**Table 5.11-1 Source Transformations**

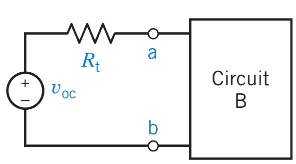
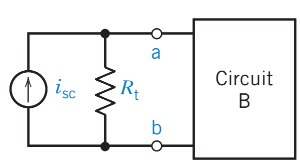
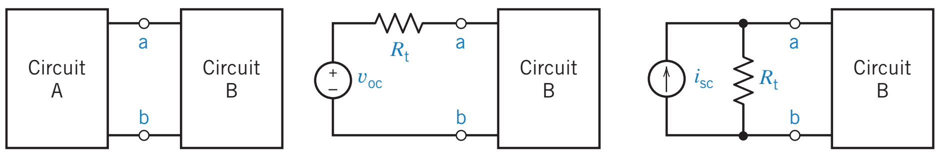
| THÉVENIN CIRCUIT | NORTON CIRCUIT |
|---|---|
|  |  |

**Table 5.11-2 Thévenin and Norton Equivalent Circuits**

| ORIGINAL CIRCUIT | THÉVENIN CIRCUIT | NORTON EQUIVALENT CIRCUIT |
|---|---|---|




## PROBLEMS

⊕ Problem available in WileyPLUS at instructor's discretion.

### Section 5.2 Source Transformations

**P 5.2-1** ⊕ The circuit shown in Figure P 5.2-1*a* has been divided into two parts. The circuit shown in Figure P 5.2-1*b* was obtained by simplifying the part to the right of the terminals using source transformations. The part of the circuit to the left of the terminals was not changed.

**(a)** Determine the values of $R_t$ and $v_t$ in Figure P 5.2-1*b*.

**(b)** Determine the values of the current $i$ and the voltage $v$ in Figure P 5.2-1*b*. The circuit in Figure P 5.2-1*b* is equivalent to the circuit in Figure P 5.2-1*a*. Consequently, the current $i$ and the voltage $v$ in Figure P 5.2-1*a* have the same values as do the current $i$ and the voltage $v$ in Figure P 5.2-1*b*.

**(c)** Determine the value of the current $i_a$ in Figure P 5.2-1*a*.

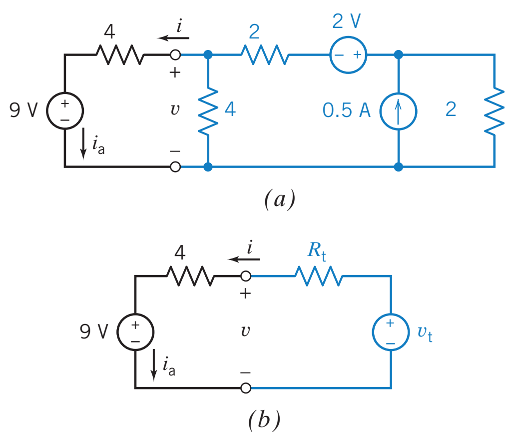



**Figure P 5.2-1**

**P 5.2-2** ⊕ Consider the circuit of Figure P 5.2-2. Find $i_a$ by simplifying the circuit (using source transformations) to a single-loop circuit so that you need to write only one KVL equation to find $i_a$.

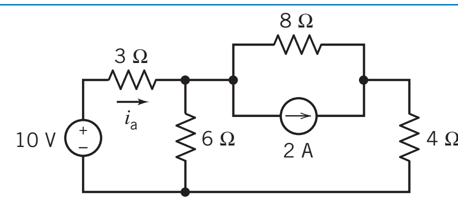



**Figure P 5.2-2**

**P 5.2-3** ⊕ Find $v_o$ using source transformations if $i = 5/2$ A in the circuit shown in Figure P 5.2-3.

***Hint:*** Reduce the circuit to a single mesh that contains the voltage source labeled $v_o$.

***Answer:*** $v_o = 28$ V

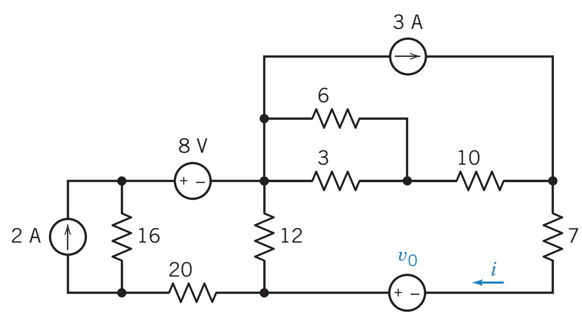



**Figure P 5.2-3**

**P 5.2-4** ⊕ Determine the value of the current $i_a$ in the circuit shown in Figure P 5.2-4.

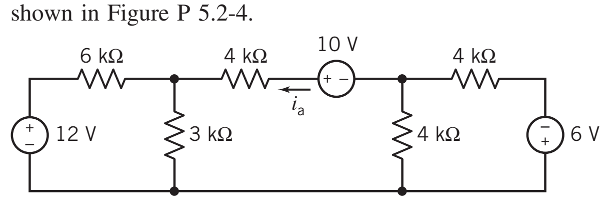



**Figure P 5.2-4**

**P 5.2-5** ⊕ Use source transformations to find the current $i_a$ in the circuit shown in Figure P 5.2-5.

***Answer:*** $i_a = 1$ A

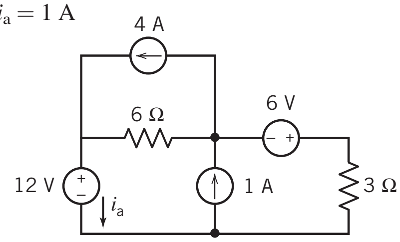



**Figure P 5.2-5**

**P 5.2-6** Use source transformations to find the value of the voltage $v_a$ in Figure P 5.2-6.

***Answer:*** $v_a = 7$ V

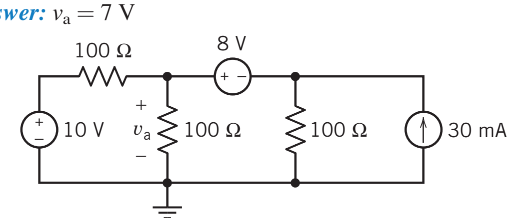



**Figure P 5.2-6**

**P 5.2-7** The equivalent circuit in Figure P 5.2-7 is obtained from the original circuit using source transformations and equivalent resistances. (The lower case letters *a* and *b* identify the nodes of the capacitor in both the original and equivalent circuits.) Determine the values of $R_a$, $V_a$, $R_b$, and $I_b$ in the equivalent circuit

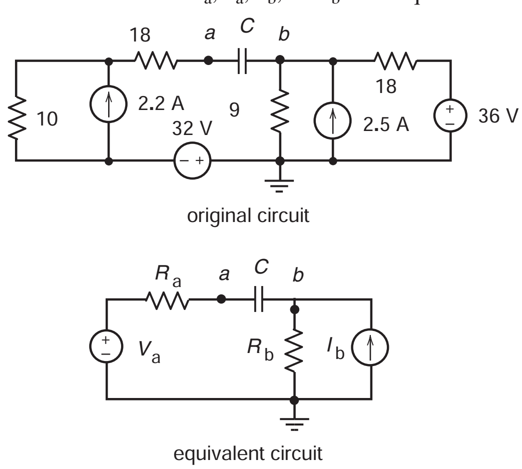



**Figure P 5.2-7**

**P 5.2-8** The circuit shown in Figure P 5.2-8 contains an unspecified resistance *R*.

**(a)** Determine the value of the current *i* when $R = 4\,\Omega$.
**(b)** Determine the value of the voltage *v* when $R = 8\,\Omega$.
**(c)** Determine the value of *R* that will cause $i = 1$ A.
**(d)** Determine the value of *R* that will cause $v = 16$ V.

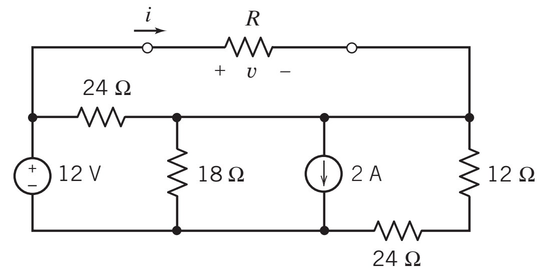



**Figure P 5.2-8**

**P 5.2-9** ⊕ Determine the value of the power supplied by the current source in the circuit shown in Figure P 5.2-9.

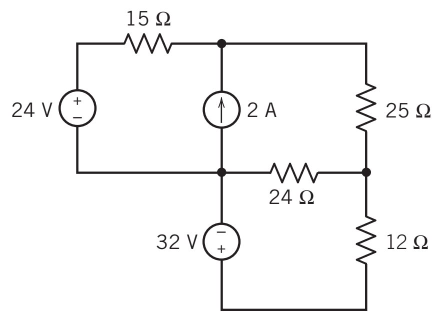



**Figure P 5.2-9**

## Section 5.3 Superposition

**P 5.3-1** ⊕ The inputs to the circuit shown in Figure P 5.3-1 are the voltage source voltages $v_1$ and $v_2$. The output of the circuit is the voltage $v_o$. The output is related to the inputs by

$$v_o = av_1 + bv_2$$

where *a* and *b* are constants. Determine the values of *a* and *b*.

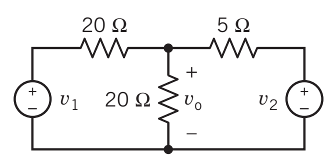



**Figure P 5.3-1**

**P 5.3-2** ⊕ A particular linear circuit has two inputs, $v_1$ and $v_2$, and one output, $v_o$. Three measurements are made. The first measurement shows that the output is $v_o = 4$ V when the inputs are $v_1 = 2$ V and $v_2 = 0$. The second measurement shows that the output is $v_o = 10$ V when the inputs are $v_1 = 0$ and $v_2 = -2.5$ V. In the third measurement, the inputs are $v_1 = 3$ V and $v_2 = 3$ V. What is the value of the output in the third measurement?

**P 5.3-3** ⊕ The circuit shown in Figure P 5.3-3 has two inputs, $v_s$ and $i_s$, and one output, $i_o$. The output is related to the inputs by the equation

$$i_o = ai_s + bv_s$$

Given the following two facts:

The output is $i_o = 0.45$ A when the inputs are $i_s = 0.25$ A and $v_s = 15$ V

and

The output is $i_o = 0.30$ A when the inputs are $i_s = 0.50$ A and $v_s = 0$ V

Determine the values of the constants $a$ and $b$ and the values of the resistances are $R_1$ and $R_2$.

***Answers:*** $a = 0.6$ A/A, $b = 0.02$ A/V, $R_1 = 30\ \Omega$, and $R_2 = 20\ \Omega$.

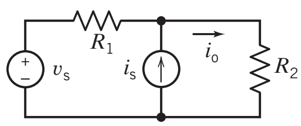



**Figure P 5.3-3**

**P 5.3-4** Use superposition to find $v$ for the circuit of Figure P 5.3-4.

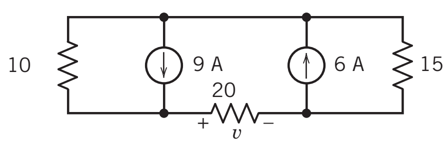



**Figure P 5.3-4**

**P 5.3-5** Determine $v(t)$, the voltage across the vertical resistor in the circuit in Figure P 5.3-5.

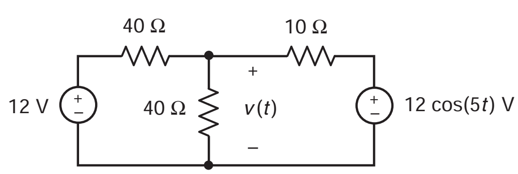



**Figure P 5.3-5**

**P 5.3-6** ⊕ Use superposition to find $i$ for the circuit of Figure P 5.3-6.

***Answer:*** $i = 3.5$ mA

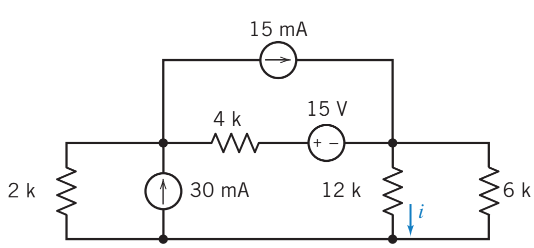



**Figure P 5.3-6**

**P 5.3-7** Determine $v(t)$, the voltage across the 40 Ω resistor in the circuit in Figure P 5.3-7.

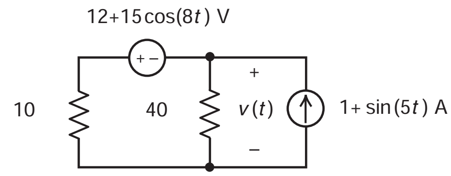



**Figure P 5.3-7**

**P 5.3-8** ⊕ Use superposition to find the value of the current $i_x$ in Figure P 5.3-8.

***Answer:*** $i_x = 1/6$ A

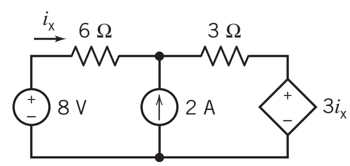



**Figure P 5.3-8**

***P 5.3-9** The input to the circuit shown in Figure P 5.3-9 is the voltage source voltage $v_s$. The output is the voltage $v_o$. The current source current $i_a$ is used to adjust the relationship between the input and output. Design the circuit so that input and output are related by the equation $v_o = 2v_s + 9$.

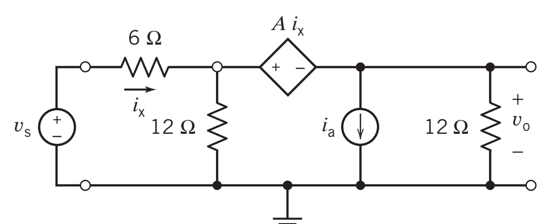



**Figure P 5.3-9**

***Hint:*** Determine the required values of $A$ and $i_a$.

**P 5.3-10** ⊕ The circuit shown in Figure P 5.3-10 has three inputs: $v_1$, $v_2$, and $i_3$. The output of the circuit is $v_o$. The output is related to the inputs by

$$v_o = av_1 + bv_2 + ci_3$$

where $a$, $b$, and $c$ are constants. Determine the values of $a$, $b$, and $c$.

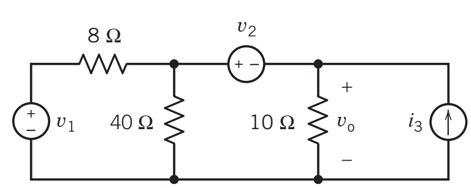



**Figure P 5.3-10**

**P 5.3-11** Determine the voltage $v_o(t)$ for the circuit shown in Figure P 5.3-11.

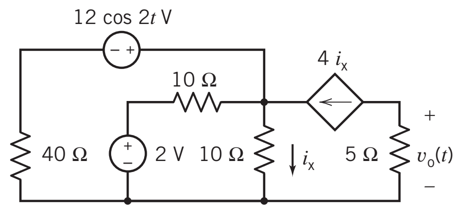



Figure P 5.3-11

**P 5.3-12** Determine the value of the voltage $v_o$ in the circuit shown in Figure P 5.3-12.

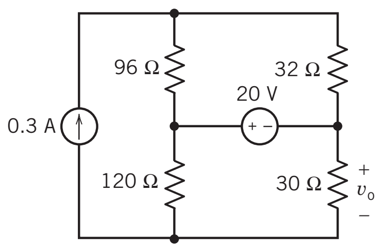



Figure P 5.3-12

**P 5.3-13** The input to the circuit shown in Figure P 5.3-13 is the current $i_1$. The output is the voltage $v_o$. The current $i_2$ is used to adjust the relationship between the input and output. Determine values of the current $i_2$ and the resistance $R$, that cause the output to be related to the input by the equation

$$v_o = -0.5 i_1 + 4$$

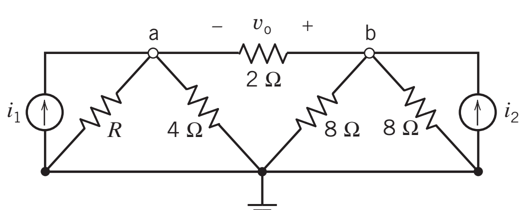



Figure P 5.3-13

**P 5.3-14** Determine values of the current $i_a$ and the resistance $R$ for the circuit shown in Figure P 5.3-14.

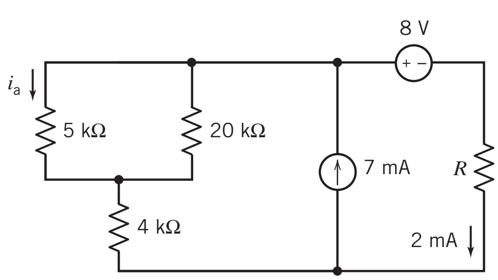



Figure P 5.3-14

**P 5.3-15** The circuit shown in Figure P 5.3-15 has three inputs: $v_1$, $i_2$, and $v_3$. The output of the circuit is the current $i_o$. The output of the circuit is related to the inputs by

$$i_1 = a v_o + b v_2 + c i_3$$

where $a$, $b$, and $c$ are constants. Determine the values of $a$, $b$, and $c$.

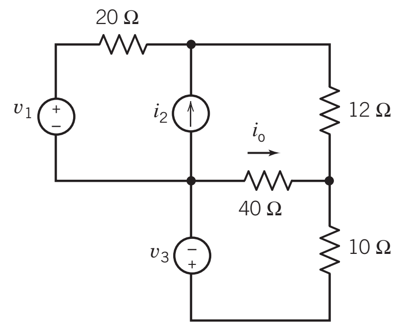



Figure P 5.3-15

**P 5.3-16** Using the superposition principle, find the value of the current measured by the ammeter in Figure P 5.3-16*a*.

*Hint:* Figure P 5.3-16*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, $i_m$.

*Answer:* $i_m = \frac{25}{3+2} - \frac{3}{2+3} 5 = 5 - 3 = 2$ A

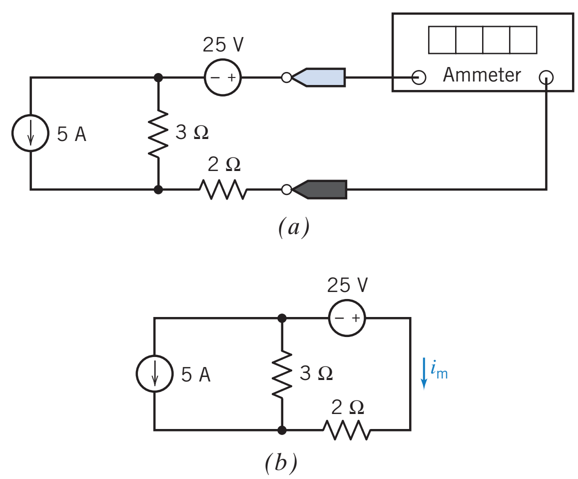


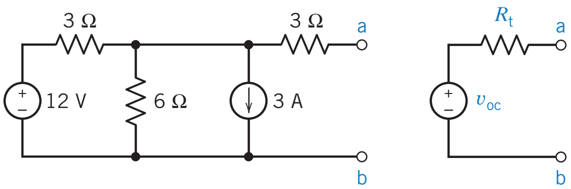

Figure P 5.3-16 (*a*) A circuit containing two independent sources. (*b*) The circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, $i_m$.

## Section 5.4 Thévenin's Theorem

**P 5.4-1** Determine values of $R_t$ and $v_{oc}$ that cause the circuit shown in Figure P 5.4-1*b* to be the Thévenin equivalent circuit of the circuit in Figure P 5.4-1*a*.

*Hint:* Use source transformations and equivalent resistances to reduce the circuit in Figure P 5.4-1*a* until it is the circuit in Figure P 5.4-1*b*.

***Answer:*** $R_t = 5\,\Omega$ and $v_{oc} = 2$ V

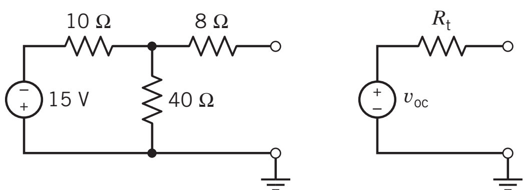



(*a*) (*b*)

**Figure P 5.4-1**

**P 5.4-2** The circuit shown in Figure P 5.4-2*b* is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-2*a*. Find the value of the open-circuit voltage $v_{oc}$ and Thévenin resistance $R_t$.

***Answer:*** $v_{oc} = -12$ V and $R_t = 16\,\Omega$

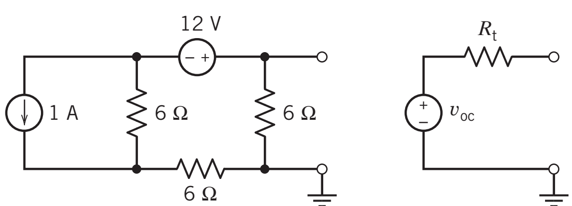



(*a*) (*b*)

**Figure P 5.4-2**

**P 5.4-3** ⊕ The circuit shown in Figure P 5.4-3*b* is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-3*a*. Find the value of the open-circuit voltage $v_{oc}$ and Thévenin resistance $R_t$.

***Answer:*** $v_{oc} = 2$ V and $R_t = 4\,\Omega$




(*a*) (*b*)

**Figure P 5.4-3**

**P 5.4-4** Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-4.

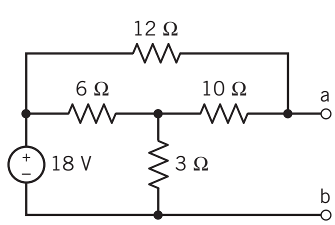



**Figure P 5.4-4**

**P 5.4-5** ⊕ Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-5.

***Answer:*** $v_{oc} = -2$ V and $R_t = -8/3\,\Omega$

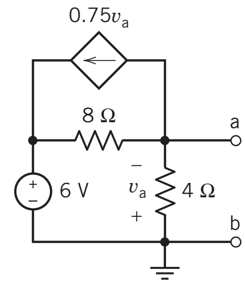



**Figure P 5.4-5**

**P 5.4-6** Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-6.

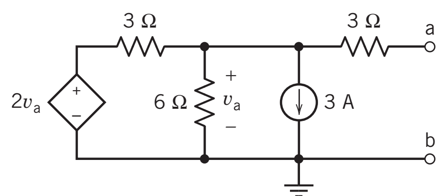



**Figure P 5.4-6**

**P 5.4-7** The equivalent circuit in Figure P 5.4-7 is obtained by replacing part of the original circuit by its Thévenin equivalent circuit. The values of the parameters of the Thévenin equivalent circuit are

$$v_{oc} = 15 \text{ V and } R_t = 60\Omega$$

Determine the following:

**(a)** The values of $V_s$ and $R_a$. (Four resistors in the original circuit have equal resistance, $R_a$.)
**(b)** The value of $R_b$ required to cause $i = 0.2$ A.
**(c)** The value of $R_b$ required to cause $v = 12$ V.

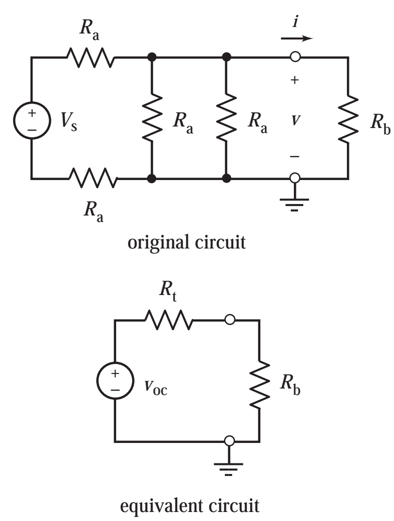
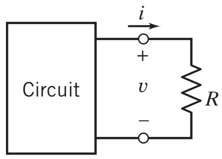



**Figure P 5.4-7**

**P 5.4-8** A resistor, $R$, was connected to a circuit box as shown in Figure P 5.4-8. The voltage $v$ was measured. The resistance was changed, and the voltage was measured again. The results are shown in the table. Determine the Thévenin equivalent of the circuit within the box and predict the voltage $v$ when $R = 8$ kΩ.



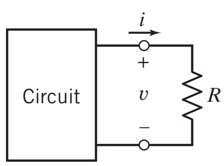

| $R$ | $v$ |
|---|---|
| 2 kΩ | 6 V |
| 4 kΩ | 2 V |

**Figure P 5.4-8**

**P 5.4-9** A resistor, $R$, was connected to a circuit box as shown in Figure P 5.4-9. The current $i$ was measured. The resistance was changed, and the current was measured again. The results are shown in the table.

**(a)** Specify the value of $R$ required to cause $i = 2$ mA.
**(b)** Given that $R > 0$, determine the maximum possible value of the current $i$.

***Hint:*** Use the data in the table to represent the circuit by a Thévenin equivalent.




| $R$ | $i$ |
|---|---|
| 2 kΩ | 4 mA |
| 4 kΩ | 3 mA |

**Figure P 5.4-9**

**P 5.4-10** For the circuit of Figure P 5.4-10, specify the resistance $R$ that will cause current $i_b$ to be 2 mA. The current $i_a$ has units of amps.

***Hint:*** Find the Thévenin equivalent circuit of the circuit connected to $R$.

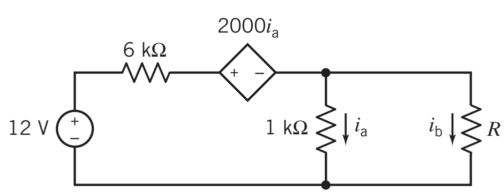



**Figure P 5.4-10**

**P 5.4-11** For the circuit of Figure P 5.4-11, specify the value of the resistance $R_L$ that will cause current $i_L$ to be −2 A.

***Answer:*** $R_L = 12\ \Omega$

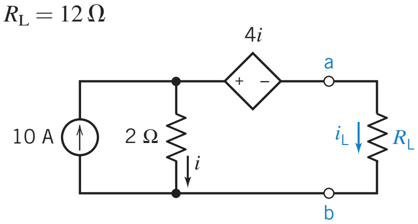



**Figure P 5.4-11**

**P 5.4-12** The circuit shown in Figure P 5.4-12 contains an adjustable resistor. The resistance $R$ can be set to any value in the range $0 \le R \le 100$ kΩ.

**(a)** Determine the maximum value of the current $i_a$ that can be obtained by adjusting $R$. Determine the corresponding value of $R$.
**(b)** Determine the maximum value of the voltage $v_a$ that can be obtained by adjusting $R$. Determine the corresponding value of $R$.
**(c)** Determine the maximum value of the power supplied to the adjustable resistor that can be obtained by adjusting $R$. Determine the corresponding value of $R$.

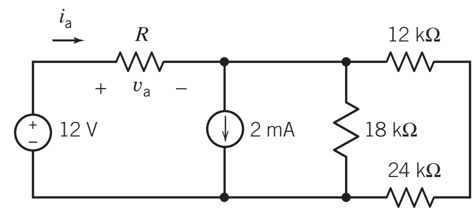



**Figure P 5.4-12**

**P 5.4-13** The circuit shown in Figure P 5.4-13 consists of two parts, the source (to the left of the terminals) and the load. The load consists of a single adjustable resistor having resistance $0 \le R_L \le 20\ \Omega$. The resistance $R$ is fixed but unspecified. When $R_L = 4\ \Omega$, the load current is measured to be $i_o = 0.375$ A. When $R_L = 8\ \Omega$, the value of the load current is $i_o = 0.300$ A.

**(a)** Determine the value of the load current when $R_L = 10\ \Omega$.
**(b)** Determine the value of $R$.

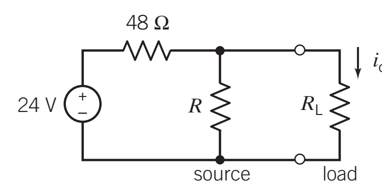



**Figure P 5.4-13**

**P 5.4-14** The circuit shown in Figure P 5.4-14 contains an unspecified resistance, $R$. Determine the value of $R$ in each of the following two ways.

**(a)** Write and solve mesh equations.
**(b)** Replace the part of the circuit connected to the resistor $R$ by a Thévenin equivalent circuit. Analyze the resulting circuit.

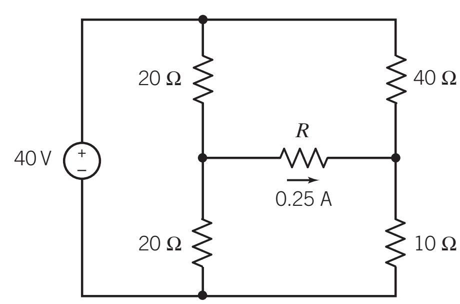



**Figure P 5.4-14**

**P 5.4-15** ⊕ Consider the circuit shown in Figure P 5.4-15. Replace the part of the circuit to the left of terminals a–b by its Thévenin equivalent circuit. Determine the value of the current $i_o$.

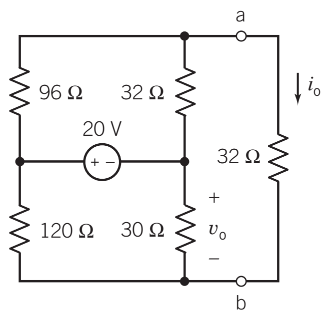



**Figure P 5.4-15**

**P 5.4-16** An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 5.4-16*a* shows a circuit with a voltmeter that measures the voltage $v_m$. In Figure P 5.4-16*b*, the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. The voltmeter measures $v_{mi}$, the ideal value of $v_m$.

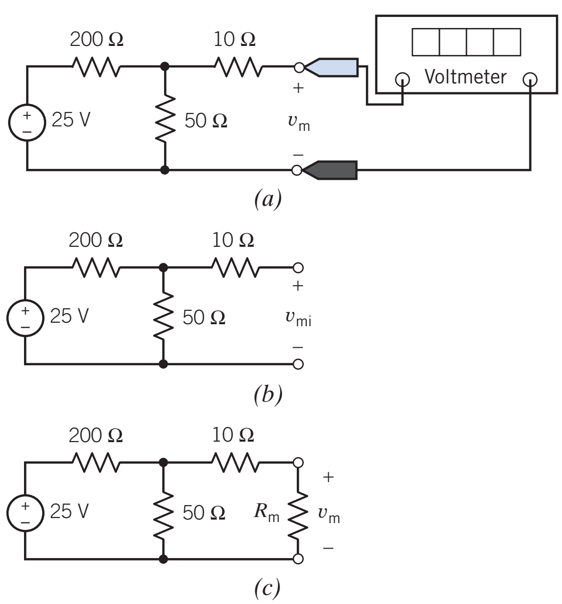



**Figure P 5.4-16**

As $R_m \to \infty$, the voltmeter becomes an ideal voltmeter and $v_m \to v_{mi}$. When $R_m < \infty$, the voltmeter is not ideal and $v_m > v_{mi}$. The difference between $v_m$ and $v_{mi}$ is a measurement error caused by the fact that the voltmeter is not ideal.

**(a)** Determine the value of $v_{mi}$.
**(b)** Express the measurement error that occurs when $R_m = 1000\,\Omega$ as a percentage of $v_{mi}$.
**(c)** Determine the minimum value of $R_m$ required to ensure that the measurement error is smaller than 2 percent of $v_{mi}$.

**P 5.4-17** Given that $0 \le R \le \infty$ in the circuit shown in Figure P 5.4-17, consider these two observations:

Observation 1: When $R = 2\,\Omega$ then $v_R = 4$ V and $i_R = 2$ A.
Observation 1: When $R = 6\,\Omega$ then $v_R = 6$ V and $i_R = 1$ A.

Determine the following:

**(a)** The maximum value of $i_R$ and the value of $R$ that causes $i_R$ to be maximal.
**(b)** The maximum value of $v_R$ and the value of $R$ that causes $v_R$ to be maximal.
**(c)** The maximum value of $p_R = i_R v_R$ and the value of $R$ that causes $p_R$ to be maximal.

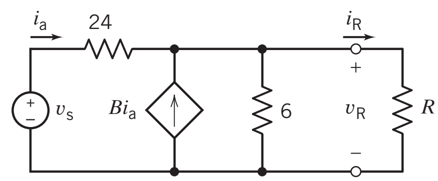



**Figure P 5.4-17**

**P 5.4-18** ⊕ Consider the circuit shown in Figure P 5.4-18. Determine

**(a)** The value of $v_R$ that occurs when $R = 9\,\Omega$.
**(b)** The value of $R$ that causes $v_R = 5.4$ V.
**(c)** The value of $R$ that causes $i_R = 300$ mA.

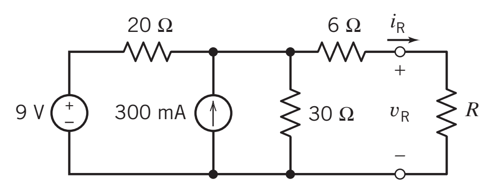



**Figure P 5.4-18**

**P 5.4-19** The circuit shown in Figure P 5.4-19*a* can be reduced to the circuit shown in Figure P 5.4-19*b* using source transformations and equivalent resistances. Determine the values of the source voltage $v_{oc}$ and the resistance $R$.

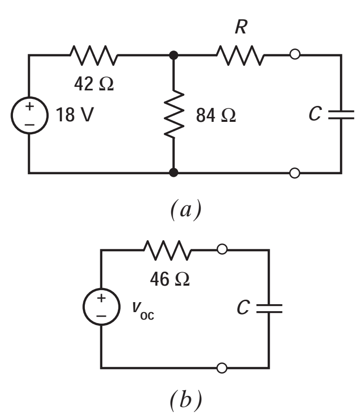



**Figure P 5.4-19**

**P 5.4-20** The equivalent circuit in Figure P 5.4-20 is obtained by replacing part of the original circuit by its Thévenin equivalent circuit. The values of the parameters of the Thévenin equivalent circuit are

$$v_{oc} = 15 \text{ V and } R_t = 60\ \Omega$$

Determine the following:

**(a)** The values of $V_s$ and $R_a$. (Three resistors in the original circuit have equal resistance, $R_a$.)
**(b)** The value of $R_b$ required to cause $i = 0.2$ A.
**(c)** The value of $R_b$ required to cause $v = 5$ V.

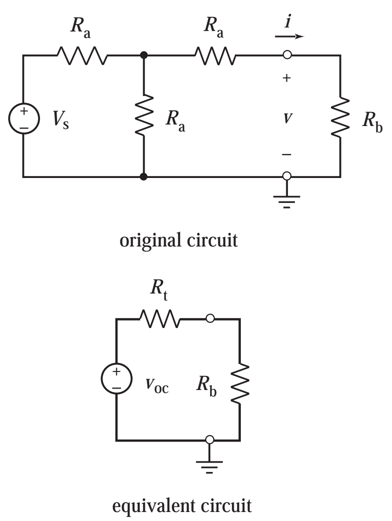



**Figure P 5.4-20**

## Section 5.5 Norton's Equivalent Circuit

**P 5.5-1** ⊕ The part of the circuit shown in Figure P 5.5-1*a* to the left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit, shown in Figure P 5.5-1*b*, will be characterized by the parameters:

$$i_{sc} = 0.5 \text{ A and } R_t = 20\ \Omega$$

**(a)** Determine the values of $v_S$ and $R_1$.
**(b)** Given that $0 \le R_2 \le \infty$, determine the maximum values of the voltage $v$ and of the power $p = vi$.

***Answers:*** $v_s = 37.5$ V, $R_1 = 25\ \Omega$, max $v = 10$ V and max $p = 1.25$ W

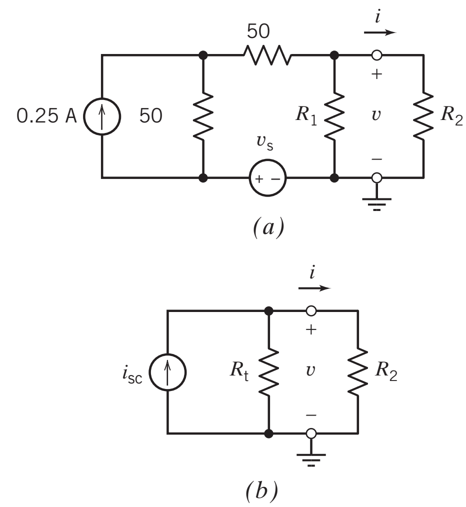



**Figure P 5.5-1**

**P 5.5-2** ⊕ Two black boxes are shown in Figure P 5.5-2. Box A contains the Thévenin equivalent of some linear circuit, and box B contains the Norton equivalent of the same circuit. With access to just the outsides of the boxes and their terminals, how can you determine which is which, using only one shorting wire?

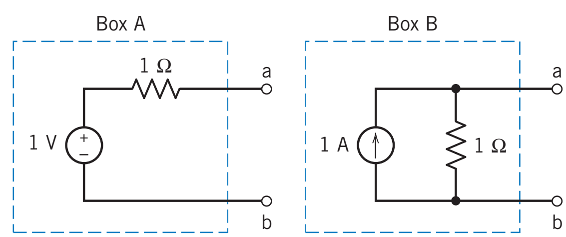



**Figure P 5.5-2** Black boxes problem.

**P 5.5-3** The circuit shown in Figure P 5.5-3*a* can be reduced to the circuit shown in Figure P 5.5-3*b* using source transformations and equivalent resistances. Determine the values of the source current $i_{sc}$ and the resistance $R$.

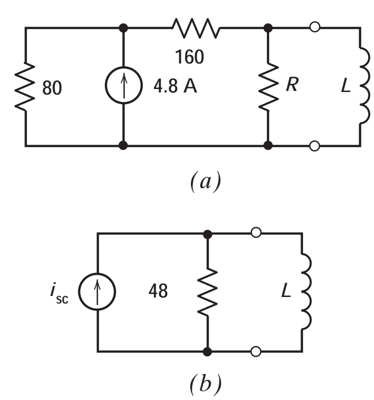



**Figure P 5.5-3**

**P 5.5-4** ⊕ Find the Norton equivalent circuit for the circuit shown in Figure P 5.5-4.

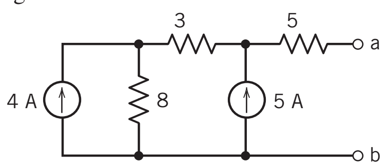



**Figure P 5.5-4**

**P 5.5-5** ⊕ The circuit shown in Figure P 5.5-5*b* is the Norton equivalent circuit of the circuit shown in Figure P 5.5-5*a*. Find the value of the short-circuit current $i_{sc}$ and Thévenin resistance $R_t$.

***Answer:*** $i_{sc} = 1.13$ A and $R_t = 7.57\ \Omega$

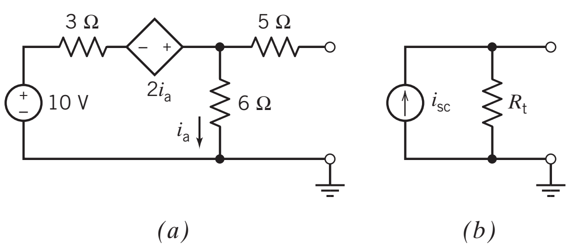



**Figure P 5.5-5**

**P 5.5-6** ⊕ The circuit shown in Figure P 5.5-6*b* is the Norton equivalent circuit of the circuit shown in Figure P 5.5-6*a*. Find the value of the short-circuit current $i_{sc}$ and Thévenin resistance $R_t$.

***Answer:*** $i_{sc} = -24$ A and $R_t = -3\ \Omega$

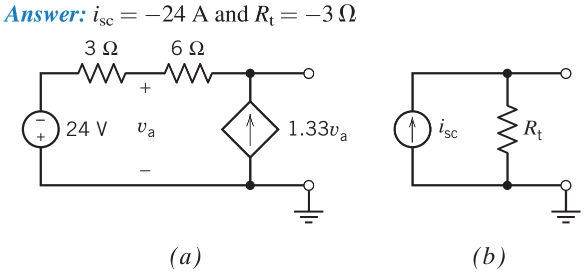



**Figure P 5.5-6**

**P 5.5-7** Determine the value of the resistance $R$ in the circuit shown in Figure P 5.5-7 by each of the following methods:

**(a)** Replace the part of the circuit to the left of terminals a–b by its Norton equivalent circuit. Use current division to determine the value of $R$.

**(b)** Analyze the circuit shown Figure P 5.5-7 using mesh equations. Solve the mesh equations to determine the value of $R$.

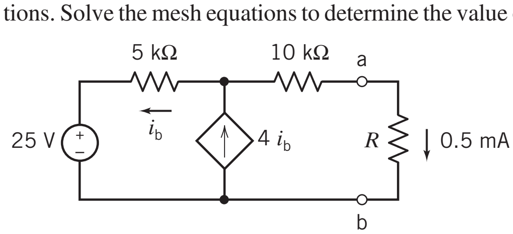



**Figure P 5.5-7**

**P 5.5-8** Find the Norton equivalent circuit for the circuit shown in Figure P 5.5-8.

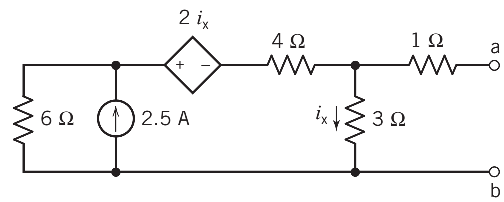



**Figure P 5.5-8**

**P 5.5-9** Find the Norton equivalent circuit for the circuit shown in Figure P 5.5-9.

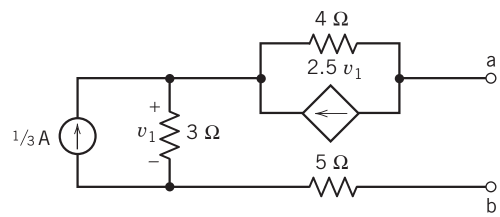



**Figure P 5.5-9**

**P 5.5-10** An ideal ammeter is modeled as a short circuit. A more realistic model of an ammeter is a small resistance. Figure P 5.5-10*a* shows a circuit with an ammeter that measures the current $i_m$. In Figure P 5.5-10*b*, the ammeter is replaced by the model of an ideal ammeter, a short circuit. The ammeter measures $i_{mi}$, the ideal value of $i_m$.

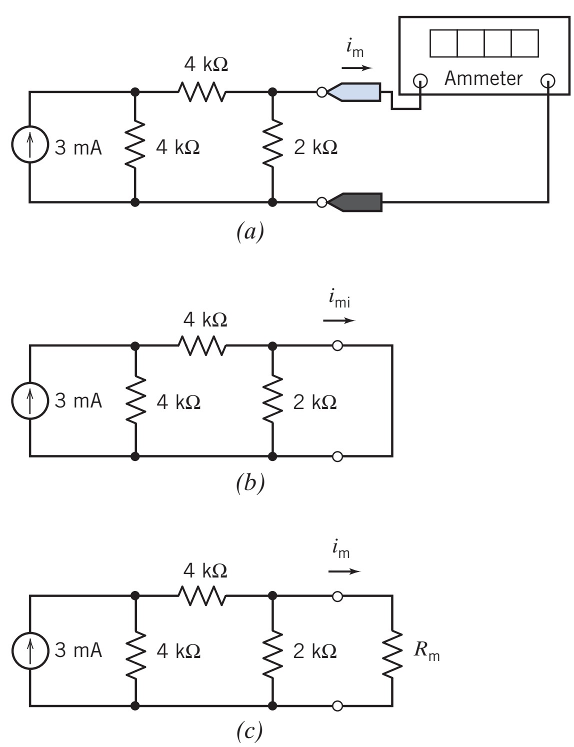


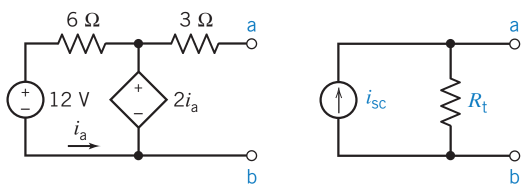

**Figure P 5.5-10**

As $R_m \to 0$, the ammeter becomes an ideal ammeter and $i_m \to i_{mi}$. When $R_m > 0$, the ammeter is not ideal and $i_m < i_{mi}$. The difference between $i_m$ and $i_{mi}$ is a measurement error caused by the fact that the ammeter is not ideal.

**(a)** Determine the value of $i_{mi}$.
**(b)** Express the measurement error that occurs when $R_m = 20\,\Omega$ as a percentage of $i_{mi}$.
**(c)** Determine the maximum value of $R_m$ required to ensure that the measurement error is smaller than 2 percent of $i_{mi}$.

**P 5.5-11** Determine values of $R_t$ and $i_{sc}$ that cause the circuit shown in Figure P 5.5-11*b* to be the Norton equivalent circuit of the circuit in Figure P 5.5-11*a*.

*Answer:* $R_t = 3\,\Omega$ and $i_{sc} = -2$ A




*(a)* *(b)*

**Figure P 5.5-11**

**P 5.5-12** Use Norton's theorem to formulate a general expression for the current $i$ in terms of the variable resistance $R$ shown in Figure P 5.5-12.

*Answer:* $i = 20/(8 + R)$ A

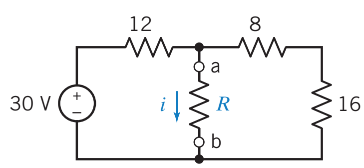



**Figure P 5.5-12**

### Section 5.6 Maximum Power Transfer

**P 5.6-1** The circuit shown in Figure P 5.6-1 consists of two parts separated by a pair of terminals. Consider the part of the circuit to the left of the terminals. The open circuit voltage is $v_{oc} = 8$ V, and short-circuit current is $i_{sc} = 2$ A. Determine the values of (a) the voltage source voltage $v_s$ and the resistance $R_2$, and (b) the resistance $R$ that maximizes the power delivered to the resistor to the right of the terminals, and the corresponding maximum power.

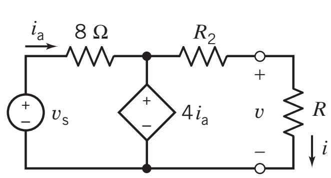



**Figure P 5.6-1**

**P 5.6-2** The circuit model for a photovoltaic cell is given in Figure P 5.6-2 (Edelson, 1992). The current $i_s$ is proportional to the solar insolation (kW/m$^2$).

**(a)** Find the load resistance, $R_L$, for maximum power transfer.
**(b)** Find the maximum power transferred when $i_s = 1$ A.

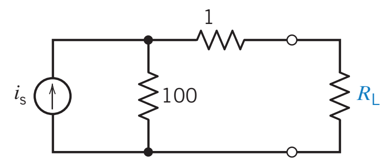



**Figure P 5.6-2** Circuit model of a photovoltaic cell.

**P 5.6-3** For the circuit in Figure P 5.6-3, (a) find $R$ such that maximum power is dissipated in $R$, and (b) calculate the value of maximum power.

*Answer:* $R = 60\,\Omega$ and $P_{max} = 54$ mW

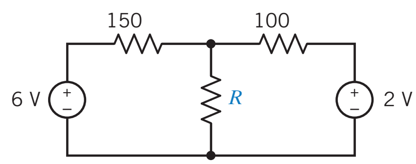



**Figure P 5.6-3**

**P 5.6-4** For the circuit in Figure P 5.6-4, prove that for $R_s$ variable and $R_L$ fixed, the power dissipated in $R_L$ is maximum when $R_s = 0$.

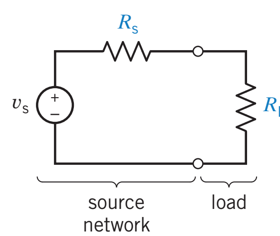



**Figure P 5.6-4**

**P 5.6-5** Determine the maximum power that can be absorbed by a resistor, $R$, connected to terminals a–b of the circuit shown in Figure P 5.6-5. Specify the required value of $R$.

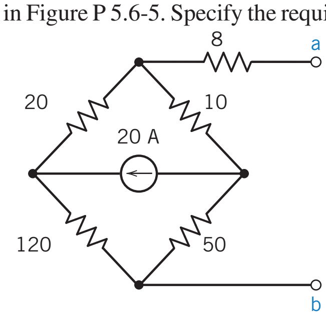



**Figure P 5.6-5** Bridge circuit.

**P 5.6-6** ⊕ Figure P 5.6-6 shows a source connected to a load through an amplifier. The load can safely receive up to 15 W of power. Consider three cases:

**(a)** $A = 20$ V/V and $R_o = 10\,\Omega$. Determine the value of $R_L$ that maximizes the power delivered to the load and the corresponding maximum load power.
**(b)** $A = 20$ V/V and $R_L = 8\,\Omega$. Determine the value of $R_o$ that maximizes the power delivered to the load and the corresponding maximum load power.
**(c)** $R_o = 10\,\Omega$ and $R_L = 8\,\Omega$. Determine the value of $A$ that maximizes the power delivered to the load and the corresponding maximum load power.

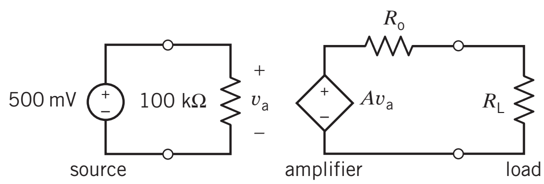



**Figure P 5.6-6**

**P 5.6-7** ⊕ The circuit in Figure P 5.6-7 contains a variable resistance, $R$, implemented using a potentiometer. The resistance of the variable resistor varies over the range $0 \le R \le 1000\,\Omega$. The variable resistor can safely receive 1/4 W power. Determine the maximum power received by the variable resistor. Is the circuit safe?

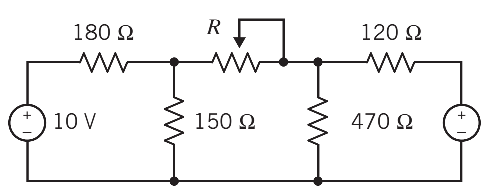



**Figure P 5.6-7**

**P 5.6-8** For the circuit of Figure P 5.6-8, find the power delivered to the load when $R_L$ is fixed and $R_t$ may be varied between $1\,\Omega$ and $5\,\Omega$. Select $R_t$ so that maximum power is delivered to $R_L$.

***Answer:*** 13.9 W

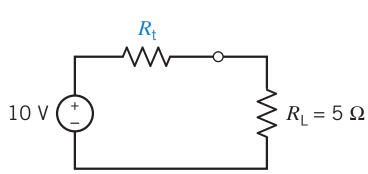



**Figure P 5.6-8**

**P 5.6-9** A resistive circuit was connected to a variable resistor, and the power delivered to the resistor was measured as shown in Figure P 5.6-9. Determine the Thévenin equivalent circuit.

***Answer:*** $R_t = 20\,\Omega$ and $v_{oc} = 20$ V

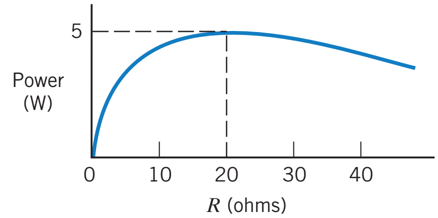



**Figure P 5.6-9**

**P 5.6-10** The part circuit shown in Figure P 5.6-10*a* to left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit, shown in Figure P 5.6-10*b*, will be characterized by the parameters:

$$i_{sc} = 1.5\text{ A and } R_t = 80\,\Omega$$

**(a)** Determine the values of $i_s$ and $R_1$.
**(b)** Given that $0 \le R_2 \le \infty$, determine the maximum value of $p = v_i$, the power delivered to $R_2$.

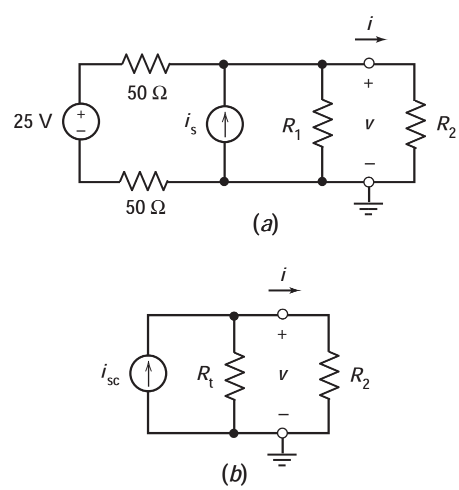



**Figure P 5.6-10**

**P 5.6-11** Given that $0 \le R \le \infty$ in the circuit shown in Figure P 5.6-11, determine (*a*) maximum value of $i_a$, (*b*) the maximum value of $v_a$, and (*c*) the maximum value of $p_a = i_a\,v_a$.

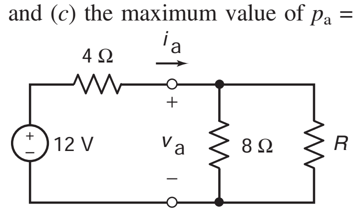



**Figure P 5.6-11**

**P 5.6-12** Given that $0 \le R \le \infty$ in the circuit shown in Figure P 5.6-12, determine value of $R$ that maximizes the power $p_a = i_a\,v_a$ and the corresponding maximum value of $p_a$.

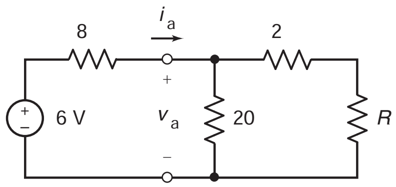



Figure P 5.6-12

## Section 5.8 Using PSpice to Determine the Thévenin Equivalent Circuit

**P 5.8-1** The circuit shown in Figure P 5.8-1 is separated into two parts by a pair of terminals. Call the part of the circuit to the left of the terminals circuit A and the part of the circuit to the right of the terminal circuit B. Use PSpice to do the following:

(a) Determine the node voltages for the entire circuit.
(b) Determine the Thévenin equivalent circuit of circuit A.
(c) Replace circuit A by its Thévenin equivalent and determine the node voltages of the modified circuit.
(d) Compare the node voltages of circuit B before and after replacing circuit A by its Thévenin equivalent.

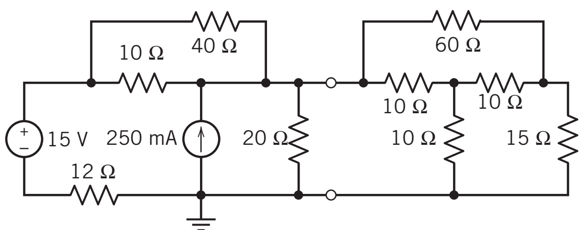



Figure P 5.8-1

## Section 5.9 How Can We Check . . . ?

**P 5.9-1** For the circuit of Figure P 5.9-1, the current $i$ has been measured for three different values of $R$ and is listed in the table. Are the data consistent?

| $R(\Omega)$ | $i$(mA) |
|---|---|
| 5000 | 16.5 |
| 500 | 43.8 |
| 0 | 97.2 |

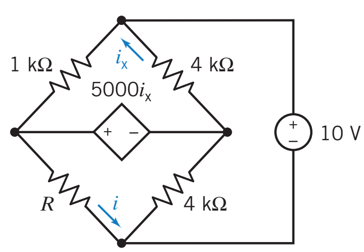
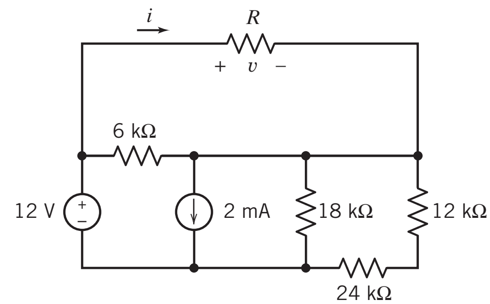



Figure P 5.9-1

**P 5.9-2** Your lab partner built the circuit shown in Figure P 5.9-2 and measured the current $i$ and voltage $v$ corresponding to several values of the resistance $R$. The results are shown in the table in Figure P 5.9-2. Your lab partner says that $R_L = 8000\ \Omega$ is required to cause $i = 1$ mA. Do you agree? Justify your answer.




| $R$ | $i$ | $v$ |
|---|---|---|
| open | 0 mA | 12 V |
| 10 kΩ | 0.857 mA | 8.57 V |
| short | 3 mA | 0 V |

Figure P 5.9-2

**P 5.9-3** In preparation for lab, your lab partner determined the Thévenin equivalent of the circuit connected to $R_L$ in Figure P 5.9-3. She says that the Thévenin resistance is $R_t = \frac{6}{11}R$ and the open-circuit voltage is $v_{oc} = \frac{60}{11}$ V. In lab, you built the circuit using $R = 110\ \Omega$ and $R_L = 40\ \Omega$ and measured that $i = 54.5$ mA. Is this measurement consistent with the prelab calculations? Justify your answers.

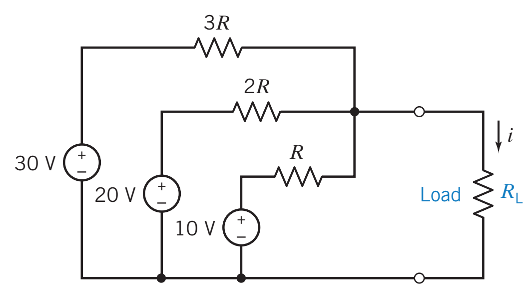



Figure P 5.9-3

**P 5.9-4** Your lab partner claims that the current $i$ in Figure P 5.9-4 will be no greater than 12.0 mA, regardless of the value of the resistance $R$. Do you agree?

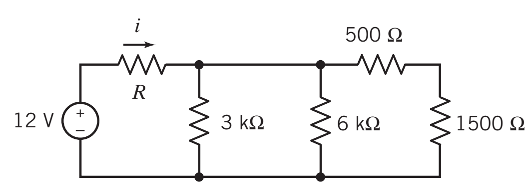



Figure P 5.9-4

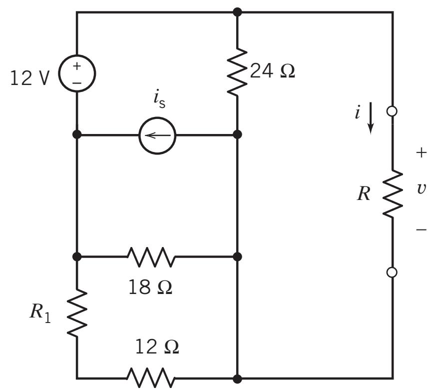
**P 5.9-5** Figure P 5.9-5 shows a circuit and some corresponding data. Two resistances, $R_1$ and $R$, and the current source current are unspecified. The tabulated data provide values of the current $i$ and voltage $v$ corresponding to several values of the resistance $R$.

**(a)** Consider replacing the part of the circuit connected to the resistor $R$ by a Thévenin equivalent circuit. Use the data in rows 2 and 3 of the table to find the values of $R_t$ and $v_{oc}$, the Thévenin resistance, and the open-circuit voltage.

**(b)** Use the results of part (a) to verify that the tabulated data are consistent.

**(c)** Fill in the blanks in the table.

**(d)** Determine the values of $R_1$ and $i_s$.




(a)

| $R, \Omega$ | $i$, A | $v$, V |
|---|---|---|
| 0 | 3 | 0 |
| 10 | 1.333 | 13.33 |
| 20 | 0.857 | 17.14 |
| 40 | 0.5 | ? |
| 80 | ? | 21.82 |

(b)

**Figure P 5.9-5**

## PSpice Problems

**SP 5-1** The circuit in Figure SP 5-1 has three inputs: $v_1$, $v_2$, and $i_3$. The circuit has one output, $v_o$. The equation

$$v_o = a\,v_1 + b\,v_2 + c\,i_3$$

expresses the output as a function of the inputs. The coefficients $a$, $b$, and $c$ are real constants.

**(a)** Use PSpice and the principle of superposition to determine the values of $a$, $b$, and $c$.

**(b)** Suppose $v_1 = 10$ V and $v_2 = 8$ V, and we want the output to be $v_o = 7$ V. What is the required value of $i_3$?

***Hint:*** The output is given by $v_o = a$ when $v_1 = 1$ V, $v_2 = 0$ V, and $i_3 = 0$ A.

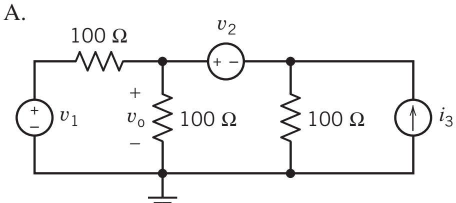



**Figure SP 5-1**

***Answer:*** (a) $v_o = 0.3333v_1 + 0.3333v_2 + 33.33i_3$, (b) $i_3 = 30$ mA

**SP 5-2** The pair of terminals a–b partitions the circuit in Figure SP 5-2 into two parts. Denote the node voltages at nodes 1 and 2 as $v_1$ and $v_2$. Use PSpice to demonstrate that performing a source transformation on the part of the circuit to the left of the terminal does not change anything to the right of the terminals. In particular, show that the current $i_o$ and the node voltages $v_1$ and $v_2$ have the same values after the source transformation as before the source transformation.

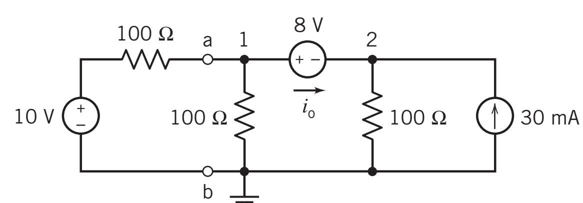



**Figure SP 5-2**

**SP 5-3** Use PSpice to find the Thévenin equivalent circuit for the circuit shown in Figure SP 5-3.

***Answer:*** $v_{oc} = -2$ V and $R_t = -8/3\ \Omega$

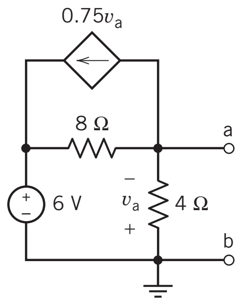
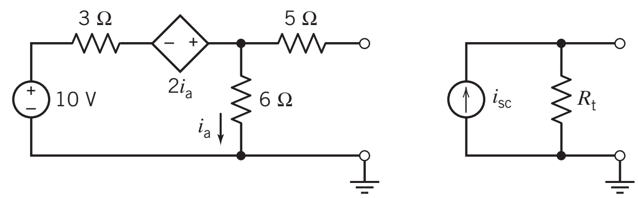



**Figure SP 5-3**

**SP 5-4** The circuit shown in Figure SP 5-4*b* is the Norton equivalent circuit of the circuit shown in Figure SP 5-4*a*. Find the value of the short-circuit current $i_{sc}$ and Thévenin resistance $R_t$.

***Answer:*** $i_{sc} = 1.13$ V and $R_t = 7.57\ \Omega$

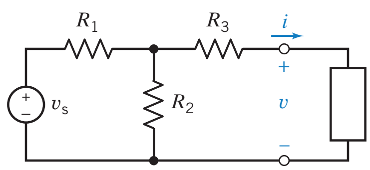



(*a*) (*b*)

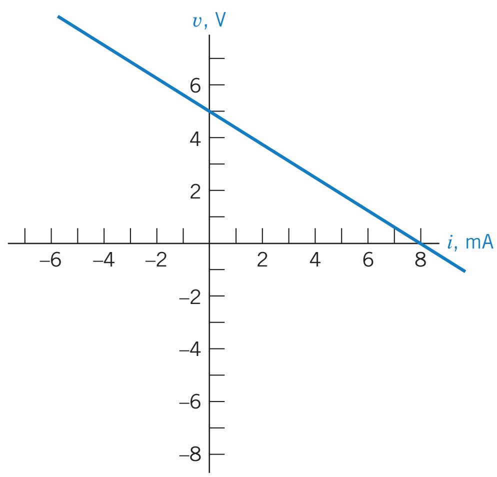
**Figure SP 5-4**

## Design Problems

**DP 5-1** The circuit shown in Figure DP 5-1*a* has four unspecified circuit parameters: $v_s$, $R_1$, $R_2$, and $R_3$. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-1*b* describes a relationship between the current $i$ and the voltage $v$.




(*a*)




(*b*)

**Figure DP 5-1**

Specify values of $v_s$, $R_1$, $R_2$, and $R_3$ that cause the current $i$ and the voltage $v$ in Figure DP 5-1*a* to satisfy the relationship described by the graph in Figure DP 5-1*b*.

***First Hint:*** The equation representing the straight line in Figure DP 5-1*b* is

$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to $-1$ times the Thévenin resistance, and the $v$-intercept is equal to the open-circuit voltage.

***Second Hint:*** There is more than one correct answer to this problem. Try setting $R_1 = R_2$.

**DP 5-2** The circuit shown in Figure DP 5-2*a* has four unspecified circuit parameters: $i_s$, $R_1$, $R_2$, and $R_3$. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-2*b* describes a relationship between the current $i$ and the voltage $v$.

Specify values of $i_s$, $R_1$, $R_2$, and $R_3$ that cause the current $i$ and the voltage $v$ in Figure DP 5-2*a* to satisfy the relationship described by the graph in Figure DP 5-2*b*.

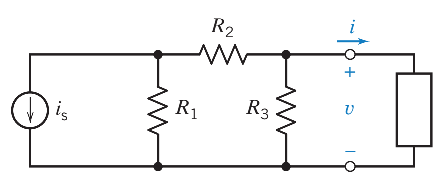
***First Hint:*** Calculate the open-circuit voltage $v_{oc}$ and the Thévenin resistance $R_t$, of the part of the circuit to the left of the terminals in Figure DP 5-2*a*.

***Second Hint:*** The equation representing the straight line in Figure DP 5-2*b* is

$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to $-1$ times the Thévenin resistance, and the $v$-intercept is equal to the open-circuit voltage.

***Third Hint:*** There is more than one correct answer to this problem. Try setting both $R_3$ and $R_1 + R_2$ equal to twice the slope of the graph in Figure DP 5-2*b*.




(*a*)

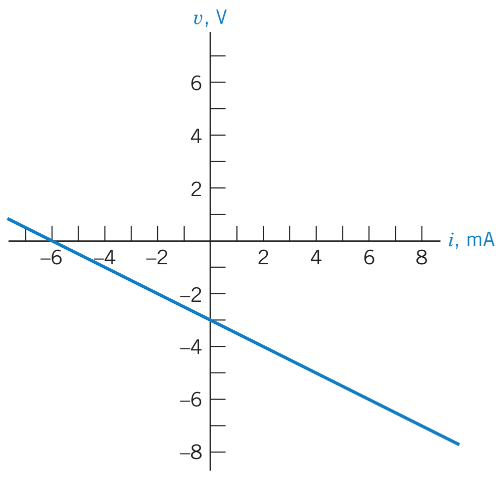



Figure DP 5-2

**DP 5-3** The circuit shown in Figure DP 5-3*a* has four unspecified circuit parameters: $v_s$, $R_1$, $R_2$, and $R_3$. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-3*b* describes a relationship between the current $i$ and the voltage $v$.

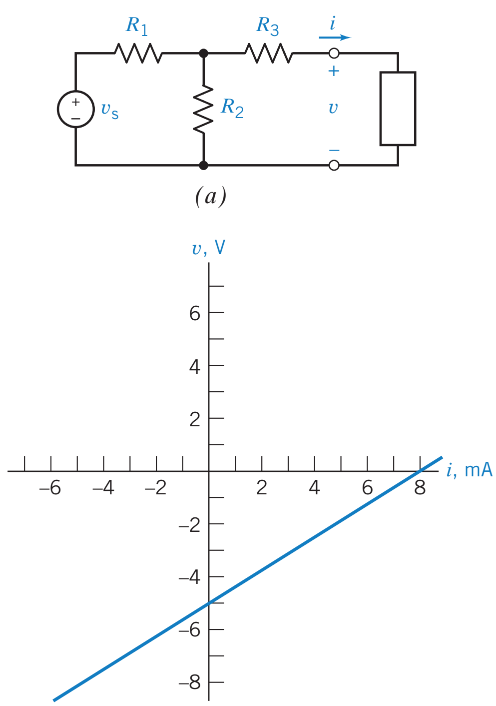



Figure DP 5-3

Is it possible to specify values of $v_s$, $R_1$, $R_2$, and $R_3$ that cause the current $i$ and the voltage $v$ in Figure DP 5-1*a* to satisfy the relationship described by the graph in Figure DP 5-3*b*? Justify your answer.

**DP 5-4** The circuit shown in Figure DP 5-4*a* has four unspecified circuit parameters: $v_s$, $R_1$, $R_2$, and $d$, where $d$ is the gain of the CCCS. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-4*b* describes a relationship between the current $i$ and the voltage $v$.

Specify values of $v_s$, $R_1$, $R_2$, and $d$ that cause the current $i$ and the voltage $v$ in Figure DP 5-4*a* to satisfy the relationship described by the graph in Figure DP 5-4*b*.

***First Hint:*** The equation representing the straight line in Figure DP 5-4*b* is

$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to $-1$ times the Thévenin resistance and the $v$-intercept is equal to the open-circuit voltage.

***Second Hint:*** There is more than one correct answer to this problem. Try setting $R_1 = R_2$.

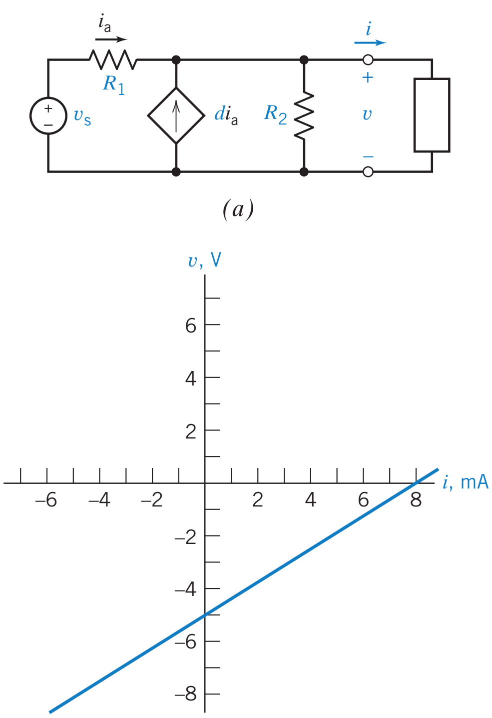



Figure DP 5-4